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When old meets young? Germany's population ageing and the current account

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Non-technical summary

Research Question

Germany has high and persistent current account surpluses since the start of the millennium. To many, (projected) population ageing in Germany is one of the most important drivers for these developments. In an ageing society, people want to prepare for increased longevity by augmenting savings. As these savings cannot all be invested domestically, net foreign assets and the current account surplus rise.

Contribution

We analyze the effects of population ageing in a three-region New Keynesian life-cycle model. Two of the regions form a monetary union. The model is calibrated to Germany, the rest of the Eurozone and the remaining OECD countries. In an additional analysis, we add China to the latter region.

Results

We find that population ageing in Germany is indeed a significant driver of net foreign asset developments. It has the potential of generating current account surpluses of up to 15% of GDP. However, this only happens when the demographic trends of the ageing region (Germany) and its trading partners are unsynchronized. Put differently, this happens when Germany ages while the demographic structure in the rest of the world remains constant. Assuming Germany's most important trading partners to be the other Euro area member states and the remaining OECD countries (and potentially China), we see that these regions face a similar, but delayed ageing process. Feeding these developments into our model reduces the average current account-to-GDP ratio from 2000 to 2018 in Germany to 2.8% (1.2% when taking into account China). It turns negative around 2035.

Nichttechnische Zusammenfassung

Fragestellung

Die deutschen Leistungsbilanzüberschüsse sind seit der Jahrtausendwende stetig gestiegen und verharren auf einem hohen Niveau. Viele machen dafür die prognostizierte Bevölkerungsalterung verantwortlich. In einer alternden Gesellschaft erhöhen Haushalte ihre Sparanstrengungen, um auch im höheren Alter keine größeren Konsumeinbußen hinnehmen zu müssen. Da die vermehrten Ersparnisse nicht alle im Inland angelegt werden können, erhöhen sich die Nettoauslandsvermögen sowie die Leistungsbilanzüberschüsse.

Beitrag

Wir analysieren die Effekte der Bevölkerungsalterung in einem drei Regionen umfassenden, neukeynesianischen Modell mit Bevölkerungsalterung. Zwei der Regionen sind in einer Währungsunion. Das Modell ist auf Deutschland, den Rest der Eurozone und die verbleibenden OECD-Länder kalibriert. In einer zusätzlichen Simulation beziehen wir China mit ein.

Ergebnisse

Wir finden, dass die Bevölkerungsalterung in Deutschland einen signifikanten Beitrag zum Aufbau von Nettoauslandsvermögen leistet. Sie hat das Potenzial, Leistungsbilanzüberschüsse bis zu einer Höhe von 15% des BIP zu generieren. Dies passiert allerdings nur, wenn wir unterstellen, dass der demographische Trend der alternden Region (Deutschland) nicht synchron mit denen der anderen Regionen ist. Anders ausgedrückt, wenn wir unterstellen, dass die deutsche Bevölkerung altert, während die der anderen Regionen konstant bleibt. Wenn wir unterstellen, dass die wichtigsten Handelspartner Deutschland die anderen Euro-Staaten sowie die verbleibenden OECD-Staaten (plus China in einer zusätzlichen Simulation) sind, sehen wir, dass dem nicht so ist. Auch diese Regionen sehen sich ähnlichen demographischen Trends ausgesetzt, allerdings etwas verzögert. Wenn wir das in unseren Modellsimulationen berücksichtigen, können wir für Deutschland in den Jahren 2000 bis 2018 lediglich eine durchschnittliche Leistungsbilanz in Höhe von 2,8% des BIP (1,2% des BIP unter Berücksichtigung Chinas) erklären. Sie dürfte gemäß der Modellsimulationen um das Jahr 2035 negativ werden.

DEUTSCHE BUNDESBANK DISCUSSION PAPER NO 33/2019

When Old Meets Young? Germany's Population Ageing and the Current Account*

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Abstract

In a three-region New Keynesian life-cycle model calibrated to Germany, the Euro area (without Germany) and the rest of the world, we analyze the impact of population ageing on net foreign asset and current account developments. Using unsynchronized demographic trends by taking those of Germany as given and assuming constant population everywhere else, we are able to generate German current account surpluses of up to 15% of GDP during the first half of this century. However, projected demographic trends from 2000 to 2080 in OECD countries (and China in an additional analysis) are much more synchronized. Feeding these into our model suggests that the average annual German current account surplus from 2000 to 2018 that should be attributed to ageing reduces to around 2.83% (1.23%) of GDP, with a maximum at 4.3% (2.7%) in 2006 (when taking into account China), turning negative around 2035.

Keywords: Population Ageing, Net Foreign Assets, Global Imbalances, DSGE Models

JEL classification: E43, E44, E52, E58.

*Contact address: Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt, Germany. E-mail: matthias.schoen1@bundesbank.de, nikolai.staehler@bundesbank.de. We would like to thank Peter Burgold, Jürgen Hamker, Brigitte Hochmuth, Matija Lozej, Karsten Wendorff, the Bundesbank's Fiscal Policy Division, participants of the 2019 CEF conference and two anonymous referees for helpful comments. The views expressed in this paper are those of the authors and do not necessarily coincide with the views of the Deutsche Bundesbank or the Eurosystem.

1 Introduction

The issue of global imbalances has returned to public (policy) debates with momentum. A prominent example often mentioned in these debates is the high and persistent German current account surplus. During the 1990s, it fluctuated around -1% of GDP. At the turn of the millennium, it started improving, reaching a level of 7% of GDP by 2008, further increased to 9% in 2015 and still remains high, currently standing at 8% of GDP (see IMF, 2019). The surplus has long been criticized within the European Commission's Macroeconomic Imbalance Procedure, the IMF's External Imbalance Assessment and by others. Many urge the German government to cut it down (see EC, 2016; IMF, 2018; The Economist, 2017). The US administration has repeatedly threatened to impose tariffs on imports from Germany (and other surplus countries) to deal with the issue.

To date, the German government has remained rather vague in dealing with its current account surplus. It argues that there are no obvious policy failures and that the surplus is an outcome of market-based adjustment to developments outside the control of the government (BMF, 2017). Population ageing is said to be one of the most important drivers of these developments (Busl, Jokisch, and Schleer, 2012; Felbermayr, Fuest, and Wollmershäuser, 2017; Bundesbank, 2018). This statement can qualitatively be supported by findings in the academic literature dealing with global imbalances, not necessarily focusing on Germany, however (see, for example, Blanchard and Milesi-Ferretti, 2010; Ferrero, 2010; Backus, Cooley, and Henriksen, 2014; Kollmann, Ratto, Roeger, in 't Veld, and Vogel, 2015; Dao and Jones, 2018). The impact of population ageing will be one of the key topics in this year's G20 meetings.

In this paper, we analyze how much population ageing in Germany contributes to its net foreign asset and current account positions. We base our analysis on a three-region New Keynesian life-cycle model. Two of the regions (Germany and the rest of the Eurozone) form a monetary union. The third region represents the rest of the world. To be more precise, the model we use for analyzing the question is a three region-version of New Keynesian OLG model in line with Fujiwara and Teranishi (2008) and Kara and von Thadden (2016). They provide a nominal extension of the Gertler (1999)-OLG model. In contrast to the standard open-economy New Keynesian modelling framework with an infinitely-lived representative agent, our OLG setting generates steady-state determinacy and stationarity of net foreign assets as well as an endogenous world interest rate (see Ghironi, 2008; Ferrero, 2010; Di Giorgio and Nistico, 2013; Di Giorgio, Nistico, and Traficante, 2018; Di Giorgio and Traficante, 2018, for an in-depth discussion). This allows for a thorough analysis of the effects of population ageing on current account and net foreign asset positions. International trade and asset flows are introduced in a similar way as in Ferrero (2010). By additionally including regionally differentiated goods and home bias in consumption and investment, we are not only able able to analyze the effects on the trade balance and net foreign assets, but we can also address the impact of ageing on international competitiveness.

We find that population ageing indeed significantly affects net foreign assets and current account positions. Taking into account projected population developments in Germany and, at the same time, neglecting population ageing around the rest of the world, our model simulations have the potential of generating surpluses of up to 15% of GDP. However, this only happens when the demographic trends of the ageing region (Germany) and its trading partners are rather unsynchronized. Put more vividly, "when old meets young".

However, based on OECD data (OECD, 2017), demographic trends in Germany and its main trading partners – assumed to be the other Euro area member states and the remaining OECD countries (and potentially China) – are far from being unsynchronized (which we will show in more detail below). All region face population ageing, and those regions that are young(er) today tend to age more quickly until 2080.[1]

Feeding these demographic trends into our model, simulations suggest that Germany will export capital (i.e. have a positive net foreign asset position) until beyond mid-century and experience a positive current account surplus until about 2035. This change thereafter, which is a result of the fact that, then, the rest of the world also faces population ageing. The average yearly current account-to-GDP ratio from 2000 to 2018 in our model simulations is about 2.83% (or 1.23% when taking into account China). Relative to the observed average value of 5.6% during that period, our model simulations thus suggest that German population ageing explains, at maximum, about one half (fifth) of the actual developments (when taking into account China).

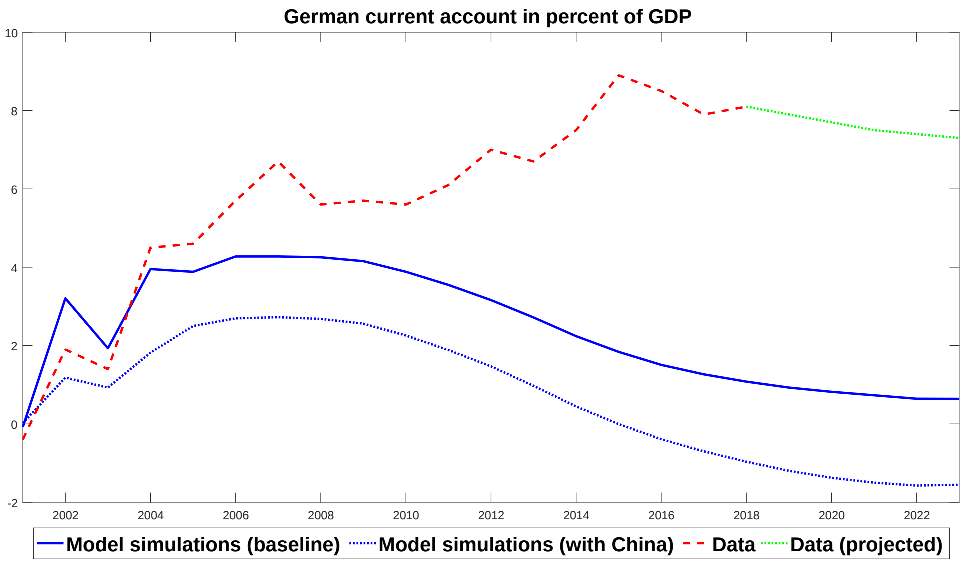

Figure 1: German current account developments

Notes: Figure plots (projected) current account balance in percent of GDP observed in the data (Source: IMF, 2019) from 2000 to 2018 (red dashed lines). Data from 2018 onward are IMF projections (greed dotted lines). The current account developments resulting from our model simulations are depicted by blue solid (dotted) lines (when taking into account China).

In Figure 1, we compare the model-implied current account developments, which are solely driven by demographics as the only exogenous process, with the data (IMF, 2019). We see that, at the beginning of the millennium, data and model-implied current account ratios both keep on rising until about 2005/2006. From 2006 onward, the model-implied

[1]What is different is that, in Germany, the baby boomer generation reaches retirement age earlier than it does in the other regions and the decrease in total population in the other regions is less severe than in Germany.

current account surplus starts falling, while it still keeps on rising in the data.[2] Taking these results seriously, our simulations suggest that population ageing alone cannot be the (main) driver of the currently observed imbalances. Hence, the main takeaway of our paper is that, while certainly important, attributing too much of the latest German current account developments to ageing may be jumping too short. Something else must have happened to boost the Germany current account surplus to its current levels, and further research to fully understand the effects at play is certainly in order.

Our paper is related to the literature dealing with the effects of ageing on households' savings decisions and the resulting consequences for the (German) current account. As shown by Carvalho, Ferrero, and Nechio (2016), the level of aggregate savings within an ageing economy tends to increase. This is because of two reasons. First, given a longer life expectancy, the time span over which households receive the relatively lower pension income increases. For consumption smoothing reasons, individuals prepare for this by increasing individual savings efforts. Second, as the composition between old and young changes, and because elderly people tend to have more assets, savings within an economy increases "by construction". Moreover, less capital is needed in societies with a declining working-age population as fewer people need less (productive) capital. In a closed economy, this leads to a fall in the "natural rate of interest" as the price of capital falls. In our model, too, these mechanisms are at play.

In an open economy, higher supply and lower demand of capital within an economy lead to capital exports. This increases net foreign assets and the current account (Blanchard and Milesi-Ferretti, 2010; Ferrero, 2010; Backus et al., 2014; Börsch-Supan, Ludwig, and Winter, 2006; Börsch-Supan, Härtl, and Ludwig, 2014; Eugeni, 2015). This is confirmed by Poterba (2001), Börsch-Supan, Heiss, Ludwig, and Winter (2003), Krueger and Ludwig (2007) and Börsch-Supan and Ludwig (2009), among others.

In a calibrated multi-region model, Brooks (2003) finds that European countries and Japan become capital exporters with respect to North American and African countries because those regions have a much younger society. Marchiori (2011) shows that this holds for Western economies vis-à-vis the rest of the world in general. Similar to our analysis, he predicts a turning point during the first half of the century, when the other economies start ageing, too. Felbermayr et al. (2017) and Bundesbank (2018) also attribute much of the German account developments to population ageing. As is discussed above, and also mentioned in Dao and Jones (2018), one prerequisite for this to happen, however, is that the demographic trends of the ageing region and its trading partners must be rather unsynchronized. As we show in this paper, the importance of population ageing for German current account developments indeed decreases significantly if we take into account population ageing of Germany's most important trading partners, too.

Therefore, other things must have contributed to Germany's current account surpluses. For many, the German labor market reforms that were initiated in 2003 played a significant role for generating to these developments. And, indeed, 2003/2004 is the year when the surplus observed in the data starts increasing sharply. Hochmuth, Moyen, and Stähler (2019) show in a model with cross-sectional heterogeneity and a precautionary savings motive that these labor market reforms could be responsible for about one fifth of the increase in the current account surplus. Kollmann et al. (2015) estimate a conventional

[2] The fit is weaker if we take into account China. Qualitatively, however, the results still hold. We discuss this in more detail in the main body of the paper.

DSGE model to assess the drivers of the German current account surplus. Using a historical shock decomposition, they find that negative wage markup shocks (which they relate to the labor market reforms) and shocks to the discount rate (which they relate to population ageing) account for about 60% of the current account developments.[3] Empirical evidence for positive current account effects of labor market liberalization is provided by Bertola and Lo Prete (2015). Other reasons could also be, for example, financial integration (Attanasio, Kitao, and Violante, 2006; Mendoza, Quadrini, and Rios-Rull, 2009) and economic growth in emerging markets (Caballero, Farhi, and Gourinchas, 2008). As shown by Ferrero (2010), fiscal consolidation – which happened in Germany at last after the financial crisis – may also contribute to capital exports due to the decrease in domestic investment opportunities (which meet with higher savings, as discussed above).

The rest of the paper is structured as follows. Section 2 describes the model. Its calibration is explained in Section 3. The analysis is undertaken in Section 4, and Section 5 concludes. An appendix with some more details on the model and its computation as well as some supplementary analyses is added.

2 The model

In this section, we build a New Keynesian three-region life-cycle model. Regions are indexed by $i = a, b, c$. Two of the regions, a and b, form a monetary union, while the third region, c, represents the rest of the world. In order to introduce population ageing, we follow Fujiwara and Teranishi (2008) and Kara and von Thadden (2016) to extend the non-monetary overlapping-generations model of Gertler (1999). Each region i produces differentiated goods that are tradeable across countries. They are purchased by households according to their preferences in their consumption and investment baskets along the lines of Di Giorgio and Nistico (2013) and Di Giorgio and Traficante (2018). Regions differ in size, their demographic developments and other structural parameters. Net foreign asset positions and the world interest rate are determined endogenously, also in steady state. The model and simulations will be solved in a fully non-linear fashion under perfect foresight.

2.1 Demographic structure

In the spirit of Gertler (1999), population in each region i consists of two distinct groups: workers, $N_t^{w,i}$, and retirees, $N_t^{r,i}$, where the superscripts w and r denote variables/parameters relevant for the corresponding group. Each individual is born as a new worker. The working-age population grows at rate $n_t^{w,i}$. Conditional on being a worker in the current period, an individual faces a probability ω_t^i of remaining a worker in the next period. With probability $(1-\omega_t^i)$, the worker becomes a retiree. Hence, $(1-\omega_t+n_t^w)$ can be interpreted as the "fertility rate". Retirees face a survival probability γ_t^i and die with probability $(1-\gamma_t^i)$. As these two states are successively reached by individuals, our model gives rise to a life-cycle pattern.

In order to facilitate aggregation within each group, we assume that the probabilities of retirement and death are independent of age (Blanchard, 1985; Weil, 1989). However,

[3] Adding up our findings and those by Hochmuth et al. (2019), these values come pretty close.

the probabilities of retirement and death as well as the working-age population growth rate can be time-varying. Hence, the laws of motion for workers and retirees in region i are

$$
\begin{aligned}
N_{t+1}^{w,i} &= \left(1-\omega_t^i + n_t^{w,i}\right)\, N_t^{w,i} + \omega_t^i\, N_t^{w,i} = \left(1+n_t^{w,i}\right)\, N_t^{w,i}, \\
N_{t+1}^{r,i} &= \left(1-\omega_t^i\right)\, N_t^{w,i} + \gamma_t^i\, N_t^{r,i}.
\end{aligned}
$$

Defining the old-age dependency ratio as $\Psi_t^i = N_t^{r,i}/N_t^{w,i}$, its law of motion can be calculated as

$$
\Psi_{t+1}^i = \frac{1-\omega_t^i}{1+n_t^{w,i}} + \frac{\gamma_t^i}{1+n_t^{w,i}}\, \Psi_t^i. \tag{1}
$$

Because growth of the labor force and ageing across regions may be different over time, the relative size of the labor force between region i and j, defined as $rs_t^{i,j} = N_t^{w,i}/N_t^{w,j}$, evolves according to $rs_{t+1}^{i,j} = (1+n_t^{w,i})/(1+n_t^{w,j})\, rs_t^{i,j}$. Relative total population size is given by $rs_t^{tot,i,j} = (1+\Psi_t^i)/(1+\Psi_t^j)\, rs_t^{i,j}$. In steady state, it must hold that $n^{w,a} = n^{w,b} = n^{w,c}$. Otherwise, one region would eventually disappear. But ω^i and γ^i can be structurally different across regions. The growth rate of the retiree population satisfies $N_{t+1}^{r,i}/N_t^{r,i} = (1+n_t^{r,i}) = (1-\omega_t^i)/\Psi_t^i + \gamma_t^i$ which, along a balanced growth path, implies $n^{w,i} = n^{r,i}$.

2.2 Decision problem of retirees and workers

Let $V_t^{z,i}$ denote the value function associated with the life-cycle states $z = \{w, r\}$ in region i. Households maximize their expected recursive life-time utility function from consumption, $c_t^{z,i}$, and leisure, $(1-l_t^{z,i})$:

$$
\begin{aligned}
V_t^{z,i} &= \left\{ \left[\left(c_t^{z,i}\right)^{\upsilon_c^i} \left(1-l_t^{z,i}\right)^{\upsilon_l^i} \right]^{\rho} + \beta^z\, E_t \left[V_{t+1}^i | z\right]^{\rho} \right\}^{\frac{1}{\rho}}, \\
\beta^w &= \beta,\ \beta^r = \beta\, \gamma_t^i, \\
E_t\left[V_{t+1}^i | w\right] &= \omega_t^i\, V_{t+1}^{w,i} + (1-\omega_t^i) V_{t+1}^{r,i}, \\
E_t\left[V_{t+1}^i | r\right] &= V_{t+1}^{r,i},
\end{aligned}
$$

where ρ determines the intertemporal elasticity of substitution and υ_c^i and υ_l^i define the marginal rate of transformation between consumption and leisure. It holds that $\upsilon_c^i + \upsilon_l^i = 1$. The conditional expectations operator E_t depends on the states $z = \{w, r\}$, and workers and retirees have different discount factors to account for the probability of death.

As discussed by, among others, Gertler (1999), Ferrero (2010) and Carvalho et al. (2016), the model is analytically tractable because the transition probabilities from working age to retirement and, then, to death are independent of age. To avoid a strong precautionary saving motive for young agents, which is at odds with data, this requires assuming a utility function similar to Epstein and Zin (1989). Recursive non-expected utility can be used to separate risk aversion from intertemporal substitution (i.e. risk-neutral preferences with respect to income fluctuations prevent counterfactual excess of young agents' savings; see Farmer, 1990; Heiberger and Ruf, 2019). Separating the coefficient of intertemporal substitution, $\sigma = 1/(1-\rho)$, from risk aversion, as done in the utility function, helps to reproduce reasonable responses of consumption and savings to interest rate variations.

Retirees: In period t, the representative retiree, indexed by j, maximizes

$$V_t^{r,i,j} = \left\{ \left[\left(c_t^{r,i,j}\right)^{\upsilon_c^i} \left(1 - l_t^{r,i,j}\right)^{\upsilon_l^i} \right]^\rho + \beta\, \gamma_t^i \left(V_{t+1}^{r,i,j}\right)^\rho \right\}^{\frac{1}{\rho}},$$

with respect to real consumption, $c_t^{r,i,j}$, labor supply $l_t^{r,i,j}$, and real assets $a_t^{r,i,j}$, subject to the nominal flow budget constraint

$$P_t^i\, c_t^{r,i,j} + P_t^i\, a_t^{r,i,j} = \frac{1 + i_{t-1}}{\gamma_{t-1}^i} \cdot P_{t-1}^i\, a_{t-1}^{r,i,j} + \xi^i \cdot P_t^i\, w_t^i \cdot l_t^{r,i,j} + P_t^i\, e_t^{i,j},$$

where P_t^i is the consumer price index (CPI) of region i. It will be derived in detail below. The retiree receives real old-age benefits $e_t^{i,j}$ and faces an effective real wage rate $\xi^i\, w_t^i$. We assume that a retiree may still be working. The parameter $\xi^i \in (0,1)$ captures the productivity difference between the old and the young. As is standard in the literature, we will choose ξ^i such that $l_t^{r,i,j}$ is close to zero. i_t^i denotes the nominal interest rate in region i.

The real return on asset investments for a retiree who has survived from period $t-1$ to t is $(1 + i_{t-1}^i)\left(P_{t-1}^i / P_t^i\right) / \gamma_{t-1}^i$. For retirees, a perfectly competitive mutual fund industry invests the proceeds and pays back a premium over the market return to compensate for the probability of death (see Yaari, 1965; Blanchard, 1985).[4] Defining r_t as the real world interest rate that clears international capital markets (therefore the lack of the superscript), dividing by P_t^i and using the Fisher relation $(1 + r_t) = (1 + i_t^i)\left(P_t^i / P_{t+1}^i\right)$, which we will verify to hold in section 2.5, yields

$$c_t^{r,i,j} + a_t^{r,i,j} = \frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot a_{t-1}^{r,i,j} + \xi^i \cdot w_t^i \cdot l_t^{r,i,j} + e_t^{i,j}.$$

The first-order condition with respect to labor is given by

$$(1 - l_t^{r,i,j}) = \frac{\upsilon_l^i}{\upsilon_c^i} \cdot \frac{c_t^{r,i,j}}{\xi^i\, w_t^i},$$

while the consumption-Euler equation of the retiree's maximization problem turns out to be

$$c_{t+1}^{r,i,j} = \left[\left(\frac{w_t^i}{w_{t+1}^i} \right)^{\upsilon_l^i \rho} \cdot \beta\, (1 + r_t) \right]^\sigma c_t^{r,i,j},$$

where $\sigma = 1/(1-\rho)$. If we define $\epsilon_t^i \pi_t^i$ as the marginal propensity of retirees to consume

[4] In our model, national funds of region i only operate in their home region. This prevents equalization of returns in the insurance market, which would otherwise dampen the effects of life expectancy differences across regions significantly (see Ferrero, 2010).

out of wealth,[5] we can derive the consumption function and the law of motion of the retiree's marginal propensity to consume:

$$c_t^{r,i,j} = \epsilon_t^i \pi_t^i \cdot \left(\frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot a_{t-1}^{r,i,j} + h_t^{r,i,j} \right),$$

and

$$\epsilon_t^i \pi_t^i = 1 - \left[\beta \cdot \left(\frac{w_t^i}{w_{t+1}^i} \right)^{\upsilon_l^i \rho} \right]^{\sigma} \cdot [(1 + r_t)]^{\sigma - 1} \cdot \gamma_t^i \cdot \frac{\epsilon_t^i \pi_t^i}{\epsilon_{t+1}^i \pi_{t+1}^i}, \tag{2}$$

where

$$h_t^{r,i,j} = \xi^i \cdot w_t^i \cdot l_t^{r,i,j} + e_t^{i,j} + \frac{\gamma_t^i}{1 + r_t} h_{t+1}^{r,i,j}$$

is the recursive law of motion of human capital (i.e. life-time income from wages and pension benefits at time t). These expressions allow us to derive an analytical expression for the value function $V_t^{r,i,j}$, which will be a key input for the decision problem of the representative worker. Getting to these expressions is described in Appendix A.1.

Workers: In period t, the representative worker, again indexed by j, maximizes

$$V_t^{w,i,j} = \left\{ \left[\left(c_t^{w,i,j} \right)^{\upsilon_c^i} \left(1 - l_t^{w,i,j} \right)^{\upsilon_l^i} \right]^{\rho} + \beta \left(\omega_t^i V_{t+1}^{w,i,j} + (1 - \omega_t^i) V_{t+1}^{r,i,j} \right)^{\rho} \right\}^{\frac{1}{\rho}},$$

with respect to real consumption, $c_t^{w,i,j}$, labor supply $l_t^{w,i,j}$, and real assets, $a_t^{w,i,j}$, subject to the flow budget constraint

$$c_t^{w,i,j} + a_t^{w,i,j} = (1 + r_{t-1}) \cdot a_{t-1}^{w,i,j} + w_t^i \cdot l_t^{w,i,j} + f_t^{i,j} - \tau_t^{i,j}.$$

In contrast to the retiree, the worker has a different discount factor (because he cannot die) and takes into account the fact that he may stay a worker or become a retiree next period. Furthermore, the return on assets is different from retirees as there is no mutual fund operated for workers (i.e. gross interest on asset investments are no longer divided by the probability of death), workers do not receive pension benefits but obtain firm profits, $f_t^{i,j}$, and they have to pay lump-sum taxes, $\tau_t^{i,j}$.[6] They also receive the full wage w_t^i. The solution of the worker's decision problem is:

$$\omega_t^i c_{t+1}^{w,i,j} + (1 - \omega_t^i) \left(\epsilon_{t+1}^i \right)^{\frac{\sigma}{1-\sigma}} \left(\frac{1}{\xi^i} \right)^{\upsilon_l^i} c_{t+1}^{r,i,j} = \left[\beta (1 + r_t) \Omega_{t+1}^i \cdot \left(\frac{w_t^i}{w_{t+1}^i} \right)^{\upsilon_l^i \rho} \right]^{\sigma} c_t^{w,i,j},$$

[5]Here, π_t^i represents the marginal propensity to consume out of wealth for workers, as we will see below. ϵ_t^i is a markup on that value for retirees. This is a standard way of presenting the problem in the literature.

[6]If we allowed retirees to also own firms, this would change the firm discount factor (derived below). However, as discussed by Fujiwara and Teranishi (2008), this assumption does not affect the results much.

as the consumption-Euler equation, with

$$\Omega^i_{t+1} = \omega^i_t + (1-\omega^i_t)\ \left(\epsilon^i_{t+1}\right)^{1/(1-\sigma)}\ \left(1/\xi^i\right)^{\upsilon^i_l}, \tag{3}$$

and the first-order condition with respect to leisure:

$$(1 - l^{w,i,j}_t) = \frac{\upsilon^i_l}{\upsilon^i_c} \cdot \frac{c^{w,i,j}_t}{w^i_t}.$$

Again, the the reader is referred to Appendix A.1 for the formal derivations.

Defining π^i_t as the marginal propensity of workers to consume out of wealth, the worker's consumption function and the law of motion of the worker's marginal propensity to consume are

$$c^{w,i,j}_t = \pi^i_t \cdot \left((1+r_{t-1}) \cdot a^{w,i,j}_t + h^{w,i,j}_t\right),$$

and

$$\pi^i_t = 1 - \left[\beta \cdot \left(\frac{w^i_t}{w^i_{t+1}}\right)^{\upsilon^i_l \rho}\right]^{\sigma} \cdot \left[(1+r_t) \cdot \Omega^i_{t+1}\right]^{\sigma-1} \frac{\pi^i_t}{\pi^i_{t+1}}, \tag{4}$$

where

$$h^{w,i,j}_t = w^i_t \cdot l^{w,i,j}_t + f^{i,j}_t - \tau^{i,j}_t + \frac{\omega^i_t}{[1+r_t]\,\Omega^i_{t+1}} h^{w,i,j}_{t+1} + \left(1 - \frac{\omega^i_t}{\Omega^i_{t+1}}\right) \frac{h^{r,i,j}_{t+1}}{1+r_t}.$$

One can show that retirees have a higher marginal propensity to consume than workers, $\epsilon^i_t > 1 \forall t$. This implies $\Omega^i_t > 1 \forall t$ which, in turn, indicates that workers discount future income streams at an effective rate $(1+r_t)\Omega^i_{t+1} > (1+r_t)$. It reflects the expected finiteness of their life and makes the future less valuable relative to a conventional New Keynesian setting with infinite lives (see, among others, Gertler, 1999, and Kara and von Thadden, 2016, for a discussion).

2.3 Aggregation of households' decisions

To characterize aggregate variables, we drop the index j and carry on using the previous notation. Given the numbers of retirees and workers in each period t, $N^{r,i}_t$ and $N^{w,i}_t$, the aggregate labor supply schedule can be derived from the individual ones as

$$l^{w,i}_t = N^{w,i}_t\, l^{w,i,j}_t = N^{w,i}_t - \frac{\upsilon^i_l}{\upsilon^i_c} \cdot \frac{c^{w,i}_t}{w^i_t}, \tag{5}$$

$$l^{r,i}_t = N^{r,i}_t\, l^{r,i,j}_t = N^{r,i}_t - \frac{\upsilon^i_l}{\upsilon^i_c} \cdot \frac{c^{r,i}_t}{\xi^i\, w^i_t}, \tag{6}$$

$$l^i_t = l^{w,i}_t + l^{r,i}_t, \tag{7}$$

where $c_t^{z,i} = N_t^{z,i}\, c_t^{z,i,j}$ with $z = \{w, r\}$ denotes aggregate consumption of workers and retirees, respectively. Using the respective equations for retirees and workers, these are given by

$$c_t^{w,i} = \pi_t^i \left[(1 + r_{t-1})(1 - \lambda_{t-1}^i)\, a_{t-1}^i + h_t^{w,i} \right], \tag{8}$$

$$c_t^{r,i} = \epsilon_t^i \pi_t^i \left[(1 + r_{t-1})\, \lambda_{t-1}^i\, a_{t-1}^i + h_t^{r,i} \right]. \tag{9}$$

We define aggregate consumption as $c_t^i = c_t^{w,i} + c_t^{r,i}$. In equations (8) and (9), we have used $a_t^i = a_t^{w,i} + a_t^{r,i}$ and the definition $\lambda_t^i = a_t^{r,i}/a_t^i$, which is the share of (financial) wealth held by retirees over total wealth.

To determine the aggregate stocks of human capital, $h_t^{r,i} = N_t^{r,i}\, h_t^{r,i,j}$ and $h_t^{w,i} = N_t^{w,i}\, h_t^{w,i,j}$, we have to take into account population dynamics described in section 2.1. This yields

$$h_t^{r,i} = \xi^i \cdot w_t^i \cdot l_t^{r,i} + e_t^i + \frac{\gamma_t^i}{(1 + n_t^{r,i})(1 + r_t)}\, h_{t+1}^{r,i}, \tag{10}$$

$$h_t^{w,i} = w_t^i \cdot l_t^{w,i} + f_t^i - \tau_t^i + \frac{\omega_t^i \cdot h_{t+1}^{w,i,j}}{(1 + n_t^{w,i})(1 + r_t)\, \Omega_{t+1}^i} - \left(1 - \frac{\omega_t^i}{\Omega_{t+1}^i}\right) \frac{h_{t+1}^{r,i}}{(1 + n_t^{r,i})(1 + r_t)\Psi_t^i}, \tag{11}$$

where $e_t^i = N_t^{r,i}\, e_t^{i,j}$, $f_t^i = N_t^{w,i}\, f_t^{i,j}$ and $\tau_t^i = N_t^{w,i}\, \tau_t^{i,j}$. These equations take into account the respective population growth rates $n_t^{r,i}$ and $n_t^{w_i}$. The absence of γ_{t-1}^i in equation (9) relative to individual human wealth for retirees reflects the competitive insurance/annuity market. As discussed in Blanchard (1985), the probability of death is relevant for the individual household j, but it does not affect the aggregate consumption of retirees.

It remains to characterize the law of motion for λ_t^i, i.e. the fraction of wealth over total wealth held by retirees. In doing so, we realize that the fraction of total wealth held by the working-age population evolves according to $(1 - \lambda_t^i)\, a_t^i = \omega_t^i \left[(1 - \lambda_{t-1}^i)(1 + r_{t-1})\, a_{t-1}^i + w_t^i \cdot l_t^{w,i} + f_t^i - \tau_t^i - c_t^{w,i} \right]$. It increases by the savings of those workers who remain workers in the next period. Analogously, the fraction of total wealth held by retirees increases by the savings of those retirees who do not die (bearing in mind that savings of those who die are redistributed through the competitive annuity market) plus the savings of those workers who become retirees: $\lambda_t^i\, a_t^i = \lambda_{t-1}^i (1 + r_{t-1})\, a_{t-1}^i + \xi^i \cdot w_t^i \cdot l_t^{r,i} + e_t^i - c_t^{r,i} + (1 - \omega_t^i) \left[(1 - \lambda_{t-1}^i)(1 + r_{t-1})\, a_{t-1}^i + w_t^i \cdot l_t^{w,i} + f_t^i - \tau_t^i - c_t^{w,i} \right]$. Combining these expressions and using equations (8) and (9), we get

$$\begin{aligned} \lambda_t^i\, a_t^i = & \omega_t^i \left\{ \left(1 - \epsilon_t^i \pi_t^i\right) \left[(1 + r_{t-1})\, \lambda_{t-1}^i\, a_{t-1}^i + h_t^{r,i} \right] - \left(h_t^{r,i} - \xi^i\, w_t^i\, l_t^{r,i} - e_t^i \right) \right\} \\ & + (1 - \omega_t^i)\, a_t^i. \end{aligned} \tag{12}$$

2.4 Production

The production side is modelled along the lines of Gertler, Gali, and Clarida (1999), Christiano, Eichenbaum, and Evans (2005), or Smets and Wouters (2003, 2007). This implies that the production sector is partitioned in a final and an intermediate goods sector. As it is standard, we will keep its description brief.

Final goods: We assume that, in each country i, there is a measure-one continuum of firms in the final goods sector. Firms are owned by the working-age population as in Fujiwara and Teranishi (2008) and Kara and von Thadden (2016). Each final goods producer purchases a variety of differentiated intermediate goods, bundles these and sells them to the final consumer under perfect competition. The producer price index (henceforth, PPI) of goods produced in country i and sold in j is defined as $P_t^{i,j}$. We assume that the law of one price holds across regions, so firms in country i set their price $P_t^{i,i}$ for all markets (Di Giorgio and Nistico, 2013; Di Giorgio et al., 2018; Di Giorgio and Traficante, 2018). Multiplying with the nominal exchange rate $S_t^{i,j}$ then yields the price of country-i goods charged in j: $P_t^{j,i} = S_t^{j,i} P_t^{i,i}$, where $S_t^{j,i}$ is defined as country-j currency per unit of country-i currency.

Within the monetary union, it holds by definition that $S_t^{b,a} = S_t^{a,b} = 1 \forall t$. It must then hold that $S_t^{c,a} = S_t^{c,b} \equiv S_t$, where S_t is the nominal exchange rate between the monetary union and the rest of the world (expressed in country-c currency per unit of the monetary union currency); see, for example, Gadatsch, Hauzenberger, and Stähler (2016) for a discussion.

Assuming a Dixit and Stiglitz (1977)-aggregator on the interval $\tilde{j} \in [0,1]$, the final good in region i is, as usual, given by $y_t^i = \left[\int_0^1 y_t^i(\tilde{j})^{(\theta_p^i-1)/\theta_p^i} d\tilde{j}\right]^{\theta_p/(\theta_p-1)}$. $\theta_p^i > 1$ is the elasticity of substitution between differentiated intermediate goods. Demand for a an intermediate good $\tilde{j}$ is given by $y_t^i(\tilde{j}) = \left[P_t^{i,i}(\tilde{j})/P_t^{i,i}\right]^{-\theta_p^i} y_t^i$. The PPI of region i is then given by $P_t^{i,i} = \left[\int_0^1 P_t^{i,i}(\tilde{j})^{1-\theta_p^i} d\tilde{j}\right]^{1/(1-\theta_p^i)}$.

Intermediate goods: The representative intermediate good producer $\tilde{j}$ operates with production technology $y_t^i(\tilde{j}) = \left[l_t^i(\tilde{j})\right]^{\alpha^i} \cdot \left[k_{t-1}^i(\tilde{j})\right]^{1-\alpha^i}$. Here, α^i is the Cobb-Douglas share of labor in production and $l_t^i(\tilde{j})$ and $k_{t-1}^i(\tilde{j})$ are the inputs of labor and capital in production by producer $\tilde{j}$. Taking prices for labor (CPI-deflated real wages w_t^i) and capital (CPI-deflated real capital interest $r_t^{k,i}$) as given, firm $\tilde{j}$'s cost minimization problem yields the following capital-to-labor ratio

$$\frac{l_t^i}{k_{t-1}^i} = \frac{\alpha^i}{1-\alpha^i} \cdot \frac{r_t^{k,i}}{w_t^i} \tag{13}$$

which, as can easily be seen, must be equal across all intermediate goods producing firms for given wages and capital interest rates (as symmetry applies, we dropped the index $\tilde{j}$ for convenience). Hence, CPI-deflated real marginal costs are given by

$$mc_t^i = \left(\frac{w_t^i}{\alpha^i}\right)^{\alpha^i} \cdot \left(\frac{r_t^{k,i}}{1-\alpha^i}\right)^{1-\alpha^i}. \tag{14}$$

Following the convention in the New Keynesian literature, we assume that, each period, a randomly chosen fraction of firms $\kappa_p^i \in [0,1)$ cannot re-optimize their price (Calvo, 1983). In a symmetric equilibrium, the price of those firms $\tilde{j}$ who can set their price in period t is equal across firms, i.e. $P_t^{i,i,*}(\tilde{j}) = P_t^{i,i,*}$. The resulting profit maximizing price is given by

$$\frac{P_t^{i,i,*}}{P_t^i} = \frac{\kappa_p^i}{\kappa_p^i - 1} \cdot \frac{\sum_{z=0}^{\infty} \left(\kappa_p^i \beta\right)^z \cdot DF_{t,t+z}^i \cdot y_{t+z}^i \cdot mc_{t+z}^i \cdot \left(\frac{P_{t+z}^{i,i}}{P_t^{i,i}}\right)^{\kappa_p^i}}{\sum_{z=0}^{\infty} \left(\kappa_p^i \beta\right)^z \cdot DF_{t,t+z}^i \cdot y_{t+z}^i \cdot \frac{P_{t+z}^{i,i}}{P_{t+z}^i} \cdot \left(\frac{P_{t+z}^{i,i}}{P_t^{i,i}}\right)^{\kappa_p^i - 1}}. \tag{15}$$

As shown by Fujiwara and Teranishi (2008), the discount factor of firms is given by $DF_{t,t+1}^i = \partial V_{t+1}^{w,i} / \partial c_{t+1}^{w,i} = \left(\pi_{t+1}^i\right)^{-1/\rho} \left(\upsilon_l^i / \upsilon_c^i / w_{t+1}^i\right)^{\upsilon_l^i}$. This is a result of the fact that we assume workers to be the owners of firms. Producer prices in region i hence evolve according to

$$P_t^{i,i} = \left[\kappa_p^i \cdot \left(P_{t-1}^{i,i}\right)^{1-\theta_p^i} + (1-\kappa_p^i) \cdot \left(P_t^{i,i,*}\right)^{1-\theta_p^i}\right]^{1/(1-\theta_p^i)}, \tag{16}$$

while $D_t^i = \kappa_p^i \cdot \left(P_t^{i,i} / P_{t-1}^{i,i}\right)^{\theta_p^i} \cdot D_{t-1}^i + (1-\kappa_p^i) \cdot \left(P_t^i / P_t^{i,i,*}\right)^{\theta_p^i}$ is the resulting measure of price dispersion, expressed recursively. Aggregate CPI-deflated firm profit are $f_t^i = \left(P_t^{i,i}/P_t^i - mc_t^i\right) y_t^i$.

2.5 Investment funds and financial market clearing

Following Fujiwara and Teranishi (2008), an investment fund in each region i collects deposits from households, a_t^i, and invests these into physical capital, domestic government bonds and international assets. Government bonds and international assets are assumed to pay a nominal interest $i_t^{G,i}$ and $i_t^{d,i}$ next period, respectively. The financial investor pays the households a real interest r_t on the deposited assets. The investment fund hence aims to maximize

$$\begin{aligned} f_t^{fund,i} =& r_{t+1}^{k,i} \cdot k_t^i + (1 - i_t^{G,i}) \frac{P_t^i}{P_{t+1}^i} \cdot b_t^i + (1 - i_t^{d,i}) \frac{P_t^i}{P_{t+1}^i} \cdot d_t^i - inv_t^i \\ &+ a_{t+1}^i - (1+r_t) a_t^i, \end{aligned}$$

where b_t^i and d_t^i are CPI-deflated real government bonds and net foreign assets, respectively, and $r_{t+1}^{k,i}$ is the ex-ante uncertain rate of return on capital. Capital follows the conventional law of motion:

$$k_{t+1}^i = (1-\delta^i) k_t^i + \left[1 - \underbrace{\frac{\kappa_{inv}^i}{2} \left(\frac{inv_t^i}{inv_{t-1}^i} - 1\right)^2}_{\equiv S_t^i(\cdot)}\right] \cdot inv_t^i, \tag{17}$$

where δ^i denotes capital depreciation and $S_t^i(\cdot)$ denote capital adjustment costs in line with Christiano et al. (2005). This implies that the conventional no-arbitrage condition must hold:

$$(1+r_t) = (1+i_t^{G,i}) \frac{P_t^i}{P_{t+1}^i} = (1+i_t^{d,i}) \frac{P_t^i}{P_{t+1}^i} = \frac{r_{t+1}^{k,i} + Q_{t+1}^i (1-\delta^i)}{Q_t^i} = (1+i_t^i) \frac{P_t^i}{P_{t+1}^i}. \tag{18}$$

Q_t^i is the shadow price of capital, also known as Tobin's Q. It evolves according to

$$1 = Q_t^i \cdot \left[1 - S_t^i(\cdot) - S_t^{i,'}(\cdot) \cdot \frac{inv_t^i}{inv_{t-1}^i}\right] + \frac{Q_{t+1}^i}{1 + r_{t+1}} \cdot S_{t+1}^{i,'}(\cdot) \left(\frac{inv_{t+1}^i}{inv_t^i}\right)^2. \tag{19}$$

Financial markets must clear, which implies that

$$a_t^i = Q_t^i k_t^i + b_t^i + d_t^i. \tag{20}$$

2.6 Fiscal policy

The government's budget constraint in region i in CPI-deflated real terms is given by

$$b_t^i + \tau_t^i = (1 + r_{t-1})\, b_{t-1}^i + \frac{P_t^{i,i}}{P_t^i} \cdot g_t^i + e_t^i, \tag{21}$$

where use has been made of equation (18). Hence, the government must finance real government expenditures, g_t^i, aggregate real pension benefits, e_t^i, and interest payments on outstanding debt, $(1 + r_{t-1})b_{t-1}^i$, by lump-sum taxes, τ_t^i, and issuance of new debt, b_t^i. Following Stähler and Thomas (2012) and Gadatsch et al. (2016), we assume full home bias in government consumption, which requires the PPI/CPI correction. The assumption is based on the observation that the import share in government consumption is, in general, significantly lower than in private consumption or investment (Brülhart and Trionfetti, 2001, 2004, and Trionfetti, 2000).[7]

The path of aggregate real pension benefits is determined by the replacement rate μ_t^i between individual benefits and real wages, that is

$$\mu_t^i = \frac{e_t^{i,j}}{w_t^i} \quad \Rightarrow \quad e_t^i = e_t^{i,j} \cdot N_t^{r,i} = \mu_t^i \cdot w_t^i \cdot N_t^{r,i}. \tag{22}$$

To close the system, we assume that fiscal policy follows a fiscal reaction function that stabilizes a certain target level of debt, say, a certain fraction $\varpi^{b,i}$ of GDP, y_t^i, by variations in the remaining free fiscal instrument τ_t^i such that

$$\log\left(\frac{\tau_t^i}{\tau^i}\right) = \rho^{\tau,i} \log\left(\frac{\tau_{t-1}^i}{\tau^i}\right) + \zeta^{b,i} \log\left(\frac{P_{t-1}^i \cdot b_{t-1}^i}{P_{t-1}^{i,i} \cdot \varpi^{b,i} \cdot y_{t-1}^i}\right), \tag{23}$$

where the omission of the time subscript t indicates steady-state values, $\rho^{\tau,i}$ is an autocorrelation parameter and $\zeta^{b,i} > 0$ is a direct feedback parameter to counteract deviations of debt from its target. Similar fiscal rules have been discussed by, among others, Schmitt-Grohe and Uribe (2007) and Kirsanova and Wren-Lewis (2012). Because real government debt is deflated by CPI, while GDP by PPI, as outlined in section 2.4, we need to correct for $P_t^i/P_t^{i,i}$. Closing the model in terms of alternative fiscal instruments (such as g_t^i or μ_t^i, for example) and target variables (for example, including reactions to output deviations)

[7] In principle, it is straightforward to explicitly model a different home bias in public consumption, too. Nevertheless, this would come at the cost of additional relative prices. Because this assumption is irrelevant for the analysis at hand, we simplify the model here.

can easily be done by replacing equation (23) accordingly (see, for example, Mitchell, Sault, and Wallis, 2000, for a discussion).

2.7 International linkages, monetary policy and market clearing

International trade in goods and assets implies that the three regions $i = a, b, c$ are linked together, which not only affects the net foreign asset position but also the market clearing conditions. Furthermore, we have to bear in mind that there is a common monetary policy for regions a and b, while the one for region c is solely undertaken for that region. We will describe these linkages in more detail in this subsection.

International trade, prices and net foreign assets: We assume that households in region i consume goods produced in any of the three regions. The corresponding consumption bundle is given by

$$c_t^i = \left[\left(\vartheta_a^i\right)^{1-\eta^i} \left(c_{a,t}^i\right)^{\eta^i} + \left(\vartheta_b^i\right)^{1-\eta^i} \left(c_{b,t}^i\right)^{\eta^i} + \left(\vartheta_c^i\right)^{1-\eta^i} \left(c_{c,t}^i\right)^{\eta^i} \right]^{\frac{1}{\eta^i}} .$$

Here, $c_{j,t}^i$ denotes goods produced in j and consumed in i and $\eta^i \in (-\infty, 1)$ governs the elasticity of substitution between these goods, which equals $1/(1-\eta^i)$. As $\eta^i \to 0$, the function boils down to a Cobb Douglas aggregator. ϑ_j^i denotes the consumption bias of region i-households towards goods produced in j. Hence, ϑ_i^i can be interpreted as the home bias of region i. We assume that $\vartheta_a^i + \vartheta_b^i + \vartheta_c^i = 1$. Cost minimization of nominal consumption expenditures, $P_t^i c_t^i = P_t^{i,a} c_{a,t}^i + P_t^{i,b} c_{b,t}^i + P_t^{i,c} c_{c,t}^i$, implies

$$c_{j,t}^i = \vartheta_j^i \left(\frac{P_t^{i,j}}{P_t^i} \right)^{-\frac{1}{1-\eta^i}} \cdot c_t^i. \tag{24}$$

The consumer price index (CPI) results to be

$$P_t^i = \left[\vartheta_a^i \cdot \left(P_t^{a,i}\right)^{-\eta^i/(1-\eta^i)} + \vartheta_b^i \cdot \left(P_t^{b,i}\right)^{-\eta^i/(1-\eta^i)} + \vartheta_c^i \cdot \left(P_t^{c,i}\right)^{-\eta^i/(1-\eta^i)} \right]^{-\frac{1-\eta^i}{\eta^i}} . \tag{25}$$

We assume that an analogous aggregator holds for investment goods such that we can derive analogous equations for inv_t^i and $inv_{j,t}^i$. CPI-deflated net exports in region i, nx_t^i, are hence given by

$$\begin{aligned} nx_t^i = & \frac{P_t^{j,i}}{P_t^i} \cdot \left(c_{i,t}^j + inv_{i,t}^j\right) + \frac{P_t^{\tilde{j},i}}{P_t^i} \cdot \left(c_{i,t}^{\tilde{j}} + inv_{i,t}^{\tilde{j}}\right) - \frac{P_t^{i,j}}{P_t^i} \cdot \left(c_{j,t}^i + inv_{j,t}^i\right) \\ & - \frac{P_t^{i,\tilde{j}}}{P_t^i} \cdot \left(c_{\tilde{j},t}^i + inv_{\tilde{j},t}^i\right), \end{aligned} \tag{26}$$

where $i, j, \tilde{j} = a, b, c$, and $i \neq j \neq \tilde{j}$. Alternatively, net exports can also be written as domestic production minus domestic demand: $nx_t^i = P_t^{i,i}/P_t^i \left(y_t^i - g_t^i\right) - c_{i,t}^i - inv_{i,t}^i$. We note that $P_t^{j,i} = P_t^{j,j}$ whenever the regions belong to the monetary union (ie $i, j = a, b$); see Section 2.4. Whenever a monetary union-country imports from the rest of the world,

$P^{i,c} = P^{c,c}/S_t$, and when the rest of the world imports from the monetary union, $P_t^{c,i} = S_t \cdot P_t^{i,i}$, with $i = a, b$.

Given net exports and using the no-arbitrage conditions (18), we get that net foreign assets in region i evolve according to

$$d_t^i = (1 + r_{t-1})\, d_{t-1}^i + nx_t^i. \tag{27}$$

Because international assets traded between regions are in zero net supply, it must hold that $P_t^a\, d_t^a + P_t^b\, d_t^b + P_t^c\, d_t^c = 0$. The current account-to-GDP ratio is thus given by $ca_t^{rat,i} = P_t^i \left(d_t^i - P_{t-1}^i / P_t^i\, d_{t-1}^i\right) / \left(P_t^{i,i}\, y_t^i\right)$.[8]

Monetary policy: Following Ghironi (2008), Di Giorgio and Nistico (2013) and Kara and von Thadden (2016), monetary policy is modelled through a Taylor-type feedback rule (Taylor, 1993). We assume monetary policy targets a gross output price inflation of one. According to the Taylor rule, the nominal interest rate set by the central bank, i_t^i, is a function of output price inflation deviations from target, $\log\left(inf_t^i\right)$, where $inf_t^i = P_t^{i,i}/P_{t-1}^{i,i}$ (the omission of the time-subscript again denotes the steady-state value), and the previous value of the nominal interest rate. Given that regions a and b form a monetary union with a common monetary policy, we assume that the monetary policy rate in the union, denoted by $i_t^u = i_t^a = i_t^b$, reacts to a population-weighted average of inflation deviations (following, among others, Stähler and Thomas, 2012, and Gadatsch et al., 2016). Denoting union-wide aggregates by the superscript u, these are given by $inf_t^u = rs_t^{a,b}\left(P_t^{a,a}/P_{t-1}^{a,a}\right) + (1 - rs_t^{a,b})\left(P_t^{b,b}/P_{t-1}^{b,b}\right)$. Hence, monetary policy in $i = u, c$ is described by

$$\log\left(\frac{i_t^i}{i^i}\right) = \rho^{mp,i} \log\left(\frac{i_{t-1}^i}{i^i}\right) + \zeta^{\pi,i} \log\left(inf_t^i\right), \tag{28}$$

where $\rho^{mp,i}$ is an autocorrelation parameter and $\zeta^{\pi,i} > 0$ is a direct feedback parameter to counteract deviations of inflation from target.

Product market clearing: Product market clearing implies that whatever is produced in region i must be consumed/used somewhere around the world. Formally, we get

$$D_t^i \cdot y_t^i = \left(c_{i,t}^i + inv_{i,t}^i\right) + \left(c_{i,t}^j + inv_{i,t}^j\right) + \left(c_{i,t}^{\tilde{j}} + inv_{i,t}^{\tilde{j}}\right) + g_t^i. \tag{29}$$

[8] At this juncture, it may be noteworthy that the standard (multi-country) representative agent model, in general, entails steady-state indeterminacy and non-stationary dynamics of net foreign assets. To overcome this problem, modelers assume additional frictions in the international financial markets (for example, a risk premium on international asset holdings or some asset adjustment costs) whenever holdings of net foreign assets exceed some exogenously fixed reference level. That introduces a link between consumption and the net foreign asset position and pins down the steady-state level of international financial assets uniquely. However, it does so independent of policy or structural economic changes. An in-depth discussion of this issue can be found in Schmitt-Grohe and Uribe (2003), Hunt and Rebucci (2005), Lubik (2007) and Benigno (2009). As discussed by, for example, Ghironi (2008), Ghironi, Iscan, and Rebucci (2008) and Di Giorgio and Nistico (2013), such an "extra" assumption is not needed in our framework. OLG models entail an elastic asset demand curve resulting from the old-age savings motive discussed in section 2.2.

2.8 General equilibrium and detrending

The previous sections complete the model description. At equilibrium, government actions and optimizing decisions of workers, retirees, investment funds and firms must be mutually consistent at the aggregate level. As we allow for exogenously given, time-varying population dynamics, the economy may be subject to ongoing exogenous growth. Following Kara and von Thadden (2016), we therefore consider a detrended version the model. Any unbounded model variable υ_t^i can be detrended through

$$\bar{\upsilon}_t^i = \frac{\upsilon_t^i}{N_t^{w,i}}, \quad \bar{\upsilon}_t^{w,i} = \frac{\upsilon_t^{w,i}}{N_t^{w,i}}, \quad \bar{\upsilon}_t^{r,i} \cdot \Psi_t^i = \frac{\upsilon_t^{r,i}}{N_t^{r,i}} \cdot \frac{N_t^{r,i}}{N_t^{w,i}},$$

where the bar denotes the detrended variable. For the labor market-related variables, we have

$$\bar{w}_t^i = w_t^i, \quad \bar{l}_t^{w,i} = \frac{l_t^{w,i}}{N_t^{w,i}}, \quad \bar{l}_t^{r,i} \cdot \Psi_t^i = \frac{l_t^{r,i}}{N_t^{r,i}} \cdot \frac{N_t^{r,i}}{N_t^{w,i}}.$$

Using these definitions allows us to express our model, described through equations (2) to (29), in terms of efficiency units per worker, where the old age dependency ratio is given by equation (1). A summary of the fully detrended system can be found in Appendix B.1.

3 Calibration

We calibrate our model to annual frequency. Individuals are born at the age of 20, stay on average $1/(1-\omega^i)$ years in the labor force and live on average $1/(1-\gamma^i)$ years after retirement. We choose ω^i such that, in steady state, individuals retire at the age of 65. We calibrate our model to Germany (region a), the rest of the Euro area (region b) and the rest of the world (region c). The latter is given by the remaining OECD countries (excluding Germany and Euro area member states).[9]

Relative working-age population size in the initial steady state is thus given by $rs^{a,b} = 0.343$, $rs^{a,c} = 0.092$ and $rs^{b,c} = 0.268$. As population growth rates will vary differently across regions in our simulations, these will change along the transition to the new steady state. We follow Kara and von Thadden (2016) and assume $n^{w,i} = 0.004$ to be the steady-state growth rate of the working-age population. As discussed above, it has to be equal across regions in steady state. Given ω^i and population growth, the survival probabilities γ^i are used to match all region-i old-age dependency ratios of the year 2000, which we take as the base year for our steady-state derivation. They are $\Psi^a = 0.2647$, $\Psi^b = 0.2670$ and $\Psi^c = 0.2001$. Population data is from OECD (2017) and the related data appendices. Table 1 summarizes our assumptions determining the demographic situation in the initial steady state.

[9] We also simulate the model when considering the rest of the world to be the remaining OECD countries plus China. As described below, population dynamics are then different (see also Figure 2). The steady-state targets when taking into account Chinese population are put in brackets in Table 1. In addition, we simulate the model assuming a world with constant population dynamics where only Germany ages. Results will be discussed below.

For the general model calibration, we follow Ferrero (2010) and set standard values from the business cycle literature (see also Cooley and Prescott, 1995). We target a world asset market-clearing real interest rate of 4%. Together with the demographic structure described above, this implies $\beta = 0.99$. We choose a labor share in production of 2/3, assume that capital depreciates at an annual rate of 10% and set the elasticity of intertemporal substitution to $\sigma = 0.5$. As discussed in Ferrero (2010), the latter somewhat low value has become standard in this class of models since Auerbach and Kotlikoff (1987). The investment adjustment cost parameter is set to 4.5, which is a standard value in the DSGE literature.[10] Following Kara and von Thadden (2016), the choice of the relative productivity parameter ξ^i as well as υ_c^i ensures that the participation rate of workers is $\bar{l}^{w,i} = 0.7$, and that the one of retirees is $\bar{l}^{r,i} = 0.01$ (remember that $\upsilon_l^i = 1 - \upsilon_c^i$). Structural parameters are summarized in Table 2.

Table 1: Initial steady-state population dynamics

Variable/Parameter	Symbol	Value		
		Germany	Rest of EA	Rest of world
Working-age population growth	n^w	0.004	0.004	0.004
Old age dependency ratio^T	Ψ	0.2647	0.2670	0.2001 (0.1500)
Retirement probabilities	$1-\omega$	0.0222	0.0222	0.0222
Survival probabilities^e	γ	0.9200	0.9208	0.9040 (0.8686)
Relative size Germany/RoEA^T	$rs^{a,b}$	0.343		
Relative size Germany/RoW^T	$rs^{a,c}$	0.092 (0.035)		
Relative size RoEA/RoW^T	$rs^{b,c}$	0.268 (0.101)		

Source: OECD (2017). The superscript T marks targets, e endogenously derived values to meet these targets. Parameters without a mark are set exogenously as described in the main text. We omit the country index i for convenience. The values in brackets are those when assuming the rest of the world to be the remaining OECD countries and China.

The replacement rate for pension benefits μ^i is set to 0.48. The government spending-to-GDP ratio is set to 0.18 in all regions. These are standard values. The debt-to-GDP ratios are 59.8%, 68.2% and 50.53% for Germany, the rest of the Eurozone and the rest of the world in line with Eurostat and OECD data for the year 2000.

As we work with a nominal extension of the Gertler (1999)-model, we also have to specify price markups and monetary policy. We assume $\theta_p^i = 10$ following Kara and von Thadden (2016). We also assume a standard Calvo parameter of 0.3. This reflects an average price duration of almost one and a half years, which falls in the range of standard calibrations for quarterly models. In the Taylor rule, we opt for an autocorrelation parameter of 0.7 and an inflation coefficient of 2. The inflation target is zero.[11] These

[10] Given our annual calibration, one could choose this value differently. However, as argued by Chen, Imrohoroglu, and Imrohoroglu (2009) and Ferrero (2010), a relatively high value can considered to be a short-cut to account for uncertainty in (potential) productivity growth.

[11] Assuming an inflation target of zero is mainly done for technical reasons. It facilitates the steady-

values hold for the monetary union as well as for the rest of the world. Autocorrelation in the fiscal policy rule is assumed to be 0.7, too, and the reaction to deviations of the debt-to-GDP ratio from target (the initial steady-state value in our baseline simulations) is 0.3. These are standard values from the literature (taking into account that we work with an annual calibration); see Schmitt-Grohe and Uribe (2007), Kirsanova and Wren-Lewis (2012) and Kara and von Thadden (2016). Policy parameters are summarized in Table 3.

Table 2: Structural parameters

Variable/Parameter	Symbol	Value		
		Germany	Rest of EA	Rest of world
Discount rate	β	0.98	0.98	0.98
Intertemporal elasticity of substitution	σ	0.5	0.5	0.5
Preference for consumption	υ_c	0.616^e	0.613^e	0.637^e
Substitution elasticity home/foreign	$1/(1-\eta)$	1.5	1.5	1.5
Bias for goods produced in Germany	ϑ_a	0.6	0.1	0.0108^e
Bias for goods produced in rest of EA	ϑ_b	0.1	0.6	0.0998^e
Bias for goods produced in rest of world	ϑ_c	0.3^e	0.3^e	0.8894^e
Cobb-Douglas share of labor	σ	2/3	2/3	2/3
Investment adjustment costs	κ_{inv}	4.5	4.5	4.5
Capital depreciation	δ	0.1	0.1	0.1
Relative productivity of retirees	ξ	0.497^e	0.499^e	0.501^e
Elasticity of demand for intermediate goods	θ_p	10	10	10
Calvo survival probability	κ_p	0.3	0.3	0.3

Source: The superscript T marks targets, e endogenously derived values to meet these targets. Parameters without a mark are set exogenously as described in the main text. We omit the country index i for convenience.

As regards international trade, we assume a substitution elasticity between home and foreign goods of 1.5, which is a standard value in the literature. This implies $\eta^i = 0.33$. In the initial steady state, relative prices between all regions equal one. Given this and the other calibration choices made so far, we can then endogenously solve for each region's net foreign asset-to-GDP ratio in the initial steady state. According to Balta and Delgado (2009), home bias for goods in a typical EU country is a bit above 60%, and the import content from other European economies amounts to about 10%. Choosing a home bias parameter of 0.6 for domestic goods (implying a domestic consumption share of about two thirds when including public consumption) and a bias towards the goods produced in the other European region of 0.1 allows us to derive the biases towards the different regional consumption/investment goods in the rest of the world that meet the net foreign

state calculation of β to match a steady-state world interest rate of 4% in a multi-country model. Given the model structure, an inflation target of close to 2% (as is announced by the ECB, for example) should not change our results, but deriving the steady state would become much more difficult.

asset positions which we just calculated. Details about how to solve for the initial steady state can be found in Appendix B.1.

Table 3: Policy parameters

Variable/Parameter	Symbol	Value		
		Germany	Rest of EA	Rest of world
Replacement rate for pension benefits^T	μ	0.48	0.48	0.48
Government spending share^T	$\bar{g}y$	0.18	0.18	0.18
Debt-to-GDP ratio^T	ω^b	0.598	0.682	0.505
Autocorrelation in fiscal rule	ρ^{tau}	0.7	0.7	0.7
Debt feedback in fiscal rule	ζ^b	0.3	0.3	0.3
Lump-sum tax^e	$\bar{\tau}$	0.3184	0.3225	0.2876
Autocorrelation in Taylor rule	ρ^{mp}	0.7	0.7	0.7
Inflation coefficient in Taylor rule	ζ^π	2.0	2.0	2.0

Source: The superscript T marks targets, e endogenously derived values to meet these targets. Parameters without a mark are set exogenously as described in the main text. We omit the country index i for convenience.

4 Analysis

In this section, we first show the (projected) demographic trends of Germany, the rest of the Euro area and the remaining OECD countries (potentially including China). Then, we describe how these trends are fed in to our model. Last, we turn to describing the results.

4.1 Demographic trends

Figure 2 plots the (projected) population developments from 2000 to 2080 in Germany, the rest of the Euro area and the remaining OECD countries (plus China). The data source is OECD (2017). Population is aged 20 to 100 (hence, we exclude younger individuals), and the old-age dependency ratio (OADR) is defined as the share of population above 65 divided by the population share between 20 and 65. We can observe the following. Societies in Germany and in the rest of the Euro area were and are indeed older than those in the remaining OECD economies. German working-age population declined since the beginning of the 2000s and is projected to steadily decline until 2080. There is a similar process in the rest of the Euro area economies starting in 2010, while a decline in the working-age population in the remaining OECD countries has not yet started. It is projected to do so by 2030, but the decline is significantly less severe. Total population falls in Germany, slightly increases in the rest of the Euro area before falling back to its initial value in 2080 and increases sharply in the remaining OECD economies. The latter, however, is largely driven by a disproportionate increase in the population aged 65 and above. This can also be seen in the OADR. It basically increases by the same amount in the remaining OECD countries (from 20% in 2000 to 56.2% in 2080) compared to the increase in Germany (from 26.5% to 62.8%) and the rest of the Euro area (from 26.7% to 63%).

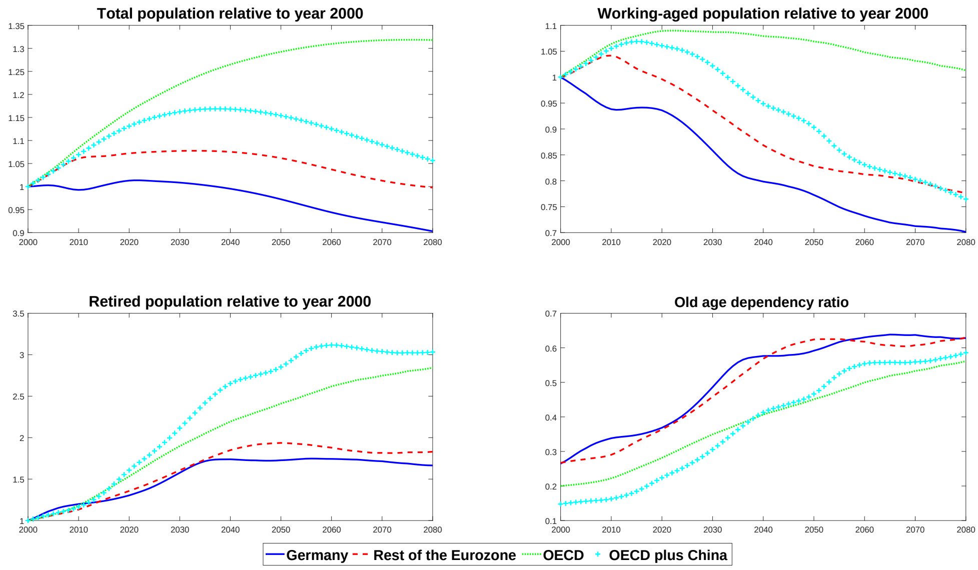

Figure 2: Population developments

Notes: Figure plots (projected) population developments for Germany (blue solid lines), the rest of the Euro area excluding Germany (red dashed lines), the remaining OECD countries (greed dotted lines) and the remaining OECD countries plus China (cyan crossed lines) from 2000 to 2080; source: OECD (2017). Population in 2000 is normalized to one in the first three subplots to make results comparable more easily. Total population was 81.488 million (Germany), 237.726 million (rest of the Euro area), 840.982 million (remaining OECD countries) and 2,124.181 million (remaining OECD countries plus China) in 2000.

The pattern of the increase in the OADR differs, however. It is rather steady in the remaining OECD economies and contains wiggles in Germany during the periods 2000 to 2010 and 2030 to 2040. The former can be explained by a relatively sharp decrease in fertility and the latter by the retiring baby boomers. This latter baby boomer effect is also present in the rest of the Euro area with a ten year delay. If we take on board China, we see that total population in the rest of the world is projected to also start falling by around 2030, working-age population is projected to start falling quickly by around 2020 and the elderly population is projected to increase even more. This can also be seen by the much faster increase in the projected OADR (from 14.8% in 2000 to 58.1% in 2080).

While the demographic trends across these regions are certainly not entirely synchronized, there is a common pattern: population becomes older. And, at least when taking into account the entire projection period until 2080, it does so equally or even more quickly for the regions that are younger today. The ageing "problem" there is just postponed.

4.2 Model implementation

Following the literature, we assume that demographic variables change unexpectedly in 2000, which is the initial steady state in our model (see, for example, Ferrero, 2010). This implies that, after the initial period, agents perfectly anticipate the evolution of the exogenous variables, which become constant again in 2080. Our model has three exogenous processes per region that allow us to imbed the demographic changes shown in Figure 2: working-age population growth, $n_t^{w,i}$, changes in survival probabilities, γ_t^i, and changes in retirement probabilities, ω_t^i.

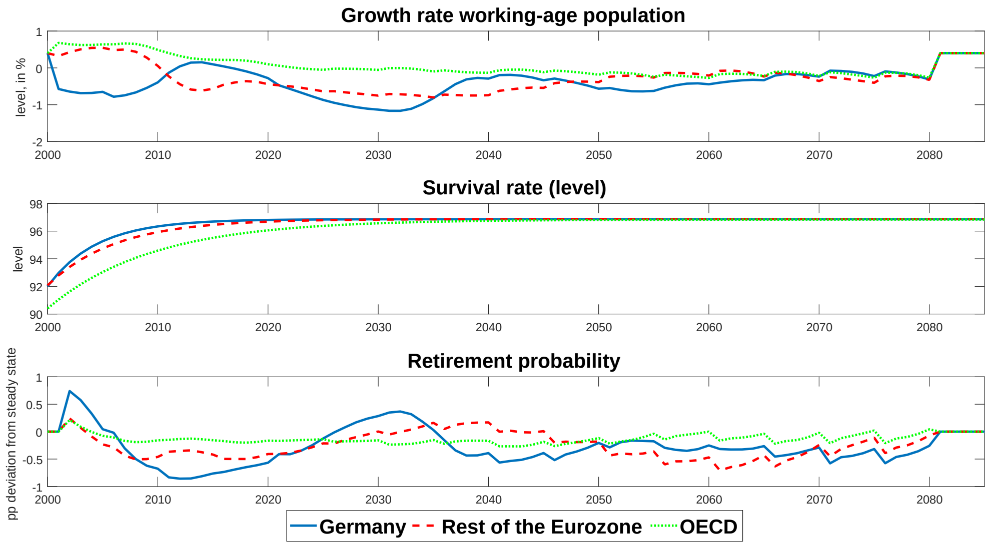

Figure 3: Exogenous demographic processes

Notes: Figure plots exogenous processes for Germany (blue solid lines), the rest of the Euro area (red dashed lines) and the remaining OECD countries (greed dotted lines) to reproduce (projected) population dynamics of Figure 2. Working-age population growth is year-to-year growth in %, the survival rates are given in levels and the retirement probabilities in percentage-point deviations from steady state.

It seems natural to use the growth rate for the working-age population to match working-age population growth. To do so, we assume that they are formally represented by $n_t^{w,i} = n^{w,i} + \epsilon_t^{nw,i}$, where $\epsilon_t^{nw,i}$ is a shock that is used to generate the growth rates observed in the data. It also seems natural to use survival probabilities to match an increase in longevity. However, it is likely that the increase in longevity is a slow and continuous process. We therefore assume that survival probabilities adjust steadily to their new value. Given the initial values for γ^i in $t = 0$, we thus assume that survival probabilities adjust to the new steady by the (exogenous) process $\gamma_t^i = (1 - \rho^{\gamma,i})\, \gamma^{i,final} + \rho^{\gamma,i}\, \gamma_{t-1}^i$. Here, $\gamma^{i,final}$ is the final steady-state value of survival probabilities generating the OADR observed in the data in 2080 (taking as given population growth and retirement probabilities). $\rho^{\gamma,a} = 0.8$, $\rho^{\gamma,b} = 0.85$ and $\rho^{\gamma,c} = 0.9$ are autocorrelation parameters reflecting the fact that population initially ages faster in Germany than it does in the other regions.[12] We

[12]This assumption is qualitatively not essential for our results. Assuming the opposite, for example, will only dampen the quantitative effects on the current account a bit.

then use the last free exogenous variable, the retirement probability ω_t^i, to match the annual OADR of each region i shown in Figure 2. Analogous to population growth, we assume a process $\omega_t^i = \omega^i + \epsilon_t^{\omega,i}$, where the shock $\epsilon_t^{\omega,i}$ is extracted to match the OADR of the data. Figure 3 plots the resulting exogenous processes.

Here, the following seems noteworthy. As can be seen in Figure 2, the projected OADR contains humps and does not increase strictly monotonously. This is especially true for Germany which is primarily a result of the baby boomer generation that starts retiring now (and reaches a retirement peak shortly after 2030; see Section 4.1). In our model, it is difficult to reproduce these humps. In a fully-fledged life-cycle model with many cohorts, a pre-determined finite life (at the age of, say, 100 years) and (potentially time-varying) age-dependent death probabilities along the lines of Auerbach and Kotlikoff (1987), these humps would automatically be generated by construction following a temporarily higher birth rate (see, for example, Schön, 2019). In our model, this does not happen given that retirement and death probabilities are independent of age (see Section 2). Hence, we have to construct the humps in the old age dependency ratio that we observe in the data by something else. In our model simulations presented in the main text, we chose a time-varying retirement probability to replicate these humps. As Figure 3 reveals, this implies fluctuations in the retirement probability that we may not observe in the data.

One could now argue that the retirement probability ω_t^i is a policy and not a "choice" variable. In order to check if this assumption is innocuous or not, we perform an analogous simulation as described above, now assuming ω^i to remain constant and having survival probabilities taking care of matching the (projected) OADR of the data. Even though survival probabilities now no longer increase steadily (but jump around quite a bit), the results remain qualitatively the same and the quantitative differences are small, too. To save space, details of this simulation are relegated to Appendix C.1.

4.3 Simulation results

We now turn to describing the simulation results. In order to assess the impact of ageing on net foreign assets and current account positions, we need to find out how (i.) population ageing affects the savings behavior of agents and how (ii.) it affects the possibility to invest these savings domestically. The latter is primarily determined by domestic capital demand in our model.[13] In order to assess how important population dynamics of Germany's trading partners are for determining the German current account, we perform two additional simulations with alternative population dynamics at the end of this section. We start off by describing the consumption/savings behavior of agents.

When households become aware of the demographic transition, they start increasing their savings effort and reduce consumption, reflected by a fall in their marginal propensity to consume (see Figure 4). This holds for both, workers and retirees. The fall in the marginal propensity to consume for retirees is stronger. As a result of the increased live expectancy (see Figure 3), retirees live longer on average. The time span they spend in retirement is extended. Anticipating this, they cut consumption today to have more to consume in the future. Because workers eventually become a retiree, this also holds for them, but to a lesser extent (in relative terms). Per-capita consumption in the economy

[13] Domestic government bonds also play a role. However, as they are determined by the fiscal rule described above, the impact of government bonds in our model simulation is of second-order importance.

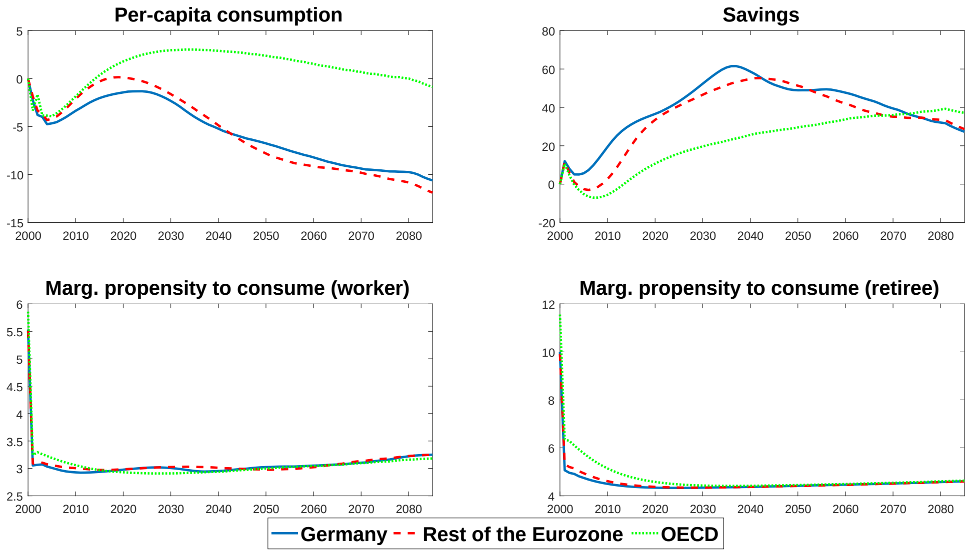

Figure 4: Consumption and savings reactions

Notes: Figure plots per-capita consumption and savings in percentage deviations from initial steady state for Germany (blue solid lines), the rest of the Euro area (red dashed lines) and the remaining OECD countries (greed dotted lines). Marginal propensities to consume are given in levels (in %).

thus falls and savings increase. The savings glut reduces the world interest rate (see Figure 7). These effects have been explained in an in-depth analysis by Carvalho et al. (2016) in a real closed-economy framework. As we can see, the mechanisms also apply in a nominal world and an open-economy framework.

Because life expectancy increases fastest in Germany, the marginal propensity to consume falls fastest in Germany (especially for retirees). Therefore, savings in Germany increase faster initially than they do in the other regions (Figure 4). Savings in the rest of the Euro area, however, pick up eventually. This is because the higher increase in life expectancy in Germany, relative to the rest of the Euro area, disappears around 2040 (see Figure 3). In addition to this, growth of the working-age population affects aggregates savings in an economy. It is negative in Germany, while it is positive in the rest of the Eurozone until about 2010 and in the remaining OECD countries until about 2020. As new workers enter the economy with zero assets, positive population growth compensates for higher individual saving efforts of the (elderly) population. Population growth is positive in the rest of the Euro area until about 2010 and in the remaining OECD countries until about 2020.

It is also interesting to have a brief look at consumption. We see that, after the initial drop resulting from the fall in the marginal propensity to consume, per-capita consumption starts rising again for some time in all regions – less so in Germany and most in the remaining OECD countries. This is a result of two factors. First, there is a composition effect. As we can see in Figure 4, the marginal propensity to consume is about twice as high for retirees than it is for workers, which remains to hold even after the the marginal propensity to consume has fallen. Hence, as the share of retirees in the economy rises, the reduction of aggregate (per-capita) consumption is mitigated by the population composition (i.e. the higher share of retirees who consume more). Second, per-period income of both, workers and retirees increases as a result of higher labor income (see Figure 6). Real wages increase because of a higher capital-to-labor ratio (which we will explain in more detail below). That induces workers and retirees to supply more labor, augmenting per-period (labor) income . Income of retirees is additionally affected positively because wages also determine pension benefits. The drop in consumption per retiree and per worker is thus less severe than what could have been expected by solely looking at the drop in the marginal propensities to consume (which determine the share of income consumed by households). Given that the increase in real wages is strongest in the remaining OECD countries, per-capita consumption actually increases above its initial steady-state level there for a while. Eventually, however, per-capita consumption starts falling in all regions, however. This is first and foremost a result of reduced GDP (which we will describe in more detail below) primarily driven by the decrease in (working-age) population.[14]

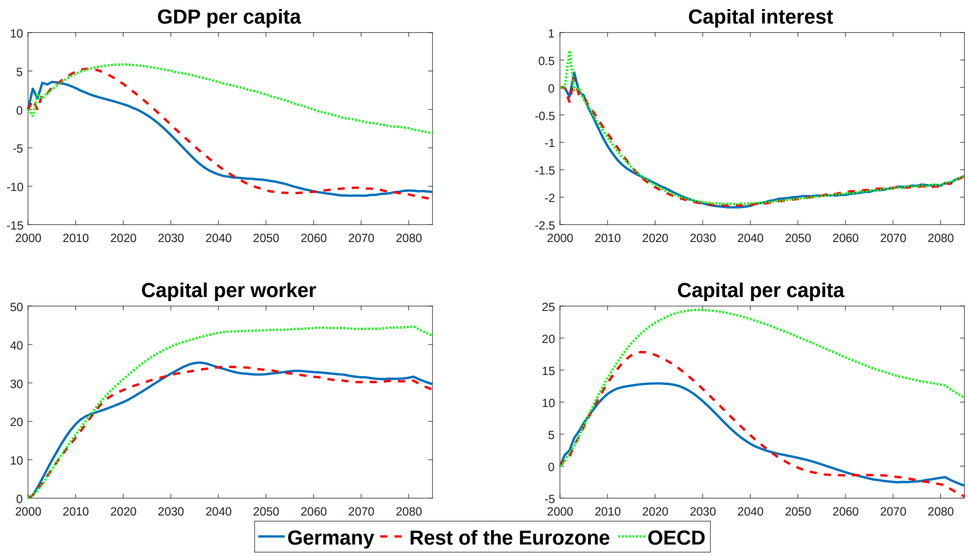

Figure 5: Production sector reactions

Notes: Figure plots production related variables in percentage deviations (percentage point deviations capital interest) from initial steady state for Germany (blue solid lines), the rest of the Euro area (red dashed lines) and the remaining OECD countries (greed dotted lines).

Having determined the savings behavior of agents, we now need find out how domestic capital demand reacts. This determines the possibility to invest the additional savings domestically. As described above, the savings glut reduces the world interest rate (Figure 7). Through the no-arbitrage condition, this translates into a reduction in capital interest (see Figure 5). The fall in the capital interest rate is roughly the same in all regions.

[14]In addition, the reduction in net foreign assets in Germany and the rest of the Euro area starting around 2040 (Figure 7) contributes to the decline in per-capita consumption in these regions then. The reason is that, then, these regions receive lower transfers from the remaining OECD regions (in terms of interest payments on the outstanding assets). For the other OECD regions, this mitigates the drop in per-capita consumption.

Hence, firms employ more capital and production becomes more capital-intense. The capital per worker ratio increases. A higher capital-to-labor ratio increases per-capita GDP. However, as population declines eventually, and because relatively more non-working retirees populate the economy, it starts falling eventually. The increase per-capita GDP and the capita per worker ratio across regions differs because the number of workers evolves differently. As population ages, the increase in capital per capita is smaller than the rise in capital intensity, and it is smallest in Germany relative to the other regions (as Germany ages faster initially).[15] A relatively lower increase in capital per capita combined with a reduction in working-age population growth in Germany reduce the possibility to invest higher German savings domestically (in relative terms).

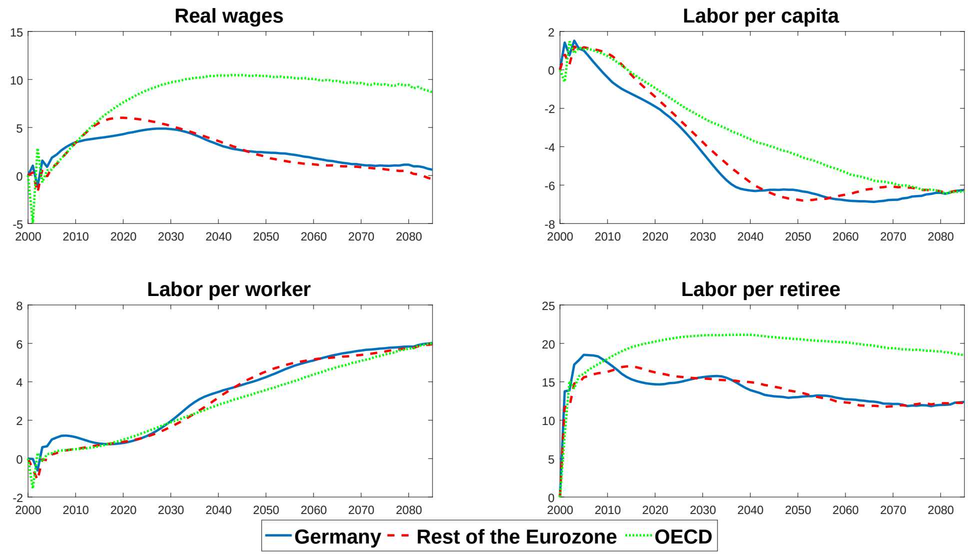

Figure 6: Labor market effects

Notes: Figure plots labor market related variables in percentage deviations (percentage point deviations for per-capita labor) from initial steady state for Germany (blue solid lines), the rest of the Euro area (red dashed lines) and the remaining OECD countries (greed dotted lines).

The combination of the savings behavior and domestic capital demand now determines current account and net foreign asset positions. In Figure 7, we show the evolution of these international variables. Due to the higher increase in individual savings to insure against longevity until beyond mid-century and the weaker increase in capital per capita (combined with lower population growth), Germany and the rest of the Euro area become capital exporters, reflected by an increase in their net foreign asset positions and the current account-to-GDP ratio. The current account is positive until around 2035 in Germany and the rest of the Euro area, which corresponds to the turning point in their net foreign asset-to-GDP positions. Because savings increase sharply (and suddenly) on impact – when people start realizing that they live in an ageing world – the drop in the

[15]Furthermore note that staggered price setting and the fact that capital investment is associated with adjustment costs imply that the reduction in the capital interest rate does not follow the reduction in the world interest immediately. Part of this is absorbed by the relative price of capital, Q_t^i; see equation (18).

world interest rate is highest on impact. Higher capital demand due to lower interest rates (Figure 5) mitigates the fall in the world interest eventually. Still, it stays at around 1.5 percentage points below its initial value in the new steady state. International competitiveness of Germany and the rest of the Euro area relative to the remaining OECD countries, measured as the relative producer price between these regions, starts falling after 2010. This is a result of relatively more younger people (workers, respectively) in the remaining OECD economies. Relatively higher labor supply reduces unit labor costs and, thus, augments international competitiveness of the remaining OECD economies. By the same token, German competitiveness vis-à-vis the Euro area falls until 2020 as German working-age population growth is negative, while it is still positive in the rest of the Euro area until 2010. As German and rest of the Euro area working-age population developments eventually converge, German competitiveness vis-à-vis the Euro area again recovers during the transition when the rest of he Euro area ages, too.

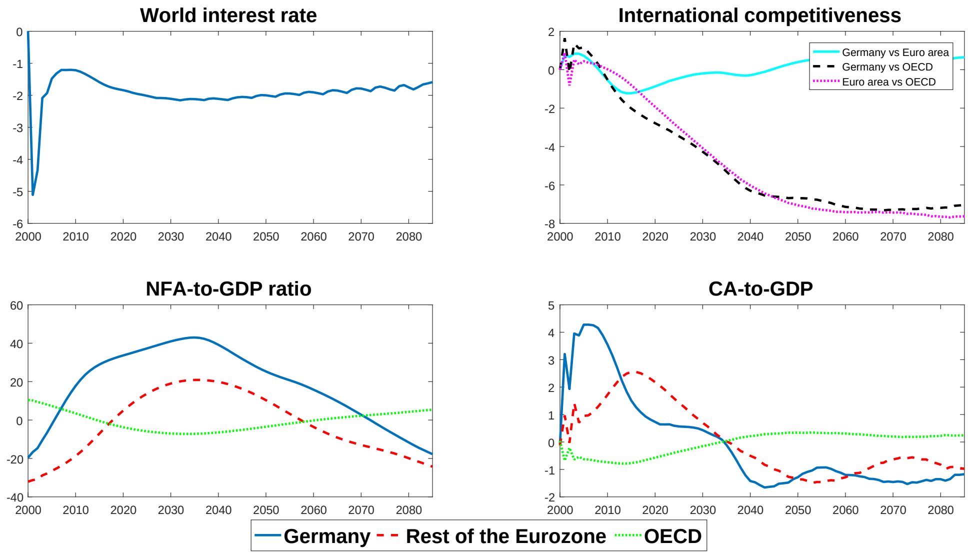

Figure 7: International transmission

Notes: Figure plots deviations of international variables from initial steady state for Germany (blue solid lines), the rest of the Euro area (red dashed lines) and the remaining OECD countries (greed dotted lines). Interest rates and international competitiveness, defined as P_t^j/P_t^i between regions i and j, are in percentage point deviations, net foreign asset and current account-to-GDP ratios in levels.

In order to check how important population dynamics of Germany's trading partners are for the evolution of net foreign assets and current account balances, we perform two additional simulations. First, as a rather extreme case, we assume that constant population dynamics around the world except for Germany (i.e. we assume population growth and the survival probabilities to remain at their initial steady-state levels in the rest of the world). Second, we feed in the population dynamics of the remaining OECD countries plus China. German current account and net foreign asset positions resulting from these simulations are presented in Figure 8. The other economic mechanisms are analogous to those described above.

As we can see, a constant world population significantly increases capital exports in

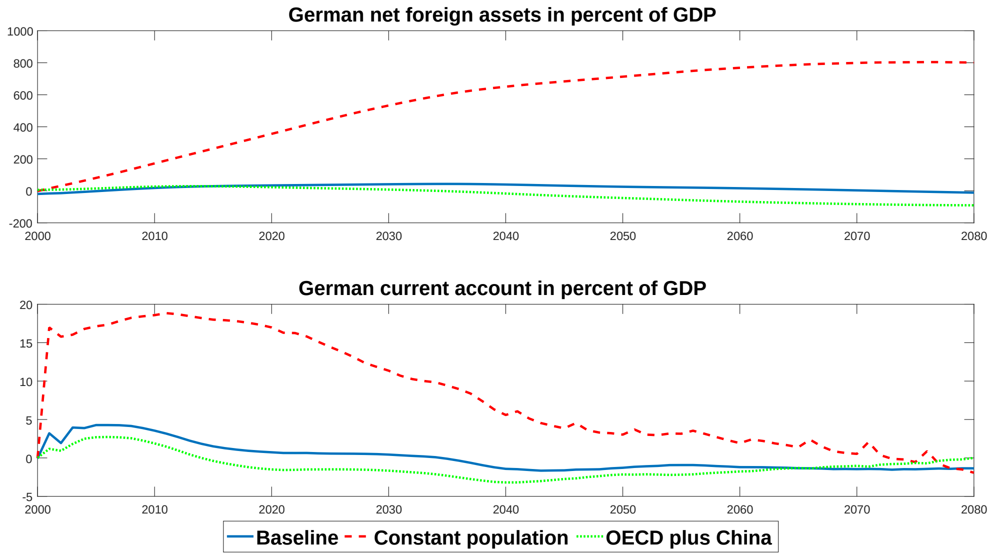

Figure 8: German current account effects for differently synchronized population dynamics

Notes: Figure plots German net foreign asset and current account-to-GDP ratios for different population dynamics and compares these to the baseline simulations that we presented above. The baseline scenario is represented by blue solid lines, constant population dynamics for Germany's trading partners by red dashed lines and population dynamics of OECD countries including China by greed dotted lines.

Germany, driving up the current account surplus to above 15% of GDP in 2010. The reason is that the incentive to save more because of ageing is only present in Germany, and not around the world. In addition, the decrease in the world interest rate (and, thus, the increase in domestic capital demand) is smaller as it is affected only by the German savings increase. Hence, as German savings increases disproportionately and domestic asset demand does not rise sufficiently, this leads to relatively higher capital exports.[16] In our second additional simulation, we take into account the quicker ageing process in China (relative to the other OECD economies in baseline simulations). Then, the opposite happens. The effects on the German current account are muted. This is a result of the fact that savings in the rest of the world now also increases sharply, driving down the world interest rate further, and leading to more negative demand effects (on the world level). The pattern of the current account-to-GDP ratio is similar to the one of our baseline simulation, but about 1.5 percentage points lower. As these simulations show, the (dis-) synchrony of the ageing process between regions is crucial for their effects on net foreign asset and current accounts. When taking the demographic trends of Germany's most relevant trading partners into account, ageing seems to only explain a smaller fraction of the German current account developments than what is sometimes claimed. According to our model simulations, the fraction of the current account that can be explained by

[16]Note that, even though the rest of the world is not subject to demographic change, its economy will also be affected. This is a result of a fall in the world interest rate and reduced demand from Germany. However, as ageing only boosts asset supply in Germany, the effects on the world interest rate will be muted. So will be the incentive to employ more capital. And, given its relative size, demand effects in German that could affect the rest of the world are also limited.

ageing (relative to what we observe since the start of the millennium in the data) amounts to about one third (fifth when taking into account China).

5 Conclusions

In this paper, we develop a three-region New Keynesian life-cycle model calibrated to Germany, the Euro area and the rest of the world to analyze the impact of population ageing in Germany on its net foreign asset and current account developments.

We are able to generate German current account surpluses of up to 15% of GDP during the first half of this century. However, this requires assuming unsynchronized demographic trends by taking those of Germany as given and assuming constant population everywhere else. Projected demographic trends from 2000 to 2080 in Germany, the Euro area and the remaining OECD countries (and potentially China) are much more synchronized. Feeding these into our model, simulations suggest that the average annual German current account surplus from 2000 to 2018 reduces to 2.83% (1.23%) of GDP (when taking into account China), turning negative around 2035.

Hence, our simulations suggest that, while important, German population ageing should not be seen as *the* most important driver of the currently observed high and persistent German account surpluses. Other possible explanations for the German current account surplus are German labor market reforms initiated in 2003, financial integration, the introduction of the Euro, economic growth in emerging markets or fiscal consolidation in Germany. Further research to fully understand the reasons and effects at play is certainly in order.

Another issue related to our analysis may be the fact that we abstract from any effect of population ageing on TFP. Some authors, such as Aksoy, Basso, Smith, and Grasl (2019), Cooley and Henriksen (2018), Hopenhayn, Neira, and Singhania (2018) or Liang, von Hui, and Lazear (2018), among others, however argue that population ageing may have a negative impact on TFP growth, especially when it is driven by a decline in the working age population. If this is true, the stronger fall in German working-age population would eventually reduce German productivity, increase relative prices for German goods and, thereby, curtail exports. This, of course, could reduce the effects of ageing on the German current account. Further research in this direction would also be interesting.

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Appendix

A.1 Deriving the workers' and retirees' consumption and value functions

As mentioned in the main text, we derive the consumption functions of the representative retiree and worker here. We start by deriving the function for the representative retiree. Maximizing

$$V_t^{r,i,j} = \left\{ \left[\left(c_t^{r,i,j}\right)^{\upsilon_c^i} \left(1 - l_t^{r,i,j}\right)^{\upsilon_l^i} \right]^{\rho} + \beta\, \gamma_t^i \left(V_{t+1}^{r,i,j}\right)^{\rho} \right\}^{\frac{1}{\rho}},$$

subject to

$$c_t^{r,i,j} + a_t^{r,i,j} = \frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot a_{t-1}^{r,i,j} + \xi^i \cdot w_t^i \cdot l_t^{r,i,j} + e_t^{i,j}$$

yields

$$\upsilon_c^i \cdot \left(c_t^{r,i,j}\right)^{\rho\upsilon_c^i - 1} \left(1 - l_t^{r,i,j}\right)^{\rho(1-\upsilon_c^i)} = \beta \cdot \gamma_t \cdot \left(V_t^{r,i,j}\right)^{\rho-1} \frac{\partial V_{t+1}^{r,i,j}}{\partial a_t^{r,i,j}}.$$

The envelope conditions are used to obtain

$$\frac{\partial V_t^{r,i,j}}{\partial a_{t-1}^{r,i,j}} = \left(V_t^{r,i,j}\right)^{1-\rho} \cdot \upsilon_c^i \cdot \frac{1 + r_{t-1}}{\gamma_{t-1}} \left(1 - l_t^{r,i,j}\right)^{\rho(1-\upsilon_c^i)} \left(c_t^{r,i,j}\right)^{\rho\upsilon_c^i - 1}.$$

Shifting forward and combining with the previously derived first-order condition for consumption yields

$$\left(c_t^{r,i,j}\right)^{\rho\upsilon_c^i - 1} \left(1 - l_t^{r,i,j}\right)^{\rho(1-\upsilon_c^i)} = \beta \cdot (1 + r_t) \cdot \left(c_t^{r,i,j} t + 1\right)^{\rho\upsilon_c^i - 1} \left(1 - l_{t+1}^{r,i,j}\right)^{\rho(1-\upsilon_c^i)}.$$

Using the first-order condition with respect to labor,

$$(1 - l_t^{r,i,j}) = \frac{1 - \upsilon_c^i}{\upsilon_c^i} \cdot \frac{c_t^{r,i,j}}{\xi^i\, w_t^i},$$

and combining with the previous equation, we get the Euler equation

$$c_{t+1}^{r,i,j} = \left[\left(\frac{w_t^i}{w_{t+1}^i} \right)^{(1-\upsilon_c^i)\rho} \cdot \beta\, (1 + r_t) \right]^{\sigma} c_t^{r,i,j},$$

where $\sigma = 1/(1-\rho)$. We now guess that consumption is a fraction $\epsilon_t^i \pi_t^i$ of total wealth,

$$c_t^{r,i,j} = \epsilon_t^i \pi_t^i \cdot \left(\frac{1 + r_{t-1}}{\gamma_{t-1}^i} \cdot a_{t-1}^{r,i,j} + h_t^{r,i,j} \right),$$

using the recursive representation of $h_t^{r,i,j}$ presented in the main text. Substituting this in the previously derived Euler equation equation, we get

$$a_t^{r,i,j} + \frac{\gamma_t}{1+r_t} \cdot h_{t+1}^{r,i,j} = \left[\left(\frac{w_t^i}{w_{t+1}^i} \right)^{(1-\upsilon_c^i)\rho} \cdot \beta \right]^{\sigma} \cdot (1+r_t)^{\sigma-1} \cdot \frac{\epsilon_t^i \pi_t^i}{\epsilon_{t+1}^i \pi_{t+1}^i} \cdot \gamma_t \times \left(\frac{1+r_{t-1}}{\gamma_{t-1}^i} \cdot a_{t-1}^{r,i,j} + h_t^{r,i,j} \right),$$

which, substituted into the budget constraint, implies equation (2) of the main text.

Moreover, conjecture that the value function is linear in consumption,

$$V_t^{r,i,j} = \Delta_t^{r,i,j} c_t^{r,i,j} \left(\frac{1-\upsilon_c^i}{\upsilon_c^i} \cdot \frac{1}{\xi^i \, w_t^i} \right)^{1-\upsilon_c^i},$$

where use has been made of the first-order condition with respect to labor. Then, from the value function of the retiree, it must be the case that

$$\left(\Delta_t^{r,i,j}\right)^{\rho} = 1 + \beta \cdot \gamma_t \cdot \left(\Delta_{t+1}^{r,i,j} \cdot \frac{c_{t+1}^{r,i,j}}{c_t^{r,i,j}} \cdot \left(\frac{w_t^i}{w_{t+1}^i} \right)^{1-\upsilon_c^i} \right)^{\rho}.$$

Using the Euler equation, we get

$$\left(\Delta_t^{r,i,j}\right)^{\rho} = 1 + \gamma_t \cdot (1+r_t)^{\sigma-1} \cdot \left(\Delta_{t+1}^{r,i,j}\right)^{\rho} \cdot \left(\beta \cdot \left(\frac{w_t^i}{w_{t+1}^i} \right)^{(1-\upsilon_c^i)\rho} \right)^{\sigma},$$

where use has been made of the fact that $\rho\sigma = \sigma - 1$. Let $\left(\Delta_t^{r,i,j}\right)^{\rho} = \left(\epsilon_t^i \pi_t^i\right)^{-1}$. Then, we again obtain equation (2) of the main text, which completes the conjectures for the representative retiree's consumption function and his value function $V_t^{r,i,j}$.

For the representative worker, who maximizes

$$V_t^{w,i,j} = \left\{ \left[\left(c_t^{w,i,j}\right)^{\upsilon_c^i} \left(1 - l_t^{w,i,j}\right)^{\upsilon_l^i} \right]^{\rho} + \beta \, \left(\omega_t^i \, V_{t+1}^{w,i,j} + (1-\omega_t^i) \, V_{t+1}^{r,i,j} \right)^{\rho} \right\}^{\frac{1}{\rho}},$$

subject to

$$c_t^{w,i,j} + a_t^{w,i,j} = (1+r_{t-1}) \cdot a_{t-1}^{w,i,j} + w_t^i \cdot l_t^{w,i,j} + f_t^{i,j} - \tau_t^{i,j},$$

we proceed analogously, taking the previously calculated retiree's value function $V_t^{r,i,j}$ as an input. The maximization with respect to consumption yields

$$\upsilon_c^i \cdot \left(c_t^{w,i,j}\right)^{\rho\upsilon_c^i - 1} \left(1 - l_t^{w,i,j}\right)^{\rho(1-\upsilon_c^i)} = \beta \cdot \left[\omega_t^i \cdot \left(V_t^{w,i,j}\right) + (1-\omega_t^i) \cdot \left(V_t^{w,i,j}\right) \right]^{\rho-1} \times \left[\omega_t^i \cdot \frac{\partial V_{t+1}^{w,i,j}}{\partial a_t^{w,i,j}} + (1-\omega_t^i) \cdot \frac{\partial V_{t+1}^{r,i,j}}{\partial a_t^{r,i,j}} \right].$$

Again applying the (analogous) envelope conditions (for the derivatives of the worker's and retiree's value functions with respect to assets), using the worker's first-order conditions with respect to labor, $(1 - l_t^{w,i,j}) = \frac{1-\upsilon_c^i}{\upsilon_c^i} \cdot \frac{c_t^{w,i,j}}{w_t^i}$, and combining all these expressions, we get

$$\begin{aligned}\left(c_t^{w,i,j}\right)^{\rho-1} =& \beta \cdot (1+r_t) \cdot \left(\frac{w_t^i}{w_{t+1}^i}\right)^{\rho(1-\upsilon_c^i)} \cdot \left[\omega_t^i \left(V_t^{w,i,j}\right) + (1-\omega_t^i)\left(V_t^{r,i,j}\right)\right]^{\rho-1} \\ & \times \left[\omega_t^i \left(c_{t+1}^{w,i,j}\right)^{\rho-1} \cdot \left(V_t^{w,i,j}\right)^{1-\rho} + (1-\omega_t^i) \cdot \left(c_{t+1}^{r,i,j}\right)^{\rho-1} \left(V_t^{r,i,j}\right)^{1-\rho} \cdot \left(1/\xi^i\right)^{\rho(1-\upsilon_c^i)}\right].\end{aligned}$$

To solve for the worker's problem, conjecture a value function $V_t^{w,i,j} = \Delta_t^{w,i,j} c_t^{w,i,j} \left(\frac{1-\upsilon_c^i}{\upsilon_c^i} \cdot \frac{1}{w_t^i}\right)^{1-\upsilon_c^i}$, where we additionally conjecture that $\Delta_t^{w,i,j} = \left(\pi_t^i\right)^{-1/\rho}$, and recall the value function for retirees (which we derived above). Substitution in the above equation then yields

$$\begin{aligned}\left(c_t^{w,i,j}\right)^{\rho-1} =& \beta \cdot (1+r_t) \cdot \left(\frac{w_t^i}{w_{t+1}^i}\right)^{\rho(1-\upsilon_c^i)} \cdot \left[\omega_t^i + (1-\omega_t^i) \cdot \left(\epsilon_{t+1}^i\right)^{-(1-\rho)/\rho} \cdot \left(1/\xi^i\right)^{1-\upsilon_c^i}\right] \\ & \times \left[\omega_t^i \left(c_t^{w,i,j}\right) + (1-\omega_t^i)\left(c_t^{r,i,j}\right) \cdot \left(\epsilon_{t+1}^i\right)^{-1/\rho} \cdot \left(1/\xi^i\right)^{1-\upsilon_c^i}\right]^{\rho-1},\end{aligned}$$

where use has been made of the fact that $\Delta_t^{r,i,j}/\Delta_t^{w,i,j} = \left(\epsilon_t^i\right)^{-1/\rho}$. Remembering that $-1/\rho = \sigma/(1-\sigma)$ and defining Ω_t^i as in equation (3) of the main text, the representative worker's consumption Euler equation is given by

$$\omega_t^i \, c_{t+1}^{w,i,j} + (1-\omega_t^i)\ \left(\epsilon_{t+1}^i\right)^{\frac{\sigma}{1-\sigma}} \ \left(\frac{1}{\xi^i}\right)^{\upsilon_l^i} \ c_{t+1}^{r,i,j} = \left[\beta\,(1+r_t)\,\Omega_{t+1}^i \cdot \left(\frac{w_t^i}{w_{t+1}^i}\right)^{\upsilon_l^i \rho}\right]^{\sigma} \ c_t^{w,i,j}.$$

Now conjecture that the worker's consumption is a fraction π_t^i of the worker's wealth,

$$c_t^{w,i,j} = \pi_t^i \cdot \left((1+r_{t-1}) \cdot a_t^{w,i,j} + h_t^{w,i,j}\right),$$

while the just retired person (a worker in the previous period) consumes

$$c_t^{r,i,j} = \epsilon_t^i \pi_t^i \cdot \left((1+r_{t-1}) \cdot a_t^{w,i,j} + h_t^{r,i,j}\right),$$

where $h_t^{w,i,j}$ and $h_t^{r,i,j}$ are defined in the main text. Substituting the conjectures in the worker's consumption Euler equation and proceeding analogously as we did for retirees, we can confirm that equation (4) of the main text holds. Hence, all conjectures add up to consistent solutions across all equations characterizing optimal decisions of workers and retirees.

B.1 Equation summary and steady-state derivation

In this appendix, we provide a **summary of the detrended model equations** as described in Section 2.8. The bar indicates the respective detrended variable. For each region $i = a, b, c$, the marginal propensities to consume are given by

$$\epsilon_t^i \pi_t^i = 1 - \left[\beta \cdot \left(\frac{\bar{w}_t^i}{(1+x^i)\,\bar{w}_{t+1}^i} \right)^{\upsilon_l^i \rho} \right]^{\sigma} \cdot [(1+r_t)]^{\sigma-1} \cdot \gamma_t^i \cdot \frac{\epsilon_t^i \pi_t^i}{\epsilon_{t+1}^i \pi_{t+1}^i}, \tag{B.1}$$

and

$$\pi_t^i = 1 - \left[\beta \cdot \left(\frac{\bar{w}_t^i}{(1+x^i)\,\bar{w}_{t+1}^i} \right)^{\upsilon_l^i \rho} \right]^{\sigma} \cdot \left[(1+r_t) \cdot \Omega_{t+1}^i \right]^{\sigma-1} \frac{\pi_t^i}{\pi_{t+1}^i}, \tag{B.2}$$

with

$$\Omega_{t+1}^i = \omega_t^i + (1-\omega_t^i)\, \left(\epsilon_{t+1}^i\right)^{1/(1-\sigma)} \, \left(1/\xi^i\right)^{\upsilon_l^i}. \tag{B.3}$$
$$\tag{B.4}$$

Consumption is given by

$$\bar{c}_t^{w,i} = \pi_t^i \left[\frac{(1+r_{t-1})}{(1+n_{t-1}^{w,i})(1+x^i)} \,(1-\lambda_{t-1}^i)\, \bar{a}_{t-1}^i + \bar{h}_t^{w,i} \right], \tag{B.5}$$

$$\bar{c}_t^{r,i} = \epsilon_t^i \pi_t^i \left[\frac{(1+r_{t-1})}{(1+n_{t-1}^{r,i})(1+x^i)} \,\lambda_{t-1}^i\, \bar{a}_{t-1}^i + \bar{h}_t^{r,i} \right], \tag{B.6}$$

$$\bar{c}_t^i = \bar{c}_t^{w,i} + \Psi_t^i \cdot \bar{c}_t^{r,i}. \tag{B.7}$$
$$\tag{B.8}$$

Human wealth is

$$\bar{h}_t^{r,i} = \xi^i \cdot \bar{w}_t^i \cdot \bar{l}_t^{r,i} + \bar{e}_t^i + \frac{(1+x^i)\cdot \gamma_t^i}{(1+r_t)} \,\bar{h}_{t+1}^{r,i}, \tag{B.9}$$

$$\bar{h}_t^{w,i} = \bar{w}_t^i \cdot \bar{l}_t^{w,i} + \bar{f}_t^i - \bar{\tau}_t^i + \frac{\omega_t^i \cdot (1+x^i) \cdot \bar{h}_{t+1}^{w,i,j}}{(1+r_t)\,\Omega_{t+1}^i} + \left(1 - \frac{\omega_t^i}{\Omega_{t+1}^i}\right) \frac{(1+x^i)\cdot h_{t+1}^{r,i}}{(1+r_t)}, \tag{B.10}$$

and financial wealth

$$\lambda_t^i \,\bar{a}_t^i = \omega_t^i \left\{ \left(1 - \epsilon_t^i \pi_t^i\right) \left[\frac{(1+r_{t-1})}{(1+n_{t-1}^{w,i})(1+x^i)} \,\lambda_{t-1}^i\, a_{t-1}^i + \Psi_t^i \bar{h}_t^{r,i} \right] - \Psi_t^i \left(\bar{h}_t^{r,i} - \xi^i \,\bar{w}_t^i \,\bar{l}_t^{r,i} - \bar{e}_t^i \right) \right\} + (1-\omega_t^i)\, \bar{a}_t^i, \tag{B.11}$$

with

$$\bar{a}_t^i = Q_t^{k,i} \,\bar{k}_t^i + \bar{b}_t^i + \bar{d}_t^i. \tag{B.12}$$

Labor supply by households is

$$\bar{l}_t^{w,i} = 1 - \frac{\upsilon_l^i}{\upsilon_c^i} \cdot \frac{\bar{c}_t^{w,i}}{\bar{w}_t^i}, \tag{B.13}$$

$$\bar{l}_t^{r,i} = 1 - \frac{\upsilon_l^i}{\upsilon_c^i} \cdot \frac{\bar{c}_t^{r,i}}{\xi^i \, \bar{w}_t^i}, \tag{B.14}$$

$$\bar{l}_t^i = \bar{l}_t^{w,i} + \bar{l}_t^{r,i} \cdot \xi^i \cdot \Psi_t^i. \tag{B.15}$$

Firms produce

$$\bar{y}_t^i = \left(\bar{l}_t^i\right)^{\alpha^i} \left(\frac{\bar{k}_{t-1}^i}{(1+n_{t-1}^{w,i})(1+x^i)}\right)^{1-\alpha^i}, \tag{B.16}$$

implying

$$(1+n_{t-1}^{w,i})(1+x^i) \cdot \frac{\bar{l}_t^i}{\bar{k}_{t-1}^i} = \frac{\alpha^i}{1-\alpha^i} \cdot \frac{r_t^{k,i}}{\bar{w}_t^i}, \tag{B.17}$$

$$mc_t^i = \left(\frac{\bar{w}_t^i}{\alpha^i}\right)^{\alpha^i} \cdot \left(\frac{r_t^{k,i}}{1-\alpha^i}\right)^{1-\alpha^i} \tag{B.18}$$

and

$$\bar{f}_t^i = \left(\frac{P_t^{i,i}}{P_t^i} - mc_t^i\right) \bar{y}_t^i \tag{B.19}$$

as the capital-labor ratio, the marginal cost function and aggregate profits, respectively. Calvo pricing gives

$$D_t^i = \frac{P_t^{i,i}}{P_t^i} = \kappa_p^i \left(\frac{P_t^{i,i}}{P_{t-1}^{i,i}}\right)^{\theta_p^i} D_{t-1}^i + (1-\kappa_p^i) \left(\frac{P_t^i}{P_t^{i,i,*}}\right)^{\theta_p^i} \tag{B.20}$$

with

$$\frac{P_t^{i,i,*}}{P_t^i} = \frac{\kappa_p^i}{\kappa_p^i - 1} \cdot \frac{q1_t^i}{q2_t^i}, \tag{B.21}$$

where

$$q1_t^i = DF_t^i \, mc_t^i \, \bar{y}_t^i + \beta \, \kappa_p^i \left(\frac{P_{t+1}^{i,i}}{P_t^{i,i}}\right)^{\theta_p^i} \cdot q1_{t+1}^i, \tag{B.22}$$

$$q2_t^i = DF_t^i \, \bar{y}_t^i \cdot \frac{P_t^i}{P_t^{i,i}} + \beta \, \kappa_p^i \left(\frac{P_{t+1}^{i,i}}{P_t^{i,i}}\right)^{\theta_p^i - 1} \cdot q2_{t+1}^i, \tag{B.23}$$

and

$$DF^i_{t,t+1} = \left(\pi^i_{t+1}\right)^{-1/\rho} \left(\upsilon^i_l/\upsilon^i_c/((1+x^i)\bar{w}^i_{t+1})\right)^{\upsilon^i_l}. \tag{B.24}$$

Capital evolves according to

$$\bar{k}^i_t = \frac{(1-\delta^i)}{(1+n^{w,i}_{t-1})(1+x^i)} \cdot \bar{k}^i_{t-1} + \left[1 - \frac{\kappa^i_{inv}}{2}\left(\frac{\bar{inv}^i_t}{\bar{inv}^i_{t-1}} - 1\right)^2\right] \cdot \bar{inv}^i_t, \tag{B.25}$$

and the no-arbitrage conditions imply

$$(1+r_t) = \frac{r^{k,i}_{t+1} + Q^{k,i}_{t+1}(1-\delta^i)}{Q^{k,i}_t} = (1+i^i_t)\,\frac{P^i_t}{P^i_{t+1}}, \tag{B.26}$$

with

$$1 = Q^{k,i}_t \cdot \left[1 - S^i_t(\cdot) - S^{i,'}_t(\cdot) \cdot \frac{\bar{inv}^i_t}{\bar{inv}^i_{t-1}}\right] + \frac{Q^{k,i}_{t+1}}{1+r_{t+1}} \cdot S^{i,'}_{t+1}(\cdot)\left(\frac{\bar{inv}^i_{t+1}}{\bar{inv}^i_t}\right)^2. \tag{B.27}$$

The fiscal authorities behave according to

$$\bar{b}^i_t + \bar{\tau}^i_t = \frac{(1+r_{t-1})}{(1+n^{w,i}_{t-1})(1+x^i)} \cdot \bar{b}^i_{t-1} + \frac{P^{i,i}_t}{P^i_t} \cdot \bar{g}^i_t + \bar{e}^i_t, \tag{B.28}$$

with

$$\bar{g}^i_t = \bar{g}^i, \tag{B.29}$$

$$\mu^i_t = \mu^i, \tag{B.30}$$

and

$$\log\left(\frac{\tau^i_t}{\tau^i}\right) = \rho^{\tau,i}\,\log\left(\frac{\tau^i_{t-1}}{\tau^i}\right) + \zeta^{b,i}\,\log\left(\frac{P^i_{t-1} \cdot \bar{b}^i_{t-1}}{P^{i,i}_{t-1} \cdot \varpi^{b,i} \cdot \bar{y}^i_{t-1}}\right), \tag{B.31}$$

where $\bar{e}^i_t = \mu^i_t\,\bar{w}^i_t\,\Psi^i_t$ as described in the main text.

For the international part, we get

$$\bar{\bar{x}}^i_t = \left[\left(\vartheta^i_a\right)^{1-\eta^i}\left(\bar{\bar{x}}^i_{a,t}\right)^{\eta^i} + \left(\vartheta^i_b\right)^{1-\eta^i}\left(\bar{\bar{x}}^i_{b,t}\right)^{\eta^i} + \left(\vartheta^i_c\right)^{1-\eta^i}\left(\bar{\bar{x}}^i_{c,t}\right)^{\eta^i}\right]^{\frac{1}{\eta^i}}, \tag{B.32}$$

$$\frac{\bar{\bar{x}}^i_{a,t}}{\bar{\bar{x}}^i_{b,t}} = \frac{\vartheta^i_a}{\vartheta^i_b}\left(\frac{P^{b,i}_t}{P^{a,i}_t}\right)^{\frac{1}{1-\eta^i}}, \tag{B.33}$$

and

$$\frac{\bar{\tilde{x}}^i_{a,t}}{\bar{\tilde{x}}^i_{c,t}} = \frac{\vartheta^i_a}{\vartheta^i_c} \left(\frac{P^{c,i}_t}{P^{a,i}_t}\right)^{\frac{1}{1-\eta^i}}, \tag{B.34}$$

where $\bar{\tilde{x}} \in \{c, inv\}$ and $P^{j,i}_t = S^{j,i}_t P^{i,i}_t$, with $S^{i,i}_t = S^{a,b}_t = S^{b,a}_t = 1$ as described in the main text. For relative prices, this implies

$$\frac{P^i_t}{P^{a,i}_t} = \left[\vartheta^i_a + \vartheta^i_b \cdot \left(\frac{P^{b,i}_t}{P^{a,i}_t}\right)^{-\eta^i/(1-\eta^i)} + \vartheta^i_c \cdot \left(\frac{P^{c,i}_t}{P^{a,i}_t}\right)^{-\eta^i/(1-\eta^i)}\right]^{-\frac{1-\eta^i}{\eta^i}}, \tag{B.35}$$

$$\frac{P^i_t}{P^{b,i}_t} = \left[\vartheta^i_a \cdot \left(\frac{P^{a,i}_t}{P^{b,i}_t}\right)^{-\eta^i/(1-\eta^i)} + \vartheta^i_b + \vartheta^i_c \cdot \left(\frac{P^{c,i}_t}{P^{b,i}_t}\right)^{-\eta^i/(1-\eta^i)}\right]^{-\frac{1-\eta^i}{\eta^i}}, \tag{B.36}$$

and

$$\frac{P^i_t}{P^{c,i}_t} = \left[\vartheta^i_a \cdot \left(\frac{P^{a,i}_t}{P^{c,i}_t}\right)^{-\eta^i/(1-\eta^i)} + \vartheta^i_b \cdot \left(\frac{P^{b,i}_t}{P^{c,i}_t}\right)^{-\eta^i/(1-\eta^i)} + \vartheta^i_c\right]^{-\frac{1-\eta^i}{\eta^i}}. \tag{B.37}$$

For net foreign assets in regions $\tilde{i} \in \{a, b\}$, we get

$$\bar{f}^{\tilde{i}}_t = \frac{(1 + r_{t-1})}{(1 + n^{w,\tilde{i}}_{t-1})(1 + x^{\tilde{i}})} \cdot \bar{f}^{\tilde{i}}_{t-1} + \frac{P^{\tilde{i},\tilde{i}}_t}{P^{\tilde{i}}_t}\left(\bar{y}^{\tilde{i}}_t - \bar{g}^{\tilde{i}}_t\right) - \bar{c}^{\tilde{i}}_t - i\bar{n}v^{\tilde{i}}_t, \tag{B.38}$$

and

$$rs^{a,c}_t \cdot \bar{f}^a_t = - rs^{b,c}_t \cdot \frac{P^b_t}{P^a_t} \cdot \bar{f}^b_t - \frac{P^b_t}{P^a_t} \cdot \bar{f}^b_t, \tag{B.39}$$

where the latter equation is a result of the fact that net foreign assets must be in zero net supply on the world level. Product market for regions $i \in \{a, b, c\}$ clearing implies

$$D^i_t \, \bar{y}^i_t = \bar{c}^i_{i_t} + i\bar{n}v^i_{i,t} + \bar{g}^i_t + rs^{j,i}_t \cdot \left(c^j_{i,t} + inv^j_{i,t}\right) + rs^{\tilde{j},i}_t \cdot \left(c^{\tilde{j}}_{i,t} + inv^{\tilde{j}}_{i,t}\right), \tag{B.40}$$

where $j, \tilde{j} \in \{a, b, c\}$ and $i \neq j \neq \tilde{j}$.

Monetary policy reacts as described in the main text (see Section 2.7 as well as equation (28)), where inflation rates are defined accordingly. Population dynamics are given by

$$\Psi^i_t = \frac{1 - \omega^i_{t-1}}{1 + n^{w,i}_{t-1}} + \frac{\gamma^i_{t-1}}{1 + n^{w,i}_{t-1}} \Psi^i_{t-1}, \tag{B.41}$$

where population growth rates as well as retirement and survival probabilities are given by the exogenou processes

$$(1 + n_t^{w,i}) =(1 + n^{w,i}) + \epsilon_t^{n^w,i}, \tag{B.42}$$

$$\omega_t^i =\omega^i + \epsilon_t^{\omega,i} \tag{B.43}$$

and

$$\gamma_t^i =\gamma^i + \epsilon_t^{\gamma,i}. \tag{B.44}$$

Here, the ϵ's represent exogenous iid shock processes. Relative (working-age) population between regions i and j evolves according to

$$rs_t^{i,j} =\frac{(1 + n_{t-1}^{w,i})}{(1 + n_{t-1}^{w,j})} \cdot rs_{t-1}^{i,j}. \tag{B.45}$$

For the relative total population size, we have to take into account the evolution of the OADR as described in Section 2.1 of the main text.

Given the targets and the parameter values described in Section 3, it is then straightforward to numerically **derive the steady state** based on the above equation system evaluated at steady state. After some rearranging and making use of the recursiveness of the above equation system, that boils down to solving a system of 12 equations in 12 unknowns. For $i \in \{a, b, c\}$, the 12 equations we need to solve for are

$$\bar{c}^i =\bar{c}^{w,i} + \Psi^i \cdot \bar{c}^{r,i}, \tag{B.46}$$

$$\epsilon^i =\frac{1 - [\beta \cdot a\bar{u}x^i]^\sigma \cdot [(1 + r)]^{\sigma-1} \cdot \gamma^i}{1 - [\beta \cdot a\bar{u}x^i]^\sigma \cdot [(1 + r) \cdot \Omega^i]^{\sigma-1}} \tag{B.47}$$

$$\bar{l}^{w,i} =1 - \frac{\upsilon_l^i}{\upsilon_c^i} \cdot \frac{\bar{c}^{w,i}}{\bar{w}^i}, \tag{B.48}$$

$$\bar{l}^{r,i} =1 - \frac{\upsilon_l^i}{\upsilon_c^i} \cdot \frac{\bar{c}^{r,i}}{\xi^i\,\bar{w}^i}. \tag{B.49}$$

The unknowns are the world interest rate r, the net foreign asset positions $\bar{f}^a$ and $\bar{f}^b$ – where, by equation (B.39) evaluated steady state, we can also solve for $\bar{f}^c$ directly –, and the markup on the propensity to consume of retirees ϵ^i, with $i \in \{a, b, c\}$. The remaining ("unknown") parameters to solve for are ξ^i and υ_l^i, making use of $\upsilon_l^i = 1 - \upsilon_c^i - \upsilon_l^i$ for the latter one. Given the recursiveness of the system of equations (B.1) to (B.45), it is straightforward and computationally not demanding to solve for all steady-state values.

C.1 Matching the OADR with changes in survival probabilities only

As stated in the main text, one could argue that the retirement probability is a policy variable and not a choice varibale. Keeping this parameter constant, and assuming the survival probability to match the observed OADR, however, yields analogous results. This is shown in Figures B.2 to B.4, which are analogous to those in the main text. Figure B.1 shows the relevant exogenous processes that we feed in to our model to match the observed OADR.

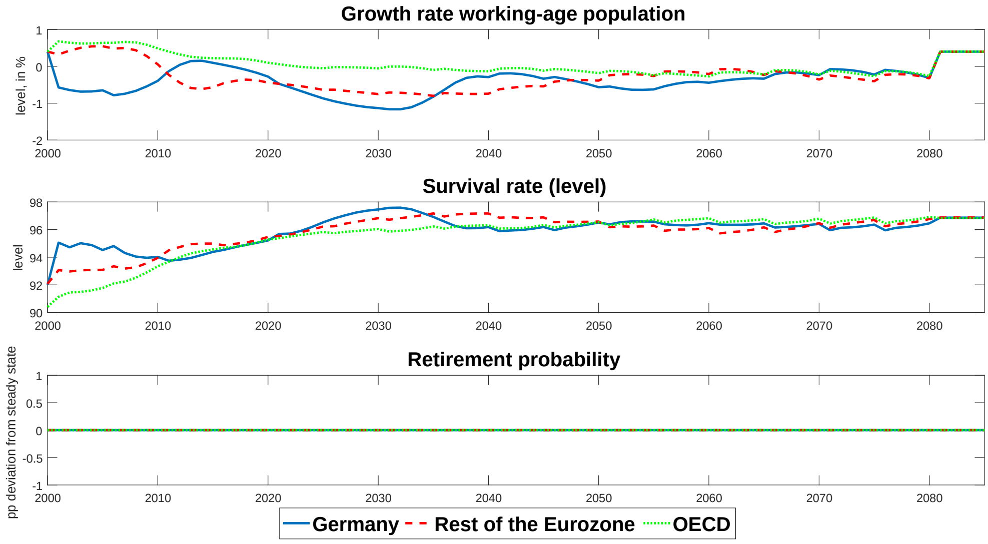

Figure B.1: Exogenous demographic processes

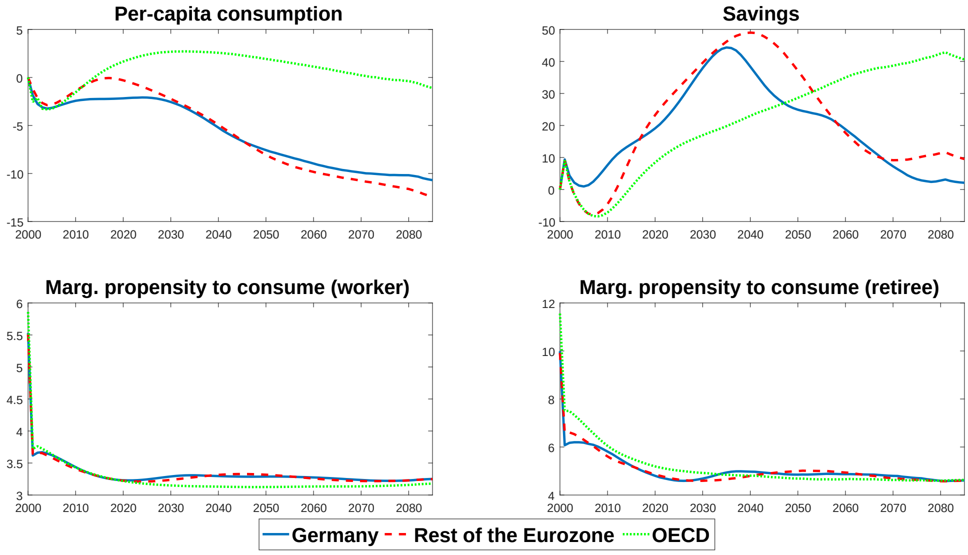

Figure B.2: Consumption and savings reactions

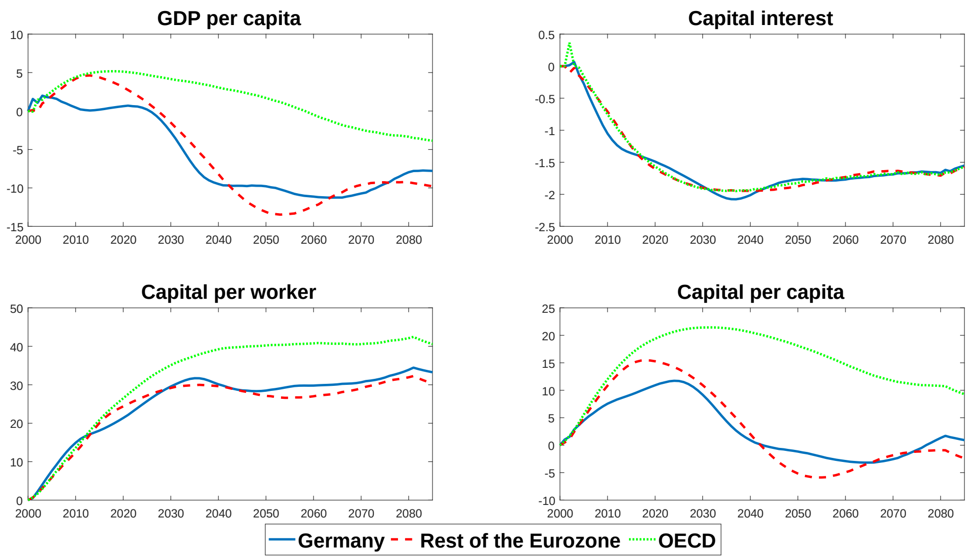

Figure B.3: Production sector reactions

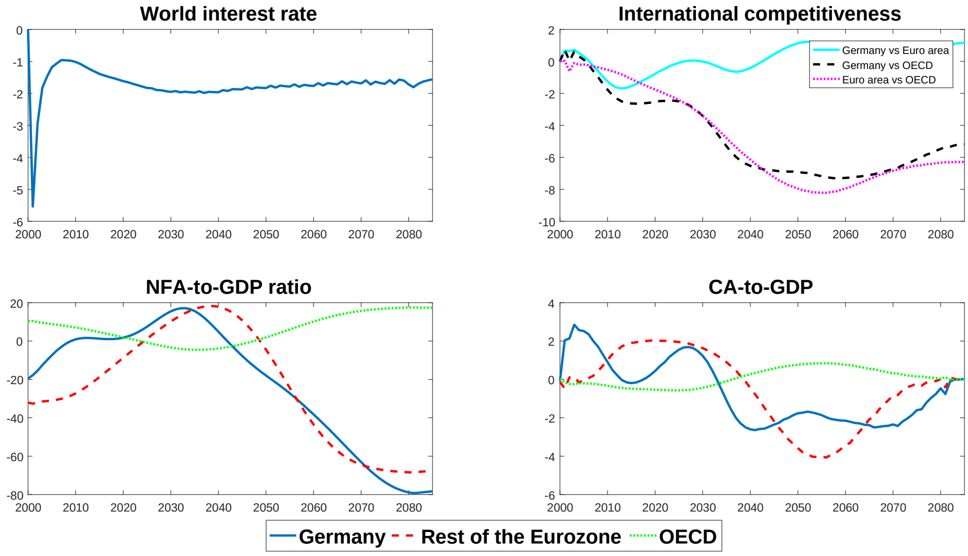

Figure B.4: International transmission