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Credit crunches from occasionally binding bank borrowing constraints

Tom D. Holden
(Deutsche Bundesbank)

Paul Levine
(University of Surrey)

Jonathan M. Swarbrick
(Bank of Canada)

Deutsche Bundesbank, Wilhelm-Epstein-Straße 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-0

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

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Non-technical summary

Research Question

During the 2007–08 financial crisis, there was a widespread credit crunch, interest rates spreads increased dramatically and real economic activity contracted. Because such events are occasional phenomena, the time series of interest spreads is very "spiky", with sharp rises occurring during downturns. These spikes in spreads are also associated with spikes in bank equity issuance. This paper investigates the model features necessary to match these features of the data.

Contribution

We show that these features of the data may be explained by a model in which financial intermediaries are borrowing constrained only some of the time. In the model, banks face both the risk of being borrowing constrained and costs of issuing equity. Financial intermediation is usually efficient, because banks can raise equity finance via retained earnings for free. Moderately large adverse shocks, however, can cause banks to become borrowing constrained. Thanks to equity issuance costs, this leads to an increase in interest spreads and a reduction in credit to the non-financial sector.

Results

The model can help interpret two stylized facts. First, as credit crunches are occasional phenomena, increases in the interest spread during recessions are much greater than the decreases occurring during booms, as observed in the data. Second, due to equity issuance costs, debt is preferred to raising equity finance under normal circumstances and banks issue new equity only when financial conditions worsen. This leads to countercyclical equity issuance, with occasional large issuances, as observed in the data, but missed in other models.

Nichttechnische Zusammenfassung

Fragestellung

Während der Finanzkrise in den Jahren 2007 und 2008 kam es zu einer breit angelegten Kreditklemme sowie zu einer deutlichen Ausweitung der Zinsspreads und einem Rückgang der realwirtschaftlichen Aktivität. Wegen der Unregelmäßigkeit solcher Ereignisse weist die Zeitreihe der Zinsspreads zahlreiche „Spitzen" und insbesondere starke Anstiege in den Abschwungphasen auf. Dieses Verlaufsmuster geht zudem mit entsprechenden Spitzen in der Emission von Bankaktien einher. Im vorliegenden Diskussionspapier wird untersucht, welche Merkmale ein Modell aufweisen muss, um die beschriebenen Eigenschaften der Daten abzubilden.

Beitrag

Es wird gezeigt, dass sich die Dateneigenschaften mithilfe eines Modells erklären lassen, in dem die Mittelaufnahme von Finanzintermediären nicht immer, sondern nur in bestimmten Zeiträumen, eingeschränkt ist. Im Modell sehen sich die Banken nicht nur dem Risiko einer beschränkten Kapitalbeschaffung gegenüber, sondern auch einer Kostenbelastung durch die Ausgabe von Aktien. Die Finanzintermediation verläuft in der Regel effizient, da Banken aufgrund der einbehaltenen Gewinne über eine kostenlose Möglichkeit verfügen, sich stärker über Eigenkapital zu refinanzieren. Moderate negative Schocks können jedoch eine Beschränkung der Refinanzierungsmöglichkeiten bewirken. In Verbindung mit den Kosten der Aktienemission ergibt sich somit eine Ausweitung der Zinsspreads und ein Rückgang der Kreditvergabe an den nichtfinanziellen Sektor.

Ergebnisse

Das Modell trägt zur Interpretation zweier stilisierter Fakten bei. Erstens: Kreditklemmen sind unregelmäßig auftretende Ereignisse, und aus den Daten geht hervor, dass sich der Zinsspread in Rezessionsphasen erheblich stärker ausweitet, als er sich in Boomphasen verringert. Zweitens: Aufgrund der mit Aktienemissionen einhergehenden Kosten bevorzugen die Banken unter normalen Bedingungen die Aufnahme von Schulden gegenüber einer Refinanzierung über Eigenkapital und geben nur dann neue Aktien aus, wenn sich die Refinanzierungsbedingungen verschlechtern. Aus den Daten geht hervor, dass dies zu einem antizyklischen Verlauf der Aktienemissionen mit gelegentlich hohen Volumina führt, die in anderen Modellen übersehen werden.

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Tom D. Holden, Paul Levine and Jonathan M. Swarbrick*†

Abstract

We present a model in which banks and other financial intermediaries face both occasionally binding borrowing constraints, and costs of equity issuance. Near the steady state, these intermediaries can raise equity finance at no cost through retained earnings. However, even moderately large shocks cause their borrowing constraints to bind, leading to contractions in credit offered to firms, and requiring the intermediaries to raise further funds by paying the cost to issue equity. This leads to the occasional sharp increases in interest spreads and the counter-cyclical, positively skewed equity issuance that are characteristic of the credit crunches observed in the data.

Keywords: Occasionally binding constraints; Credit crunches; Financial crises; Spreads; Dividends; Equity; Banking

JEL classification: E22, E32, E51, G2.

*Tom D. Holden is Researcher in the Research Department, Deutsche Bundesbank (thomas.holden@gmail.com). Paul Levine is a Professor of Economics in the School of Economics, University of Surrey (p.levine@surrey.ac.uk). Jonathan M. Swarbrick is a Senior Economist in the Canadian Economic Analysis department, Bank of Canada (jswarbrick@bankofcanada.ca).

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1 INTRODUCTION

Economic downturns are usually accompanied by sharp increases in interest spreads as the effects of financial frictions worsen. This is particularly true during banking crises when the financing costs faced by intermediaries rise dramatically.[1] In this paper, we present a model in which financial intermediaries face occasionally binding borrowing constraints that cause spreads to rise when the value of assets declines sufficiently, thanks to the costs these intermediaries face in issuing new equity. The increased spread between the savings rate and the return on capital implies a drop in the marginal efficiency of investment, generating declines in aggregate investment relative to the efficient benchmark, and introducing asymmetries in macroeconomic time series. However, in our model, in the vicinity of the steady state, financial constraints are slack and financial intermediation is efficient. This allows for the characterization of *normal times* and *credit crunches*. Our model differs importantly from the existing literature in this dimension. Whereas the presence of occasionally binding constraints usually depends on calibration,[2] in our model, borrowing constraints are always occasionally binding, essentially irrespective of parameter values. This holds since financial intermediaries, henceforth simply known as "banks," choose to borrow to the edge of the constrained region.[3]

The model is in the spirit of the banking model proposed in Gertler and Kiyotaki (2010) (henceforth GK), but while these authors prevent equity issuance to ensure banks are always financially constrained,[4] endogenous dividend payments and equity issuance costs in our model imply that the financial constraint is only occasionally binding. In order to raise funds when further debt finance is unavailable, banks can reduce dividend payments and use retained profits for free. However, if banks are unable to raise sufficient funds via retained earnings, they are restricted to costly equity issuance. This introduces a spread between the risk-free saving rate, and the risky return to capital, often described as an investment or

[1] See Babihuga and Spaltro (2014) for a discussion of bank funding costs during the 2007–08 financial crisis.

[2] Compare, for example, Gertler and Kiyotaki (2010) with Bocola (2016). The constraint is always binding in the former but only binds occasionally in the latter due to different calibrations of banking sector parameters.

[3] As discussed below, we consider this to be appealing feature given that financial crises occur across many countries and under a range of banking regulations.

[4] There is an extension discussed in GK, pursued further in Gertler, Kiyotaki and Queralto (2012) (henceforth GKQ), that introduces bank equity issuance by extending the same agency problem in debt finance to equity finance by differentiating between inside and outside shareholders. But in doing so, the set-up generates counter-factual dynamics with respect to equity issuance. Specifically, equity issuance is procyclical whereas the data indicate that this is countercyclical.

capital wedge.[5] Because the investment wedge appears only during downturns, the effects of the financial friction are inherently asymmetric. This enables our model to better explain a number of key facts as compared with other models such as GK. In particular, we are able to match the large positive skewness in spreads and to provide an explanation for the observation that crises are occasional phenomena during which the adverse effects of financial frictions worsen significantly. Furthermore, because it is desirable to issue equity only when all other sources of finance are exhausted, bank equity issuance is strongly countercyclical, consistent with the data,[6] but missed in other models of bank equity issuance, such as GKQ.[7] Additionally, modelling occasionally binding financial constraints eliminates the financial accelerator mechanism during normal times, in line with the evidence that models without a financial accelerator perform better in normal times (Del Negro, Hasegawa and Schorfheide, 2016); in our model, only during sufficiently deep downturns do the financial constraints bind, further amplifying the recession. However, once allowing banks to choose the level of dividend payments and issue new equity, we find that the financial accelerator mechanism is significantly dampened, even during crisis episodes.

As well as modelling the financial structure of banks more realistically, we improve upon the GK agency problem. The GK borrowing constraint emerges due to limited contract enforceability; banks have an outside option to divert assets and declare bankruptcy. By parameterizing the proportion of assets that can be reclaimed by creditors, the authors set the outside option to a fixed amount of the current value of bank assets. In our model, we carefully specify off-equilibrium play and use U.S. bankruptcy law to implement the amount recoverable by creditors. In particular, whereas GK place timing restrictions on when banks can choose to default in order to prevent banks making a large, unrecoverable dividend at the end of one period before defaulting in the next, the restriction is not required in our approach as, according to U.S. bankruptcy law, the amount paid out would also be liable to be reclaimed by the courts. This mechanism also gives an additional motive for dividend payments; since recent dividend payments are reclaimable during bankruptcy, dividend payments act to relax the present and future borrowing constraint, and consequently can sometimes be paid even if the bank is issuing equity, helping to explain a long-discussed puzzle (see, e.g., Myers,

[5]See Chari, Kehoe and McGrattan (2007) as an example of the former, and Hall (2010) of the latter.

[6]It is widely accepted that equity issuance by most non-financial firms is procyclical; however, recent studies have shown that bank equity issuance is countercyclical (see, e.g., Baron, 2017).

[7]As mentioned in footnote 4. The introduction of differing costs of equity and debt is similar to that proposed in Jermann and Quadrini (2012) who include tax benefits of debt finance; however, in our model, the tightness of the borrowing constraint is endogenous, and only occasionally binding.

1984; Loderer and Mauer, 1992; Fama and French, 2005). In particular, while the borrowing constraint in GK is given by:

$$V_t^j \geq \theta A_t^j,$$

for bank j where V_t^j is the value of bank equity and A_t^j the value of bank assets, in our model, this becomes:

$$V_t^j \geq \theta_{1,t}^j A_t^j - \theta_{2,t} \bar{D}_{t-1}^j,$$

where $\bar{D}_{t-1}^j$ is a weighted average of previous, reclaimable divided payments. This captures both a time-varying, bank-specific, weight on bank assets, and the role of past dividends in relaxing the constraint.[8]

We compare our model both with the standard real business cycle (henceforth RBC) model, which provides an efficient benchmark, and with the always-binding borrowing constraints model of GK with the equity issuance extension. In their model, to ensure that the borrowing constraint is always binding, bankers exit with a fixed probability. This is set to 2.5 percent per quarter and is described as a turnover between workers and bankers. However, as this is treated as a payment to the representative household, it is equivalent to a fixed dividend rate, which, at 10 percent per annum, seems implausibly high.[9] This high dividend payment rate ensures debt is always the cheapest source of finance in GK, which, in combination with the calibration of the proportion of divertable assets, implies the borrowing constraint is always binding. By contrast, in our model, the borrowing constraint binds when demand for funds increases without an equivalent rise in the value of future discounted dividends. This can occur following an adverse supply-side shock to capital, which increases the demand for investment. Such a shock implies a reduction in the bank's future profit stream and so an increase in the marginal value of the bank cashing-out, i.e., diverting assets and defaulting.

We examine model dynamics in the presence of investment adjustment costs and capital quality shocks, which, following GK, may be thought of as modelling the economic obsolescence of capital, rather than its physical destruction. This introduces an exogenous variation to the value of capital. As a source of occasional disasters, this shock is particularly relevant given the events of late 2007 in the U.S., when a huge amount of value was knocked off bank assets, leading to the banking crisis.

We differ from the GK set-up with the use of the household preferences proposed in

[8] See Appendix A for further details.

[9] Between 1965 and 2013, dividend payments made by the largest 20 U.S. banks averaged 5.15 percent (using the data set constructed in Baron, 2017).

Jaimovich and Rebelo (2009). This allows for the parameterization of the strength of the short-run wealth effect on labour supply. By choosing a weak wealth effect, positive (negative) news about the future can generate a rise (fall) in labour supply, so producing co-movement in consumption and investment following capital quality shocks.[10] Furthermore, a small short-run wealth effect can be motivated by the observation that a large proportion of households have very little or no net wealth, with just a small few owning a disproportionate share of total wealth (see, e.g., Mankiw, 2000).[11]

In the remainder of the article, we discuss the support for the chosen model of financial constraints before briefly outlining the related literature on financial frictions and occasionally binding financial constraints. We then proceed to describe in detail the derivation of equilibrium conditions that characterize the behaviour of the economy and discuss some key analytical results. We end with a discussion of the main numerical results.

1.1 Our model of financial constraints

In order to generate crisis periods, our model must feature an aggregate occasionally binding constraint. We argue that the most appropriate location for this occasionally binding constraint is on debt finance, since under normal circumstances, debt is preferred to equity due to equity issuance costs. Prior to the financial crisis of 2007–08, the banking system had built up a reliance on short-term debt finance.[12] Following the bursting of the U.S. subprime mortgage bubble, there was a sharp contraction in the money markets cumulating in the collapse of the shadow banking system. While debt finance had been relatively unconstrained prior to the financial crises, bank borrowing constraints began to bind as the value of assets plummeted.

In a study of U.S. commercial banks between 1925 and 2012, Baron (2017) finds that bank equity issuance has been countercyclical. This observation seems self-evident in Figure 1, which plots new equity issuance for the largest U.S. commercial banks since the Great Depression.[13] The implication is that banks switch from

[10] We analyze various utility functions, forms of investment and capital adjustment costs, and habits in consumption and leisure, finding that the key results are unchanged.

[11] GKQ employ GHH preferences that are quantitatively very similar to our model but inconsistent with balanced growth.

[12] This issue is discussed at length in Shin (2009); explaining the financial crisis as a bank run, the author highlights the rising importance of alternative sources of debt finance such as money market funds.

[13] Data as described in Baron (2017) and kindly provided by the author. New equity issuance is derived from bank level net issuances, adjusting for dilutions and stock splits. Following Jagannathan, Stephens and Weisbach (2000), net issuances are decomposed onto new issuance

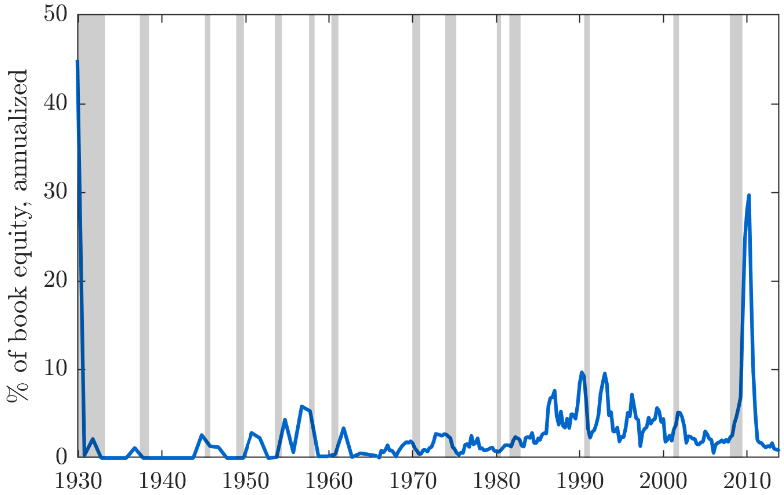

Figure 1: New equity issuance aggregated over the 20 largest U.S. commercial banks with NBER recession bands.

debt finance to equity finance during periods of financial stress as the marginal value of finance rises higher than equity issuance costs. In our model, this occurs when bank borrowing constraints tighten. In another empirical study of bank equity, Black, Floros and Sengupta (2016) find that both private sources of equity finance and government programs (e.g., TARP in the U.S.) were important during and after the financial crisis and that liquidity needs were an important factor, particularly for larger banks. Begenau, Bigio and Majerovitz (2017) present a discussion of the institutional context and provide evidence that banks seek to meet a leverage target but face equity adjustment costs. The authors argue that before the crisis, banks adjusted assets in order to de-lever whereas raised equity finance after, either through issuances or retained earnings.

One dimension in which debt finance is preferable to equity is due to a tax advantage (see, e.g., Jermann and Quadrini, 2012), but a number of studies have also estimated the transaction costs associated with equity issuances (e.g., underwriter fees, legal costs). These estimates lie between 5 and 7 percent on average and fall in the size of offering (see Lee et al., 1996; Altinkiliç and Hansen, 2000; Hennessy and Whited, 2007). As well as these explicit costs, raising equity finance is plagued by agency problems (see, e.g., Jensen and Meckling, 1976; Myers and Ma-

= max(net issuance, 0) and repurchases = min(net issuance, 0). Baron (2017) hand-collects the 1930–1965 data from Moody's Bank and Finance Manuals for the largest 15 U.S. banks and takes 1965–2014 data from Compustat for the 20 largest banks.

jluf, 1984; Miller and Rock, 1985; Asquith and Mullins, 1986).[14] These frictions result in implicit issuance costs that can be estimated by observing the change in share price following an offering. These observed declines in value have been estimated to be anything between 0.4 and 9.9 percent following offerings (Jensen, 1986) with a mid-point of around 3 percent (Mann and Sicherman, 1991; Altinkiliç and Hansen, 2003). Furthermore, whereas the transactional costs fall in the size of issuance, the implicit costs have been found to rise. Altinkiliç and Hansen (2000) find evidence in support of U-shaped total implicit and explicit issuance fees; the initial decline driven by falling transactional fees, and the subsequent rise due to the agency frictions. In this paper, the borrowing constraint is endogenous and equity issuance costs are exogenously imposed. Following Altinkiliç and Hansen, these costs increase in aggregate equity issuance, acting as a congestion charge. This can be motivated by increases in agency costs following a large cross-sector equity issuance due, for instance, to costly monitoring and downward pressure on the issuance price as the market is flooded with new equity. Black, Floros and Sengupta (2016) verify this increase in the cost of issuing equity; studying the role of bank equity finance between 2001 and 2014, the authors find that capital constraints were tighter during the crisis episode, despite the larger-than-normal issuances.

1.2 Related literature

A starting point for the model is the agency problem proposed in Kiyotaki and Moore (1997) and extended in the GK banking model. The authors introduce limited contract enforceability on bank borrowing that results in a financial friction between banks and households. It is assumed that banks can default on their debts and exit the market, so, as the courts can only reclaim a proportion of outstanding debts, endogenous borrowing limits arise. However, unlike other models of financial frictions, such as that of Bernanke, Gertler and Gilchrist (1999), there is no default in equilibrium, since households will only loan to a bank that has no incentive to default. This constraint on debt introduces a wedge between the risk-free rate and the expected discounted return on capital that fluctuates due to movements in the value of bank assets. Related papers include: Gertler and Karadi (2011) who assess the role of unconventional monetary in the GK framework; Gertler, Kiyotaki and Queralto (2012) who build on GK to differentiate between outside and inside bank equity and study the role of credit policies; and Gerali, Neri, Sessa and Signoretti (2010) who estimate a model with monopolistic competition in a banking sector using euro area data, finding that banking sector shocks were more important than other macroeconomic shocks (see also Iacoviello, 2015). Further papers studying

[14] For example, costs resulting from asymmetric information leading to principle-agent problems and the implied dilution of current shareholders' value.

the relationship between bank leverage and macroeconomic outcomes include Chen (2001), Meh and Moran (2010) and Kiley and Sim (2014).

There is a growing literature looking at models with occasionally binding financial constraints. For instance, He and Krishnamurthy (2013) propose an occasionally binding constraint on equity, rather than on debt, in which interest premia rise sharply when the constraint binds, deepening downturns. In related work, Brunnermeier and Sannikov (2014) propose a model of constrained equity issuance that leads to non-linear dynamics; most fluctuations can be absorbed by the intermediaries' balance sheets but larger negative shocks might lead to unstable, volatile episodes. The evidence, however, indicates that debt, rather than equity, is subject to occasionally binding constraints (see, e.g., Kashyap and Stein, 2000; Calomiris and Mason, 2003; Ivashina and Scharfstein, 2010). Because both He and Krishnamurthy, and Brunnermeier and Sannikov impose constraints on the issuance of equity rather than debt, when these constraints tighten during crises, leverage increases and intermediaries reduce rather than increase equity issuance. This runs counter to the empirical evidence on the cyclical properties of bank equity issuance, discussed in the previous section, and evidence that bank leverage is pro-cyclical. While the empirical cyclical properties of leverage depend on factors such as the institution type and whether the leverage is calculated from banks' market or book value, using book value calculations Nuño and Thomas (2017) provide evidence that leverage is pro-cyclical across different types of financial institutions (see also Adrian and Shin, 2010). In our model, equity issuance is counter-cyclical and bank leverage mildly pro-cyclical as intermediaries de-lever following a credit tightening.

In other models of occasionally binding financial constraints, Akinci and Queralto (2014) and Bocola (2016) present occasionally binding extensions to GK, the latter to study the pass-through of sovereign risk. In both these papers, whether the constraint is occasionally binding depends crucially on model calibration, unlike our model. By removing the exogenous bank exit common to these studies, and allowing them to choose dividend payments, banks will borrow to the edge of the constraint, but can always raise equity finance for free in the vicinity of the steady state. It follows that credit crunches are occasional phenomena, in contrast to Akinci and Queralto, and Bocola, where it is implied that banks are constrained in the steady state. In related work, Jermann and Quadrini (2012) present a model that differentiates between the costs of debt and equity finance. This results in countercyclical equity issuance via a similar mechanism to our model, however, as with GK, the financial constraint is always binding and not subject to the endogenous variation that our model implies.

2 THE MODEL

The model features a household and firm sector common to the real business cycle literature, with the banking sector acting to intermediate funds between these two sectors.

2.1 Households

The representative household maximizes expected lifetime utility:

$$\max_{C_{t+s},H_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} U\left(C_{t+s}, H_{t+s}, X_{t+s}\right) \tag{1}$$

subject to the budget constraint:

$$C_t + B_t = W_t H_t + R_{t-1} B_{t-1} + D_t - E_t + \Pi_t - T_t, \tag{2}$$

where C_t is consumption, H_t is hours worked, X_t is a habit stock, W_t is the wage rate, and B_t is deposits with the bank that pay interest rate R_t in the following period. D_t and Π_t are dividends paid and any other profits, respectively; E_t is bank equity purchased; and T_t represents lump-sum taxes. We assume that households cannot lend directly to firms, so the intermediation provided by banks is necessary to provide funding to firms.

To achieve co-movement between investment and consumption, we employ the preferences proposed in Jaimovich and Rebelo (2009), which allow for the control of the short-run wealth effect on labour supply. In particular, we suppose that the period utility takes the form:

$$U\left(C_t, H_t, X_t\right) = \frac{\left[C_t - \varrho H_t^{1+\psi} X_t\right]^{1-\sigma_c} - 1}{1-\sigma_c}, \tag{3}$$

where:

$$X_t = C_t^{\gamma} X_{t-1}^{1-\gamma}, \tag{4}$$

where $\sigma_c > 0$ is the intertemporal elasticity of substitution, $\varrho > 0$ is the utility weight on leisure, $\psi > 0$ controls the elasticity of labour supply, and $0 < \gamma \leq 1$ controls the wealth effect. When $\gamma = 0$, the preferences are equivalent to those of Greenwood, Hercowitz and Huffman (1988) (GHH) with no wealth effect on labour supply.[15]

[15] Jaimovich and Rebelo (2009) preferences benefit from being compatible with balanced

Household optimization leads to the following Euler equation and labour supply condition:

$$1 = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \right] R_t \tag{5}$$

$$-\frac{U_{H,t}}{\lambda_t} = W_t, \tag{6}$$

where $U_{H,t}$ is the marginal utility of labour, and λ_t^C is the Lagrange multiplier on the household budget constraint, i.e., the marginal value of income. λ_t^C is given by:

$$\lambda_t^C = U_{C,t} + \gamma \mu_t \frac{X_t}{C_t}, \tag{7}$$

where μ_t is the Lagrange multiplier on equation (4), which is given by:

$$\mu_t = U_{X,t} + \beta (1 - \gamma) \mathbb{E}_t \left[\mu_{t+1} \frac{X_{t+1}}{X_t} \right]. \tag{8}$$

2.2 The banking sector

Banks in the model are owned by households. As a result, they maximize their expected value, i.e., the expected present discounted value of net dividend payments, $D_t - E_t$. In treating equity issuance as a negative dividend payment, we are following, for example, Miller and Rock (1985). However, raising equity financing from households will be costly.

The relationship between banks and households is subject to an agency problem, which arises due to imperfect contract enforcement; banks are able to declare bankruptcy and exit with creditors able to reclaim only a proportion of the outstanding debt. This follows the collateral constraints model of Kiyotaki and Moore (1997), and more closely the extension to the banking sector by GK. However, whereas GK assume an exogenous bank exit rate that fixes the dividend rate and ensures the borrowing constraint is always binding, our model relaxes this assumption so that net dividend payments are endogenous and the borrowing constraint is only occasionally binding. While it is possible to parameterize the GK model to produce an occasionally binding borrowing constraint, the range of parameters for which this is true is narrow. We consider this to be a drawback of the GK approach as financial crises, and sharp increases in spreads more generally, occur across many different countries and times, with different policies towards banks

growth, unlike the GHH preferences used in GKQ, which are not.

and bankruptcy. If crises only occurred in a narrow range of the parameter space, then one would think it unlikely that we would observe spikes in spreads across such a wide range of situations. Our approach avoids this problem, as, endogenously, in the steady state the bank will always be just on the edge of the constraint binding, irrespective of parameters. We also give the derivation of the borrowing constraint a more careful treatment, based on U.S. bankruptcy law.

Bank j raises debt finance B_t^j promising to repay $R_t^j B_t^j$ the following period. The bank will pay dividends D_t^j and raise equity E_t^j. While making dividend payments is costless, we assume there are transactional costs involved in issuing equity. To bank j the cost is exogenous and linear in equity issuance, being equal to $\kappa_t E_t^j$. However, we model κ_t as an increasing function of aggregate equity issuance. Assuming that costs increase in aggregate equity follows empirical evidence (see Altinkiliç and Hansen, 2000) and can be motivated by increases in agency costs following a large cross-sector equity issuance due, for instance, to costly monitoring and downward pressure on the issuance price as the market is flooded with new equity.[16] Specifically, we let:

$$\kappa_t \equiv \bar{\kappa}\left[1 - \exp\left(-\nu \frac{E_t}{V_t}\right)\right], \tag{9}$$

where V_t is the value of the entire banking sector, so E_t/V_t is the aggregate rate of equity issuance, and where $\bar{\kappa} \in (0,1)$ gives the maximum cost of equity issuance and ν is a parameter that determines the velocity at which κ_t converges to $\bar{\kappa}$.

Banks raise debt and equity finance in order to lend to the production sector. The lending channel is characterized by perfect monitoring and perfect contractual enforcement. Therefore, banks frictionlessly lend to firms against their future profits, and firms offer banks fully state-contingent debt, or, equivalently, equity. We denote by S_t^j the number of firm shares held by bank j at t, and we assume that each share delivers a gross return of R_t^K per unit. We will normalize the units of these shares such that one share entitles the owner to the gross return from the ownership of one unit of capital.

The book value of bank j at time t is given by:

$$\hat{V}_t^j \equiv \left[R_t^K S_{t-1}^j - R_{t-1} B_{t-1}^j\right] \frac{1}{1-\kappa_t}. \tag{10}$$

[16] While fixed equity costs may seem an appealing choice, if these are high, equity is never issued, but if very low, the financial friction is dampened significantly. Costs that are increasing in aggregate issuance allow us to better fit the observed data on interest spreads and equity issuance.

To interpret the book value, it is the cost that households would have to pay in order to create a "copy" of bank j. Were equity issuance impossible (i.e., were $\kappa_t = 1$), then creating a "copy" of a bank with positive net worth would be impossible, or infinitely expensive. We assume that once equity is in the banking system, it may be transferred between banks without incurring additional costs. Thus $\hat{V}_t^j$ is also the maximum amount that another bank would be prepared to pay in order to purchase bank j. As such, $\hat{V}_t^j$ gives a "cash-out" value of the bank.[17]

A bank that decides not to exit next period will face the budget constraint:

$$D_t^j + S_t^j + R_{t-1} B_{t-1}^j \leq B_t^j + (1 - \kappa_t) E_t^j + R_t^K S_{t-1}^j. \tag{11}$$

The objective of bank j is to maximize its expected value. Additionally, we suppose that while the household is indifferent between dividends being paid today or in future, the bank has a preference toward paying dividends now. This may capture agency problems within the bank that lead to an excess focus on short-term returns, or it may reflect a remote fear of forced nationalization. In particular, the bank solves:

$$V_t^j = \max_{B_t^j, S_t^j, E_t^j, D_t^j} \left\{ D_t^j - E_t^j + (1 - \iota) \mathbb{E}_t \left[\Lambda_{t,t+1} V_{t+1}^j \right] \right\}, \tag{12}$$

subject to the budget constraint (11) and the borrowing constraint, which is still to be derived, for $\iota \to 0^+$, where $\Lambda_{t,t+1} \equiv \beta \lambda_{t+1}^C / \lambda_t^C$ is the stochastic discount factor of the shareholders and V_t^j is the value of the bank. The term $(1 - \iota)$ is superficially similar to the exogenous bank exit rate in GK but, since preferences are under the limit as $\iota \to 0^+$, its only impact is to capture banks' arbitrarily weak preference toward paying dividends sooner rather than later.[18] If the (arbitrarily small) additional discounting is interpreted as an idiosyncratic bank exit shock, then a crucial difference between our approach and that of GK is that whereas the owners of our banks do not gain any value after the exit shock (e.g., because the bank has been forcibly nationalized), in GK, dividends are paid only after the bank is hit with such a shock.

2.3 Bank exit and default

We now consider the default decision and other aspects of off-equilibrium play that are nonetheless critical for equilibrium outcomes. If bank j fails to repay

[17] In equilibrium, there is no such transfer of equity between banks. However, making this assumption is necessary to determine the value of exit, and is implicitly assumed in GK.

[18] $\iota > 0$ is required by our numerical strategy, as we take a perturbation approximation around the deterministic steady state, which would otherwise be indeterminate. Subject to numerical accuracy limits though, ι may be set arbitrarily small. This is discussed further in section 4.

outstanding debts in period t, any remaining assets are seized and sold at market value. If this is enough to repay $R_{t-1}B^j_{t-1}$, any remaining assets are paid to shareholders as a final dividend; otherwise, the court can attempt to recover previously paid dividends plus interest.[19] It is assumed that recovering payments is a costly process due, for instance, to costs associated with tracking down shareholders, and so the court is able to recover only a fraction $(1-\theta)$ of the total amount sought, where $\theta \in (0,1)$. If the amount recovered is sufficient to cover $R_{t-1}B^j_{t-1}$ then any remaining funds are returned to shareholders; otherwise, the creditors take a haircut. We assume that the amount of past dividends that can be partially recovered, $\bar{D}^j_t$, follows the law of motion:

$$\bar{D}^j_t = \left(\rho \bar{D}^j_{t-1} + D^j_t\right) R_t \tag{13}$$

This captures the idea that dividend payments made longer ago are more costly to recover. Parameter ρ can be calibrated so that 2-years worth of dividend payments can be recovered on average, in-line with U.S. law.[20]

Bank exit can occur either by deciding in period t to exit the same period or via planning a future default. If existing assets are greater than liabilities, unplanned exit will occur only if the value of continuing $V^j_t < 0$, in which case the bank exits without default. If the bank is insolvent and defaults, the maximum creditors can reclaim from previous dividends is

$$(1-\theta)\,\bar{D}^j_{t-1}. \tag{14}$$

It follows that bank j will only exit if $V^j_t < -(1-\theta)\,\bar{D}^j_{t-1}$. If this occurs for bank j on the equilibrium path, then, by symmetry, all banks will default. However, we find that the probability of all banks wishing to exit is extremely low in our calibration.[21]

We now move on to consider whether in period t a bank might like to plan to

[19] Following U.S. law surrounding chapter 7 bankruptcy (title 11 U.S.C. §548), if the value of a bank's liabilities were greater than the value of its assets at the point of payment, or the bank had "unreasonably small capital" when a dividend was paid in the two years prior to bankruptcy, then the dividend would be deemed fraudulent. Following the legal definition of "unreasonably small capital", payments would be considered fraudulent if it later transpired the firm was left with insufficient capital to repay creditors, in which case the court is able to recover dividend payments plus interest owed even if the bank was not strictly insolvent when the payment was made. See Wittstein and Douglas (2014) for further discussion.

[20] Allowing this decay rather than an exact 2-year cut-off for claims again past dividend payments makes little difference to the numerical results (w **?**)hich follows this alternative approach]Holden2017see.

[21] Nonetheless, it may be ruled out completely by a conditional government guarantee that we describe in more detail in Appendix B.

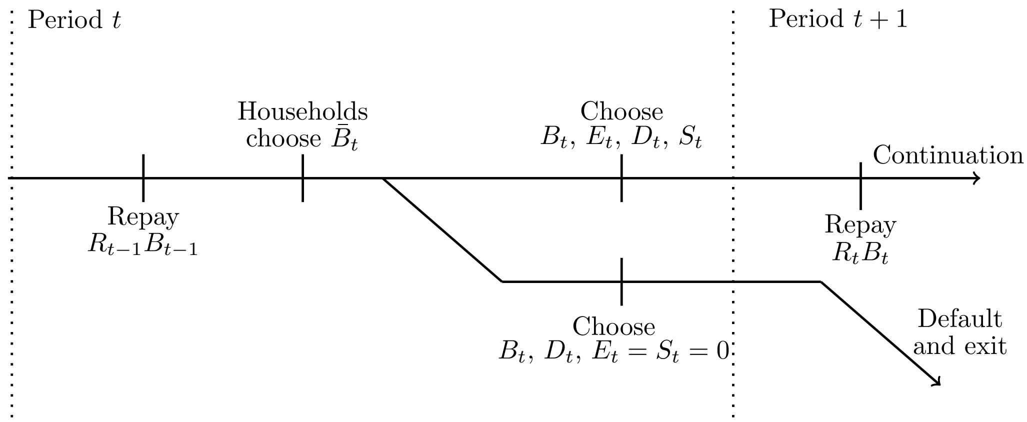

Figure 2: Timing of bank decisions and planned exit.

default in period $t + 1$. The timing is shown in Figure 2. Although government insurance can prevent unplanned default due to tail shock realizations, this is not sufficient to rule out defaults in which a bank deviates from the equilibrium path in advance of their eventual default. It is to avoid such planned defaults that households will restrict their lending to banks, leading to the borrowing constraint.

At this point, it is important to clarify the order of moves so as to correctly specify this off-equilibrium play. In particular, we assume that households observe all bank and aggregate variables from $t - 1$ but only the period t aggregate shocks before choosing the maximum amount they are prepared to deposit at the bank, $\bar{B}_t$, in period t. The bank then chooses its individual variables subject to the implied borrowing constraint. The choice of this ordering is important; if households could observe bank behaviour in advance of borrowing decisions, then they would not lend to any bank that took an off-equilibrium action, as this would be interpreted as a preparation for default.

Now, the value of bank j at time t of preparing to default in $t + 1$ is given by:

$$V_t^X = D_t^j - E_t^j - (1 - \iota)\, \mathbb{E}_t \left[\Lambda_{t,t+1}\right] (1 - \theta)\, \bar{D}_t^j, \tag{15}$$

Suppressing the bank indices for neatness, using a guess-and-verify approach, we postulate that the borrowing constraint takes the form:

$$B_t \leq \mathcal{A}_t \hat{V}_t + \mathcal{F}_t \bar{D}_{t-1}, \tag{16}$$

for some values independent of the decisions of the bank in question, $\mathcal{A}$ and $\mathcal{F}_t$. A linear borrowing constraint follows from the linearity of the budget constraint and a conjecture that the solution to the bank problem is linear in the state. The household will choose the limit on B_t so that the bank weakly prefers not to deviate from the equilibrium path by planning to default. Maximizing the value of exit subject to the borrowing constraint and the budget constraint implies that the borrowing constraint will bind, the bank will make no further investments (i.e., $S_t = 0$), and will issue no equity (i.e., $E_t = 0$). We write the value function:

$$V_t = \mathcal{M}_t \hat{V}_t + \mathcal{N}_t \bar{D}_{t-1}, \tag{17}$$

for some values independent of the decisions of the bank in question, $\mathcal{M}_t$ and $\mathcal{N}_t$.

To prevent default, the household must ensure that $V_t \geq V_t^X$. The weakest condition ensuring this implies:

$$\mathcal{A}_t = \frac{\mathcal{M}_t}{1-(1-\iota)(1-\theta)} - (1-\kappa_t), \tag{18}$$

$$\mathcal{F}_t = \frac{\mathcal{N}_t + \rho(1-\iota)(1-\theta)}{1-(1-\iota)(1-\theta)}. \tag{19}$$

The bank maximizes objective (12) subject to the borrowing constraint (16), the budget constraint (11), and positivity constraints on D_t and E_t, where the value and book value of the bank are given by equations (17) and (10), respectively. By taking first-order conditions, substituting these first-order conditions back into the problem's Lagrangian and then matching the terms in each state variable, we arrive at:

$$(1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\tfrac{1-\kappa_t}{1-\kappa_{t+1}}\tfrac{\mathcal{M}_{t+1}}{\mathcal{M}_t}R_t\right] = \left(1-\tfrac{\lambda_t^B}{(1-\kappa_t)(1-(1-\iota)(1-\theta))}\right), \tag{20}$$

$$\mathcal{N}_t = \frac{\rho(1-\iota)\left(\lambda_t^B(1-\theta)+(1-\kappa_t)\,\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right]\left[1-(1-\iota)(1-\theta)\right]\right)}{(1-\kappa_t)(1-(1-\iota)(1-\theta))-\lambda_t^B}, \tag{21}$$

where λ_t^B is the Lagrange multiplier on the borrowing constraint. The first condition gives the law of motion for the marginal value of the bank book value; the second for the marginal value of past dividend payments. Defining:

$$\mathcal{H}_t \equiv \lambda_t^B + \mathcal{M}_t\left(1-\frac{\lambda_t^B}{(1-\kappa_t)(1-(1-\iota)(1-\theta))}\right), \tag{22}$$

and:

$$\Xi_{t,t+1} \equiv (1-\iota)\Lambda_{t,t+1}\frac{1-\kappa_t}{1-\kappa_{t+1}}\frac{\mathcal{M}_{t+1}}{\mathcal{H}_t}, \tag{23}$$

equation (20) and the first-order conditions for dividends, equity and shares can be written as:

$$\lambda_t^B = \mathcal{H}_t \left(1 - \mathbb{E}_t \left[\Xi_{t,t+1} R_t\right]\right) \geq 0, \tag{24}$$

$$\lambda_t^D = \mathcal{H}_t - (1 - \iota)\left(1 - \kappa_t\right) \mathbb{E}_t \left[\Lambda_{t,t+1} \mathcal{N}_{t+1} R_t\right] - \left(1 - \kappa_t\right) \geq 0, \tag{25}$$

$$\lambda_t^E = 1 - \mathcal{H}_t \geq 0, \tag{26}$$

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} R_{t+1}^K\right], \tag{27}$$

where λ_t^D and λ_t^E are the Lagrange multipliers on the positivity constraints on dividend payments and equity issuance, respectively. The final equation implies that $\Xi_{t,t+1}$ is the pricing kernel (or stochastic discount factor) for firm equity.

2.4 Firms

The final good is produced by a perfectly competitive industry with access to the technology:

$$Y_t = \left(A_t H_t\right)^{1-\alpha} {K_{t-1}}^{\alpha}, \tag{28}$$

where A_t is a stationary stochastic process. Firms producing the final good choose the amount of labour, H_t, and capital, K_{t-1}, to hire in order to maximize their profits, which are given by $Y_t - W_t H_t - Z_t K_{t-1}$, where Z_t is the rental rate of capital. Hence, from the first-order conditions, we have the usual marginal product conditions:

$$W_t = (1 - \alpha)\frac{Y_t}{H_t}, \tag{29}$$

$$Z_t = \alpha \frac{Y_t}{K_{t-1}}. \tag{30}$$

The capital stock is owned by firms in a perfectly competitive industry with access to the following technology for producing the next period's installed capital from investment and the previous period's capital:

$$K_t = \left[1 - \Phi\left(\frac{I_t}{I_{t-1}}\right)\right] I_t + (1 - \delta) K_{t-1}, \tag{31}$$

where I_t is investment (of the final good), δ is the depreciation rate and Φ governs the Christiano, Eichenbaum and Evans (2005) style of investment adjustment costs, where $\Phi(1) = \Phi'(1) = 0$ and $\Phi''(\cdot) = \phi > 0$. Since these capital-producing firms are owned by banks, they choose investment to maximize:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=0}^{s-1} \Xi_{t+k,t+k+1} \right] (Z_{t+s} K_{t+s-1} - I_{t+s}). \tag{32}$$

Therefore, from the capital producers' first-order conditions:

$$1 = Q_t \left(1 - \Phi\left(\tfrac{I_t}{I_{t-1}}\right) - \Phi'\left(\tfrac{I_t}{I_{t-1}}\right) \tfrac{I_t}{I_{t-1}}\right) + \mathbb{E}_t \left[\Xi_{t,t+1} Q_{t+1} \Phi'\left(\tfrac{I_{t+1}}{I_t}\right) \left(\tfrac{I_{t+1}}{I_t}\right)^2\right], \tag{33}$$

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t}\right], \tag{34}$$

where Q_t is the Lagrange multiplier on equation (31), i.e., the value of a unit of installed capital. From comparing the second equation with equation (27), we see that the gross rate of return on shares in capital producers must be given by $R_t^K \equiv [Z_t + (1-\delta)Q_t] / Q_{t-1}$ (i.e., the gross return on capital), since all capital producer returns are transferred to the bank in all states of the world.

Finally, the model is closed with the resource constraint

$$Y_t = C_t + I_t. \tag{35}$$

3 THEORETICAL RESULTS

Before turning to numerical results, we will discuss a some theoretical properties of the model. All proofs are contained in Appendix C. We begin by focusing on the Lagrange multipliers and the coefficients of the bank's value function, as these offer insight into the importance of the financial constraints.

PROPOSITION 1. $\forall t, \lambda_t^E = 0$*: that is, the positivity constraint on equity issuance never binds.*

This result suggests that it can be optimal for banks to simultaneously issue equity and make dividend payments, thanks to the "signalling" value of dividend payments. Note that we are not using "signalling" in the typical asymmetric information sense here. Rather, the bank's decision to pay dividends communicates to households that they are unlikely to default in future, as dividend payments can be partially recovered following default, leading households to raise the borrowing limit. Without this channel, households would care only about $D_t - E_t$, and so, since issuing equity is costly, it could never be optimal to pay dividends while issuing equity.

To understand when simultaneous dividend and equity issuance might occur, recall that:

$$\lambda_t^D = \kappa_t - (1-\iota)\,(1-\kappa_t)\,\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right] \geq 0. \tag{36}$$

This tells us that if the marginal "signaling" value of paying a dividend is positive (i.e., if $\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right] > 0$), then it must be the case that $\kappa_t > 0$, which in turn implies that $E_t > 0$, as κ_t is an increasing function of E_t, with $\kappa_t = 0$ when $E_t = 0$. Furthermore, since issuing equity is costly, the total amount issued will be as low as possible. Therefore, if the bank has no other reason to issue equity, as the borrowing constraint is not binding, then it will be the case that $\lambda_t^D = 0$, implying that dividends payments are being funded by equity issuance. Such a situation is not implausible, as $\mathcal{N}_t > 0$ if $\Pr_t(\lambda_{t+k}^B > 0) > 0$ for any $k \geq 0$. It follows that there is always a signalling value of making dividend payments, and as such equity will be issued every period. Furthermore, it follows that the more likely that the constraint will binding in the future, the higher the equity-financed dividend payments will be. That said, if $\kappa_t'\left(E_t\right)$ is sufficiently high in the region of $E_t = 0$, then the amount of equity issued will be very low and could disappear entirely were there also fixed costs of issuance in our model.

Proposition 1 also implies that $\mathcal{H}_t = 1$, and so the stochastic discount factor applied to firms becomes:

$$\Xi_{t,t+1} \equiv (1-\iota)\Lambda_{t,t+1}\frac{1-\kappa_t}{1-\kappa_{t+1}}\mathcal{M}_{t+1}. \tag{37}$$

From this, it is easy to see that if the marginal value of an additional unit of funding is equal to one, and if the cost of equity issuance is constant, then in the limit as $\iota \to 0^+$, equation (37) will equal the household discount factor; that is to say, financial intermediation would be efficient.

PROPOSITION 2. $\lambda_t^B = 0 \iff \mathcal{M}_t = 1$ *and* $\lambda_t^B > 0 \iff \mathcal{M}_t > 1$. *That is, the marginal value of bank finance is greater than one if and only if the borrowing constraint is binding.*

It follows that the borrowing constraint is slack only if $\mathcal{M}_t = 1$. We referred to $\mathcal{M}_t$ as the marginal value of the bank book value, but it can also be described as the shadow price of bank finance; it is intuitive that this increases above unity as the bank becomes financially constrained.

The spread between the savings rate and the expected return on equity gives a measure of the current strength of the financial friction. We are particularly interested in the component of the spread that emerges from the agency problem, rather than the risk premium component. This component is captured by the Lagrange multiplier on the borrowing constraint, λ_t^B. To see this, note that from equations (24) and (27), we have:

$$\lambda_t^B = \mathbb{E}_t \left[\Xi_{t,t+1} \left(R_{t+1}^K - R_t \right) \right]. \tag{38}$$

The size of the spread depends crucially on the cost of issuing equity; if the cost were always zero, there would be no financial friction as banks would issue equity until their borrowing constraint was slack. In the benchmark GK case, equity finance is ruled out entirely, which sets $\kappa_t = 1$ for all t. (GK also propose an extension in which equity finance can be issued but is subject to the same type of friction as debt finance.) Our approach highlights the role that costly equity issuance plays when debt finance is constrained. The marginal value of bank finance, $\mathcal{M}_t$, is the value of one extra dollar of finance on the balance sheet of the bank; if the bank can raise finance via reductions in dividend payments or increased borrowing, then this will equal one dollar. As equity is issued and κ_t increases, $\mathcal{M}_t$ rises above unity. An additional dollar of finance reduces the need to raise costly equity by one dollar today, and by lowering the leverage of the bank, will relax the borrowing constraint in this and future periods.

We can show that around the deterministic steady state, in the limit as $\iota \to 0^+$, banks are not financially constrained but just at the edge of the constrained region. It follows that financial intermediation is efficient in the limit, and, close to the steady state, the borrowing constraints model replicates the standard RBC model. Throughout this paper, values without time subscripts will refer to steady-state values.

PROPOSITION 3. *The borrowing constraint is slack in the steady state only if $\iota = 0$. The banking sector is at the edge of the constrained region in the steady state in the limit as $\iota \to 0^+$.*

This result shows that steady-state financial intermediation is essentially efficient in the limit as $\iota \to 0^+$, as in the standard RBC model.[22] Of course, one might wonder how relevant the deterministic steady state is to the numerical results in the presence of uncertainty. Although banks are leveraged up to the constraint, they can absorb the majority of adverse shocks by reducing dividend payments. As well as weakening the GK financial accelerator mechanism, it follows that banks' precautionary behavior in the region of the steady state is limited, and consequently, financial intermediation is efficient most of the time.

[22] See also Corollaries 1–4 in Appendix C which indicate that in the deterministic steady-state, as $\iota \to 0^+$, the borrowing constraint becomes slack, the marginal value of dividend payments at any horizon goes to zero, the marginal value of bank finance goes to unity, the value of the bank descends to its book value, equity issuance falls to zero, and the return on shares falls to the gross real interest rate.

4 NUMERICAL ANALYSIS

To analyze the quantitative results, we begin by calculating a second-order pruned perturbation approximation to the model, and then use news shocks to impose the inequality constraints, following the algorithm of Holden (2017*a*).[23] We experimented with accurate simulations accounting for precautionary behavior associated with the bound, but found that the effects are not overly important. This is due to banks absorbing most shocks by cutting dividend payments to avoid becoming more highly leveraged; it follows that the financial constraint does not bind frequently and so precautionary motive is muted. However, performing calibration and producing average impulse responses at this high level of accuracy are computationally difficult as the constraint is so close to binding in the steady state. Thus, for consistency, we treat the bounds in a perfect-foresight manner throughout, that is, we approximate by assuming that the model's agents act today as if they were certain in which future periods the constraint would be binding.[24] Since we have a second-order solution to the underlying model, we will still capture precautionary effects stemming from the model's other non-linearities.

Because we perturb around the non-stochastic steady state, a strictly positive ι is necessary. To see this, suppose that both in this period and in the next, the borrowing constraints were slack. Then, a unit increase in dividend payments could be paid for by a unit increase in deposits now followed by a reduction in dividend payments of R_t in the next period. Thus, by the household Euler equation, households are indifferent about the level of dividends in this case.[25] Including $\iota > 0$ in the banker's discounting resolves this indeterminacy, and pins down the deterministic steady state. In practice, we set $\iota = 10^{-8}$ to minimize the departure from the $\iota \to 0^+$ world of our theoretical results, without introducing numerical problems.

[23]The algorithm is implemented in the "DynareOBC" toolkit, which extends Dynare (Adjemian et al., 2011) to solve models featuring inequality constraints. This is available at `https://github.com/tholden/dynareOBC`. Holden (2017*b*) provides the theoretical foundations for this method.

[24]An identical perfect-foresight assumption is made in the solution algorithm of Guerrieri and Iacoviello (2015), but their algorithm works only with a first-order approximation to the underlying model, whereas the algorithm of Holden (2017*a*) can handle higher-order approximations. The Holden (2017*a*) algorithm also allows us to be sure that when there is multiplicity, we are choosing the solution that escapes the bound as soon as possible.

[25]More generally, households cannot be sure that the bank's borrowing constraint will be slack next period, and so they might strictly prefer one unit of dividends today to R_t units next period.

4.1 Model parameters

We compare our numerical results to two benchmarks. A standard RBC model with $\mathbb{E}_t\left[\Lambda_{t,t+1}R^K_{t+1}\right] = \mathbb{E}_t\left[\Lambda_{t,t+1}\right] R_t$ so financial intermediation is efficient, and the GK borrowing constraints model with equity issuance, as outlined in Appendix D. These two benchmarks provide a never-binding financial friction in the case of the RBC model, and an always-binding financial friction in the case of the GK model.

Parameters common to the RBC literature are chosen to target a number of long-run ratios consistent with the literature. A discount factor $\beta = 0.995$ is chosen to achieve an average yearly real interest rate close to 2 percent; capital depreciates at 2.5 percent per quarter and the capital share is chosen to be $\alpha = 0.3$ as is standard in the literature. We choose $\varrho = 2.6$ to target a steady state value of hours to equal about one-third. Following Jaimovich and Rebelo (2009), we choose $\gamma = 0.001$, so it is small and positive, and choose $\psi = 0.4$, which corresponds to a Frisch elasticity of 2.5 when preferences take the GHH form. The second derivative of the investment adjustment cost is set as $\phi = 4$ and the (inverse) intertemporal elasticity of substitution is chosen as $\sigma_c = 2$, both within typical ranges from the literature. For the equity issuance costs, we choose a value for $\bar{\kappa}$ of 10 percent and set $\nu = 400$, which, in a fully non-linear solution, would imply that the costs would converge to the maximum for very small issuances. In our numerical simulations, the issuance costs typically fall in the 3 to 8 percent range.

The standard deviation of the total factor productivity shock, $\sigma_a = 0.0061$, is calibrated to hit a standard deviation of output of 1.015 percent,[26] and the persistence $\rho_a = 0.95$ is chosen to target a first-order output autocorrelation of 0.86.[27] We choose the proportion of assets that are unrecoverable after default, $\theta = 0.67$, to target a standard deviation of the spread between the deposit rate and the risky return on capital of 0.18 percentage points quarterly.[28] As the spread is close to zero in the unconstrained economy, the volatility of the spread is a natural choice for an additional target; in the absence of features such as liquidity premia, differing tax treatments and true default risk, the model inevitably underpredicts the mean spread.[29]

[26] This requires $\sigma_a = 0.0061$ in the RBC model and 0.0058 in the GK model.

[27] Target values are taken from empirical time series: non-banking data is 1983Q3–2016Q3 U.S. time series from `https://fred.stlouisfed.org`: GDP, FPI and PCEC for output, investment and consumption respectively, deflated using GDPDEF with CNP160V to convert to per capita. The Hodrick-Prescott filter is applied to these time series. The spread is that between Moody's Seasoned BAA and AAA Corporate Bond yields. New equity issuance is as described in Baron (2017) for the largest 20 U.S. commercial banks. For dividend payments, we sum dividends and share repurchases from Baron's (2017) data.

[28] θ is calibrated to 0.85 in the GK model with the same target.

[29] In the GK model, there are two additional parameters that control the survival rate of

4.2 Impulse response functions and simulations

In order to assess the propagation of shocks and the role of the financial constraints, we compute the average impulse response functions for shocks to productivity and capital quality.[30] This follows GK, who argue that negative capital quality shocks should not be considered physical depreciation of capital, but rather represent some form of economic obsolescence; they also suggest a possible micro-foundation. As in GK, the inclusion of the capital quality shocks allows for the characterization of occasional "disaster" shocks. In particular, we will examine the impact of a 5 percent unanticipated decline in capital quality. This gives a shock to which we can compare the model predictions to the observed macroeconomic time-series following the events of late 2007.

Let us consider the role of the borrowing constraint following such a disturbance. When either the banks' demand for funding increases, or the borrowing constraint tightens due to a relative decline in banks' expected future profits, the banks must raise equity finance. If the bank is unable to raise sufficient finance through retained earnings, they must sell equity, paying issuance costs that rise in the volume of issuance. This causes the expected marginal value of bank finance, $\mathcal{M}_{t+1}$, to increase above unity. As dividend payments relax the borrowing constraint, it is optimal for the bank to keep paying dividends even as they begin to issue equity. Indeed, past dividends become particularly important to the bank once financially constrained; the lower the past dividend payments, the tighter the borrowing constraint. This is also true for the interest rate; the lower the interest rate over the previous two years, the tighter the constraint.

Now, recall that households discount using the stochastic discount factor $\Lambda_{t,t+1}$, whereas equity is priced using $\Xi_{t,t+1}$. The latter augments the former with the marginal value of bank finance, implying that in the unconstrained case, $\mathbb{E}_t\left[\Lambda_{t,t+1}\right] = \mathbb{E}_t\left[\Xi_{t,t+1}\right]$, while $\mathbb{E}_t\left[\Lambda_{t,t+1}\right] < \mathbb{E}_t\left[\Xi_{t,t+1}\right]$ when there is a positive probability of financial constraints binding. The augmented stochastic discount factor is asymmetric as $\mathcal{M}_t \geq 1$, and has higher volatility than the household stochastic discount factor; if the expected marginal utility of future consumption increases relative to

bankers and the amount transferred to new bankers, as well as parameters controlling outside equity issuance. The banker survival rate is equivalent to a dividend rate but has to be set high to ensure an always-binding constraint. We follow GK and set this to 0.975, which is equivalent to an expected survival rate of 10 years, and set the proportion of bank equity transferred to the new "start-ups" equal to 0.3. These allow a mean spread approximately equal to the observed 0.57 percentage points and a bank leverage ratio close to the average of 4, targeted in GK. We follow GKQ with our choice of equity issuance parameter values.

[30] We take the median of the difference between 2000 pairs of 550 period simulation runs, where each pair of runs has identical shocks, apart from one additional impulse in period 400 for the first of each pair.

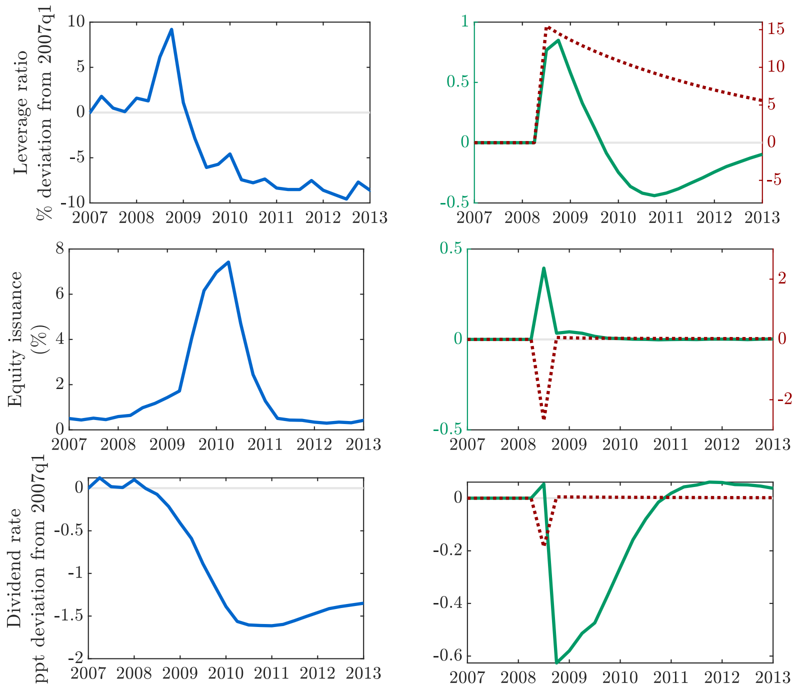

Figure 3: Empirical time-series (left panel) against model impulse response simulations (right panel) to a negative 5% capital quality shock in our model (green solid – left scale) and GK (red dots – right scale).

that of current consumption, as would be expected following an adverse shock, then $\mathbb{E}_t[\Lambda_{t,t+1}]$ would increase. Because the expected value of $\mathcal{M}_{t,t+1}$ is also likely to rise, $\mathbb{E}_t[\Xi_{t,t+1}]$ rises further still. This introduces a hedging value of debt finance that increases as the financial constraint tightens. Because of this, when a bank experiences a balance sheet shock that reduces the value of assets, such as a capital quality shock, the value of equity falls relative to debt and the leverage of the bank will increase. This results in a further tightening of the borrowing constraint. In the periods following the shock, banks respond to tight credit conditions by raising equity finance, initially through an issuance and then retained earnings, as the bank de-lever, leverage falls below the long-run average for some periods due to reduced investment and the slow rebuilding of capital. This is highlighted in Figure 3 which shows the path of asset to book equity leverage ratio, new equity issuance and dividend payments for U.S. commercial banks over the crisis period together with impulse responses to a 5 percent reduction in capital quality implied by our model and GK.[31] While our model under-predicts the size

[31]Empirical leverage from Federal Financial Institutions Examination Council (US), Total Equity to Total Assets for Banks [EQTA], retrieved from FRED, Federal Reserve Bank of St. Louis. New equity issuance and dividend payments as described in footnote 13.

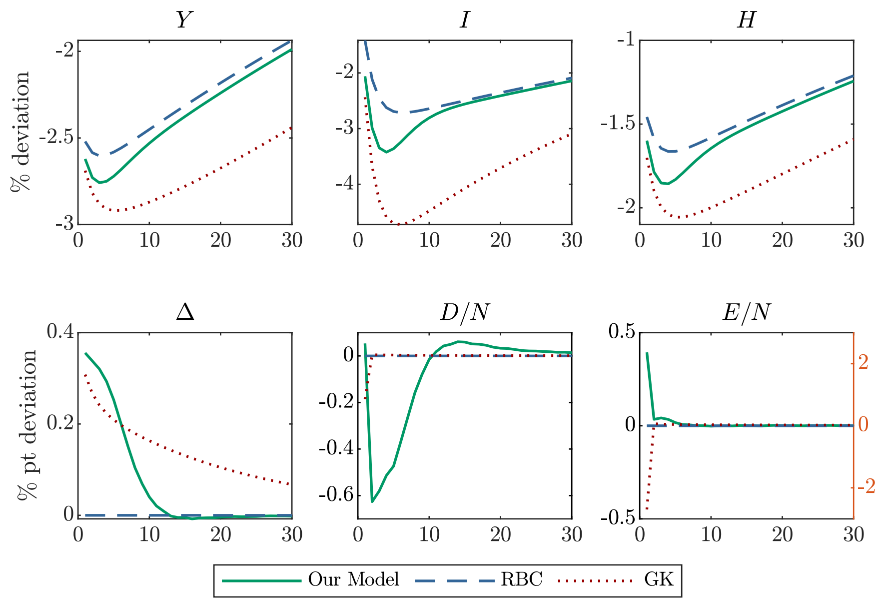

Figure 4: Average impulse responses to a negative 5 percent capital quality shock. Plots show deviations from the ergodic mean. The left axes of the last plot corresponds to our model, the right to GK.

of deviations, it performs qualitatively very well. In GK, the financial constraint is always binding and leverage remains counter-factually elevated for several years following the shock. This drives the deeper declines in output and investment as shown in Figure 4 which plots impulse response functions for these variables together with labour, the investment wedge ($\Delta_t \equiv \mathbb{E}_t[R^K_{t+1} - R_t]$), and rates of dividend payment and equity issuance across the three models. [32] Because equity finance in GK is also subject to the same friction as debt, issuance declines following the shock, whereas it increases in our model, consistent with the data. The decline in investment in our model is close to that of the GK model on impact, but begins to converge back to the RBC model after about five quarters. Nonetheless, the episode of constrained finance is persistent, with the investment

[32]Leisure is a normal good, so the presence of short-run wealth effects would imply that the adverse shock to household wealth would decrease demand for leisure and increase labour supply. Under most model specifications, this would imply an increase in investment following the shock as the poorer households consume less, and work and save more. This is overturned by reducing the short-run wealth effect on labour supply as the lower real wage rate causes a reduction in labour supply. Investment does fall in both GK and our model on impact with standard King-Plosser-Rebelo (KPR) preferences as financial constraints tighten, but quickly rebounds, leading to an investment boom. The increase in investment can also be overturned with habits in consumption (see, e.g., Cochrane and Campbell, 1999) as the substitution between consumption and savings become costly. We choose the Jaimovich-Rebelo approximation to GHH preferences, as both habits in consumption and KPR preferences imply a counterfactual increase in labour.

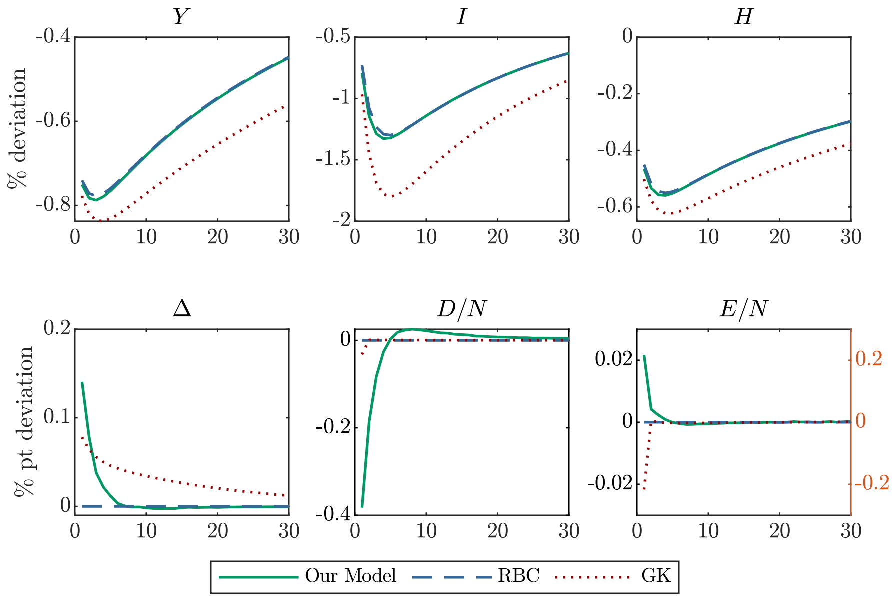

Figure 5: Average impulse response functions to a 1-standard deviation, negative productivity shock. Plots show deviations from ergodic mean. The left axes of the last plot correspond to our model, the right to GK.

wedge taking around three years to return to normal levels. Due to the signalling role of dividend payments in relaxing the borrowing constraint in our model, it is unnecessary for payments to cease before banks begin to issue equity. Indeed, for several periods, the banks simultaneously pay dividends and issue equity. Due to the costs of equity issuance, the marginal bank funding cost increases above the savings rate; this is the force behind the sharp rise in the interest spread and the deeper fall in investment relative to the RBC model.

A striking result of Figure 4 is the dampened financial accelerator in our model compared to GK. In the latter, the financial constraint applies to both debt and outside equity issuance, and so both sources of finance are tightened following the shock. Because banks are unable to raise any further finance, the financial constraint has a much larger effect on the real economy. Once the bank has access to inside equity finance, first by reducing dividend payments for free, and then paying a cost to issue shares, the financial accelerator is much weaker.

Figure 5 shows the response to a negative productivity shock. In this case, the financial constraint tightens because the value of future profits falls. In GK, investment falls much further as both debt and equity finance contract. In our model, even though debt falls, banks can raise further equity finance by retaining earnings. Because lowering dividend payments tighten the constraint further, the

		Y	I	C	D	E	Δ
Correlation with Y	Data	1	0.879	0.882	0.335	-0.279	-0.393
	Our model	1	0.958	0.985	0.326	-0.182	-0.412
	RBC	1	0.951	0.983	–	–	–
	GK	1	0.949	0.969	0.264	0.315	-0.574
Standard deviation	Data	1.06	4.51	0.92	3.83	4.57	0.18
	Our model	1.06	1.65	0.90	0.705	0.036	0.18
	RBC	1.06	1.63	0.91	–	–	0
	GK	1.06	2.12	0.80	0.027	0.195	0.18
Skewness	Data	-0.240	-0.606	-0.315	0.34	3.60	1.67
	Our model	-0.009	-0.038	0.013	1.39	1.20	0.91
	RBC	-0.007	-0.029	-0.004	–	–	–
	GK	-0.008	-0.065	0.020	-0.05	-0.24	0.20

Table 1: Simulated and empirical moments. Standard deviation in percent except D, E and Δ, which are in percentage points.

banks pay to issue equity while reducing but still paying out dividends. Even though this causes a sharp increase in the interest spread, there is only a small expected impact on the real economy relative to the RBC model. That said, the impulse response functions are asymmetric and non-monotonic; the financial accelerator effects increase as the size of adverse shocks rise, and are all but absent for shocks of the opposite sign. We illustrate this with further impulse responses in Appendix E.

4.2.1 Simulated moments

Table 1 reports simulated moments and cross-correlations for the three models together with those computed from the data. Our model introduces significant skewness in the interest spread that is entirely missing from the GK models, as well as skewness in equity issuance that arises due to occasional episodes of sharp issuances. Furthermore, when repurchases are included in the measure of gross dividends, as we do here, our model does well at predicting the cyclicality of both dividend payments and equity issuance. It also captures some of their volatility. Without stock repurchases, dividend payments in the data are actually more stable than in our model, but the inclusion of stock repurchases substantially increases their volatility.[33]

[33] This discrepancy suggests the presence of additional factors, such as other agency problems, missed by the model. Dividend payments alone are acyclical or slightly countercyclical in the data and, given that banks appear to vary stock repurchases rather than dividend payments, the empirical time series suggest that dividends are used during downturns either as a signalling device to indicate the strength of the individual bank, or as a result of the reduced number of

Volatility in investment is lower than in the data due partly to the household preferences, and partly to the capital adjustment costs. This is higher in the GK model, resulting from the financial accelerator mechanism introduced by the borrowing constraint. Volatility of investment is between the RBC and GK models as the financial accelerator is in effect only when the borrowing constraints are binding.

5 CONCLUSION

This paper embeds a model of banking into a real business cycle framework, resulting in a model that generates occasional endogenous credit crunches. In the vicinity of the steady state, the model behaves much like a standard RBC model: financial intermediation is efficient and the interest rate spread is equal to the standard risk premium. Credit crunches are precipitated by sufficiently large adverse shocks that cause the bank financing constraint to bind. This is the result of an increased incentive for banks to divert funds and declare bankruptcy caused by a reduction in expected bank profits. Banks are able to issue equity when debt finance is constrained, but issuance costs introduce a wedge between the risk-free rate and the risky return to capital, resulting in reduced investment and output.

By removing the exogenous bank exit rate common to many similar models and allowing endogenous dividend payments, we find that the borrowing constraint is always occasionally binding, independent of calibration. We consider this to be an appealing aspect of the model given the observation that financial crises, and sharp increases in spreads more generally, occur across many different countries, with different and time-varying policies towards banks and bankruptcy. If crises only occurred in a narrow range of the parameter space, then one would think it unlikely that we would observe spikes in spreads across such a wide range of situations. Furthermore, in our model, credit crunches are truly an occasional phenomena in contrast to the majority of existing models in which financial constraints bind in steady state. A key contribution is a careful treatment of the Kiyotaki and Moore (1997) agency problem extended in GK. By modeling the U.S. law relating to bankruptcy, we reveal a potentially important signaling role for dividends in acting to relax the borrowing constraint. However, once we allow endogenous dividend payments and equity issuance, even at cost, the financial accelerator mechanism is significantly dampened compared to other models, such as GK. Finally, our model gives a number of improvements in the empirical fit of simulated time series. Notably, we capture the strong positive skewness in the interest spread and bank equity issuance that are missing in the standard RBC and GK models. We also replicate the countercyclical bank equity issuance and pro-cyclical bank leverage

profitable investment opportunities.

observed in the data, contrary to other papers, such as GK, which predict the opposite.

APPENDIX A: REARRANGING THE BORROWING CONSTRAINT

Substituting the (binding) bank budget constraint (11) into the bank book value (10) yields:

$$B_t^j = S_t^j + D_t^j - \hat{V}_t^j (1 - \kappa_t) - E_t^j (1 - \kappa_t) \tag{A1}$$

Substituting this and the bank value function (17) into the borrowing constraint (16) yields:

$$V_t^j \geq \frac{\mathcal{M}_t}{1 - \kappa_t - \mathcal{A}_t} \left(S_t^j + D_t^j - E_t^j (1 - \kappa_t) \right) - \sum_{i=1}^{\tau - 1} \left(\frac{\mathcal{M}_t}{1 - \kappa_t - \mathcal{A}_t} \mathcal{F}_{i,t} - \mathcal{N}_{i,t} \right) D_{t-i}^j \tag{A2}$$

Noting that bank assets, denoted A_t^j, are equal to firm equity, S_t^j, and letting

$$\theta_{1,t}^j \equiv \frac{\mathcal{M}_t}{1 - \kappa_t - \mathcal{A}_t} \left(1 + \frac{D_t^j}{A_t^j} - \frac{E_t^j}{A_t^j} (1 - \kappa_t) \right) \tag{A3}$$

$$\theta_{2,i,t}^j \equiv \left(\frac{\mathcal{M}_t}{1 - \kappa_t - \mathcal{A}_t} \mathcal{F}_{i,t} - \mathcal{N}_{i,t} \right), \tag{A4}$$

we can write the borrowing constraint:

$$V_t^j \geq \theta_{1,t}^j A_t^j - \sum_{i=1}^{\tau} \theta_{2,i,t} D_{t-1}^j. \tag{A5}$$

APPENDIX B: BANKING SECTOR COLLAPSE AND GOVERNMENT INSURANCE

Let us consider the bank's decision in period t whether to exit that same period. First, note that for the bank to fully meet its liabilities prior to an exit without default would require households to contribute $\max\{0, -\hat{V}_t^j\}$, since $\hat{V}_t^j$ includes the costs of equity issuance. Indeed, since bank j can always sell itself to another bank and receive $\hat{V}_t^j$, the bank can always receive $\hat{V}_t^j$ by a default-free exit in period t, represented by option (b) in Figure B1. As a result, it must always be the case that $V_t^j \geq \hat{V}_t^j$. Alternatively, the bank can decide to exit via default (option (a) in Figure B1). The maximum amount that can be recouped from previous dividend payments is:

$$(1 - \theta) \bar{D}_{t-1}^j. \tag{B1}$$

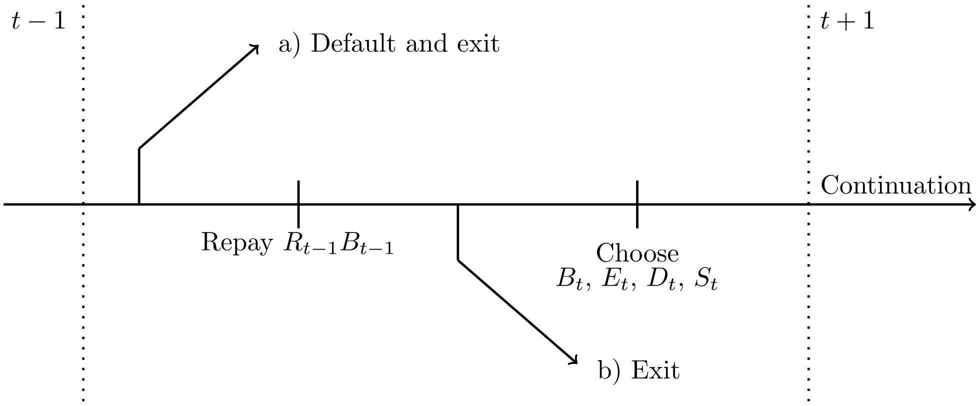

Figure B1: Timing of bank decisions and unplanned exit.

Consequently, the value of a bank exiting in period t is:

$$\max\left\{\hat{V}_t^j, -(1-\theta)\,\bar{D}_{t-1}^j\right\}. \tag{B2}$$

Thus, as $V_t^j \geq \hat{V}_t^j$, the bank will default if and only if:

$$V_t^j < -(1-\theta)\,\bar{D}_{t-1}^j. \tag{B3}$$

If this occurs for bank j on the equilibrium path, then, by symmetry, all banks will default. In this case, the value of continuation is lower than the cost of the amount that could be recovered from shareholders upon default. So, to prevent a financial collapse, it would be rational for the government to bail out the banks in this extreme tail situation. Specifically, a government guarantee on household savings mean that banks actually need repay only $\left(R_t^j - \mathcal{G}_{t+1}\right)B_t^j$ where $\mathcal{G}_{t+1}$ is only non-zero in the face of an extreme adverse shock that would otherwise cause a systemic banking collapse. The government funds this insurance via lump-sum taxes on households. We assume the government performs the smallest possible bail-out to avoid such a collapse, by choosing $\mathcal{G}_t$ such that the following complementarity condition holds:

$$\min\left\{\mathcal{G}_t, V_t + (1-\theta)\,\bar{D}_{t-1}\right\} = 0. \tag{B4}$$

Thus, the government is effectively offering free insurance on firm equity to banks. Although this policy rules out bank default along the equilibrium path, it will lead to risk being underpriced relative to the efficient benchmark, since banks internalize the insurance against tail events that the government is providing. However, without artificial constraints on when banks can default, such insurance is inescapable, as banks are undertaking risky investments but promising safe returns.

In practice, under our calibrations, the probability of this event is extremely low so the impact on the price of risk is negligible.

APPENDIX C: FURTHER PROPOSITIONS, COROLLARIES AND PROOFS

COROLLARY 1. *If $\iota > 0$, then $\mathcal{M} > 1$ and $\mathcal{N} > 0$.*
COROLLARY 2. $\lim_{\iota\to 0^+}\mathcal{M} = 1$ *and* $\lim_{\iota\to 0^+}\mathcal{N} = 0$.
COROLLARY 3. *If $\iota > 0$, $V > \hat{V}$ and $D > 0$. $\lim_{\iota\to 0^+} V = \hat{V}$, and $\lim_{\iota\to 0^+} E = 0$.*
COROLLARY 4. *If $\iota > 0$, $R^K > R$. $\lim_{\iota\to 0^+} R^K = R$.*
PROPOSITION 4. *If we take the limit as $\iota \to 0^+$ and either $\kappa \to 0^+$ or $\theta \to 0^+$, then the model converges to the standard real business cycle model.*

Proof of Proposition 1. Substituting equation (26) into (25) gives

$$\lambda_t^D = 1 - \lambda_t^E - (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(1-\kappa_t\right)\mathcal{N}_{t+1}R_t\right] - \left(1-\kappa_t\right). \tag{C1}$$

Suppose that $\lambda_t^E > 0$. Then $E_t = 0$ by complementary slackness, so, from the definition of κ_t, the previous equation becomes:

$$\lambda_t^D + \lambda_t^E + (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right] = 0, \tag{C2}$$

and so $\lambda_t^D = \lambda_t^E = (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right] = 0$ giving the required contradiction. □

Proof of Proposition 2. Substituting $\mathcal{H}_t = 1$ into equation (22) leads to:

$$\mathcal{M}_t = \frac{\left(1-\lambda_t^B\right)\left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right)}{\left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right)-\lambda_t^B} \tag{C3}$$

Since $0 \le \left(1-\kappa_t\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right) < 1$, it follows that $\mathcal{M}_t = 1$ if and only if $\lambda_t^B = 0$. Given that $\mathcal{M}_t \ge 1$ as a bank can always sell itself to another bank for $\hat{V}_t$, independent of its history of dividend payments, this also implies that $\mathcal{M}_t > 1$ if and only if $\lambda_t^B > 0$. □

Proof of Proposition 3. Using equation (20), we have that the steady-state value of λ_t^B is given by:

$$\lambda^B = \iota\left(1-\kappa\right)\left(1-\left(1-\iota\right)\left(1-\theta\right)\right) \in (0,1), \tag{C4}$$

where throughout this document, values without time subscripts will refer to steady-states. This implies that the borrowing constraint binds with positive ι but $\lim_{\iota\to 0^+} \lambda^B = 0$. As λ^B is the Lagrange multiplier on the borrowing constraint, the claim follows. □

Proof of Corollaries 1 and 2. The results for $\mathcal{M}$ in Corollaries 1 and 2 follow immediately from Proposition 2. Indeed, from equation (C3), we find:

$$\mathcal{M} = \frac{1 - \iota\,(1-\kappa)\,[1-(1-\iota)\,(1-\theta)]}{1-\iota} > 1 \tag{C5}$$

and so in the limit as $\iota \to 0^+$, we have $\mathcal{M} \to 1$. The same is true for $\mathcal{N}$ as:

$$\mathcal{N} = \frac{\rho}{1-\rho}\iota\,(1-\theta) > 0 \tag{C6}$$

and as $\iota \to 0$, $\mathcal{N} \to 0$. □

Proof of Corollary 3. The value of the bank is given by:

$$V = \mathcal{M}\hat{V} + \mathcal{N}\bar{D}. \tag{C7}$$

where $\bar{D} = D/(\beta - \rho)$. Hence, the value of a bank is always greater than its book value for $\iota > 0$, but $\lim_{\iota\to 0^+} V = \hat{V}$.

Now, banks must pay dividends in steady state, at least with $\iota > 0$, for, suppose they did not. Then, their steady-state value would be zero, by the definition of bank value, and so since book value is always weakly below value, their steady-state book value would be non-positive. However, since equity issuance is always strictly positive with $\iota > 0$, steady-state book-value would be infinite without dividend payments, giving the required contradiction. Consequently:

$$\lambda^D = \kappa - (1-\iota)\mathcal{N}\,(1-\kappa) = 0, \tag{C8}$$

so:

$$\kappa = \frac{(1-\iota)\mathcal{N}}{1+(1-\iota)\mathcal{N}} > 0. \tag{C9}$$

It follows from $\lim_{\iota\to 0^+} \mathcal{N} = 0$, that $\lim_{\iota\to 0^+} \kappa = 0$ and so there is no equity issuance in the limit. □

Proof of Corollary 4. Note:

$$R = \left(1 - \iota\left(1 - \kappa\right)\left[1 - \left(1 - \iota\right)\left(1 - \theta\right)\right]\right) R^K, \tag{C10}$$

so $R^K > R$ but $\lim_{\iota \to 0^+} R^K = R$. □

Proof of Proposition 4. First suppose that $\kappa = 0$. In this case, the first order condition with respect to dividend payments becomes:

$$\lambda_t^D = -(1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{t+1}R_t\right]. \tag{C11}$$

Now, it follows from the definition of $\mathcal{N}_t$ in equation (21), that $\mathcal{N}_t \geq 0$ for all t, since $\lambda_t^B \in [0, 1]$, by equation (24). Hence, since $\lambda_t^D \geq 0$, equation (C11) implies that $\lambda_t^D = \mathcal{N}_t = 0$ for all t. Consequently, again by equation (21), we must also have that $\lambda_t^B = 0$ for all t, which in turn implies that $\mathcal{M}_t = 1$ for all t, by Proposition 2. Using this in the definitions of the pricing kernels for bank and firm equity, we find that when $\iota = 0$ as well, $\Lambda_{t,t+1} = \Xi_{t,t+1}$ for all t, so financial intermediation is efficient. The bank is never financially constrained as they can always raise equity finance at no cost.

Next, suppose that $\theta = 0$. Recall the borrowing constraint is of the form:

$$B_t \leq \mathcal{A}\hat{V}_t + \mathcal{F}_t\bar{D}_{t-1}. \tag{C12}$$

If $\theta = 0$, then as $\iota \to 0$, it follows from the solutions of the coefficients in equations (18) and (19), that $\mathcal{A}_t, \mathcal{F}_t \to \infty$. So in the limit as $\iota \to 0$, borrowing becomes unlimited. As in the previous case, it follows that for all t, $\lambda_t^B = 0$, $\mathcal{M}_t = 1$ and $\Lambda_{t,t+1} = \Xi_{t,t+1}$ if $\iota = 0$, and so financial intermediation is efficient. □

APPENDIX D: GERTLER & KIYOTAKI (2010) MODEL

We describe a version of the GK model extension with equity issuance. The household and firm sectors are identical to our model, the difference is on the intermediation of funds between these two sectors. Every period, banks face a constant probability, $1 - \sigma_B$, of exiting and paying the household a dividend. No dividend is paid if the bank continues, the bank decides on debt and outside equity finance and issues loans to non-financial firms. When a bank exits, a new bank takes is place and is transferred a fraction ξ_B of the exiting banks' net worth. Bank activity is subject to financial constraints as the inside shareholders can divert assets. In particular bank j solves

$$V_t^j = \max_{S_t, B_t, E_t} \mathbb{E}_t\{ (1-\sigma_B) N_{t+1}^j + \sigma_B \Lambda_{t,t+1} V_{t+1}^j \} \tag{D1}$$

$$\text{s.t.} \quad V_t^j \geq \Theta\left(x_t^j\right) S_t^j \tag{D2}$$

$$N_t^j = R_t^K S_{t-1}^j - R_t^E Q_{t-1}^E E_{t-1}^j - R_{t-1} B_{t-1}^j \tag{D3}$$

$$S_t^j = B_t^j + Q_t^E E_t^j + N_t^j \tag{D4}$$

where E_t is the stock of outside equity, rather than new issuance of inside equity as in our model, Q_t^E is the price of equity, and R_t^E is the rate of return on outside equity. Where each unit of $E_t Q_t^E$ is a claim on one unit of S_t, itself a claim on a unit of $Q_t K_t$. The proportion of divertable assets, θ is a quadratic function of $x_t \equiv Q_t^E E_t / S_t$:

$$\theta\left(x_t^j\right) = \bar{\theta}\left(1 + \epsilon x_t + \frac{\kappa^{GK}}{2} x_t^2\right) \tag{D5}$$

Dropping bank indices, this leads to demand equations for debt and equity finance

$$\nu_t^b = \phi_t \left(\theta\left(x_t\right) - \left[\mu_t^s + \mu_t^e x_t\right]\right) \tag{D6}$$

$$\mu_t^e = \left[\mu_t^s + \mu_t^e x_t\right] \frac{\theta'\left(x_t\right)}{\theta\left(x_t\right)} \tag{D7}$$

with $\phi_t \equiv S_t / N_t$ and where

$$\Omega \equiv 1 - \sigma_B + \sigma_B \theta\left(x_t\right) \phi \tag{D8}$$

$$\mu_t^s \equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^K - R_t\right)\right] \tag{D9}$$

$$\nu_t^b \equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1} R_t\right] \tag{D10}$$

$$\mu_t^e \equiv \mathbb{E}_t\left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_t - R_{t+1}^E\right)\right] \tag{D11}$$

Finally, the demand for outside equity must satisfy

$$1 = \mathbb{E}_t\left[\Lambda_{t,t+1} R_{t+1}^E\right]. \tag{D12}$$

APPENDIX E: ADDITIONAL IMPULSE RESPONSE FUNCTIONS

In addition to the impulse response function to an adverse capital quality shock in the paper, here we show a positive capital quality shock that highlights the asymmetry. Plots of the responses to a positive 5 percent capital quality shock are shown in Figure E1. The same is true for shocks to total factor productivity. As a negative productivity shock decreases the continuation value of the bank, or the value of future profits, the constraint tightens. As there is also a decline in the value of bank assets, which acts in the opposite direction, a large shock is required to cause the borrowing constraint to bind sufficiently to have a large impact. There is a small financial accelerator for adverse shocks, but as shown in Figure 5, this effect is not persistent and the model converges quickly to the RBC model. As shown in Figure E2, for positive technology shock there is little difference between our model and the RBC model.

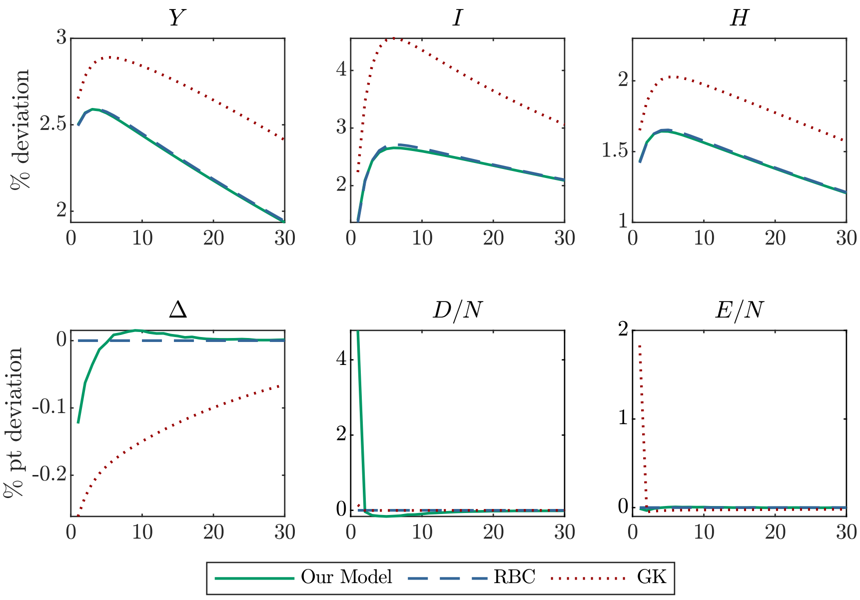

Figure E1: Average impulse responses to a positive 5 percent capital quality shock. Plots show deviations from the ergodic mean. The left axes of the last plot corresponds to our model, the right to GK.

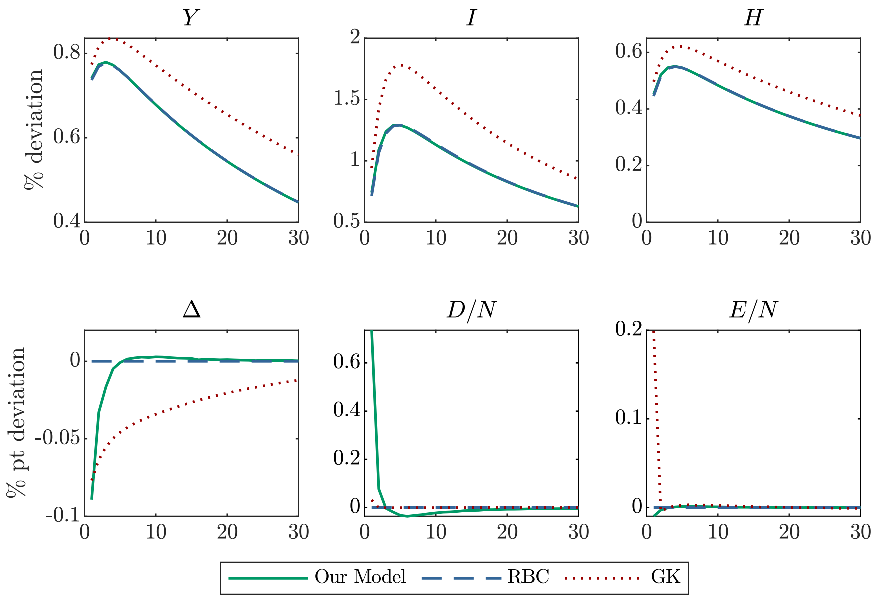

Figure E2: Average impulse response functions to a 1-standard deviation, positive productivity shock. Plots show deviations from ergodic mean.

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