# The Term Structure of Expectations and Bond Yields 

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Trinity Network Webinar

August 31, 2017

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## The Term Structure of Interest Rates

- ... is key to understanding macro-financial interactions.
- Much of macroeconomics thinks about interest rates in terms of the expectations hypothesis (EH):

Yields on default-free government bonds equal average short rate investors expect to prevail over bond's life.

- A long literature in finance (e.g. Fama-Bliss (1987), Campbell-Shiller (1991)) has shown that the EH does not hold empirically. It is now widely accepted that bond yields can be decomposed into average expected short rates and a time-varying term premium.
- Important to identify and quantify both components. Literature treats them as unobserved and uses models to measure them.


## Models Will Lead You Only So Far

- Decomposition is highly dependent on model specification:
- What is driving the short rate in the model?
- Which variables are included, yield curve factors only or macro-factors?
- Which sample period is considered?
- Other limitations:
- Most models do not capture structural change and learning.
- Econometrically challenging to deal with the lower bound.
- Model-implied long-run short rate expectations typically converge to sample mean of short rate.


## This Paper

- We start with the identity that:

```
yield on a n-period bond
= expected average path of short rates over next n periods
    + term premium for the n-period bond
```

- First paper to measure expected path of short rates directly and exclusively from survey data.
- Do not need to specify stochastic discount factor or DGP for yields.
- Do not need to make any assumptions about right set or number of state variables.
- Do not make any assumptions about dynamics of term premiums across time and maturities.


## This Paper (cont'd)

- Obtain term premiums as difference between observed yields or forwards and the corresponding expected average short rates.
- Because term premiums are residual between yields and expectations we can remain agnostic about what they specifically represent.
- For example, they might reflect:
- Shifts in investors' risk attitudes.
- Differences between the expectations of the marginal investor and consensus expectations.
- Frictions in the bond market which prevent the elimination of arbitrage opportunities.
- Need not make any assumptions about rationality/bias of expectations. Our measure of the term premium can also reflect serially correlated forecast errors.


## Provide Answers to Following Questions

- Do expected rates or term premiums explain the variation in yields?
- Do expected rates or term premiums explain the co-movement across yields?
- Are term premiums informative about future economic conditions?
- Do term premiums respond to economic shocks in meaningful ways?


## What We Find

(1) Medium-to-long-term expectations of the short-term nominal and real rate are fairly volatile. However...
(2) ...term premiums, not expected rates, are the predominant driver of bond yields.
(3) Term premiums, not expected rates, are the dominant source of co-movement across yields.
(9) Term premiums, not expected rates, predict real output growth.
(5) Macroeconomic shocks, especially policy and demand shocks, are important drivers of term premiums.

## Literature

(1) Survey data to discipline models of expectation formation:

- Friedman (1979), Froot (1989), Mankiw, Reis \& Wolfers (2003), Patton \& Timmerman (2010), Coibion \& Gorodnichenko (2012), Andrade \& Le Bihan (2013), Lahiri \& Sheng (2008) Andrade, Crump, Eusepi \& Moench (2016),
(2) Slow-moving drifts in macro-variables:
- Kozicki \& Tinsley $(2001,2005)$, Stock \& Watson $(1989,2007)$, Laubach \& Williams (2003), Cogley, Primiceri \& Sargent (2010), ...
(3) Term structure models:
- General: Too many to list ...
- Survey data: Kim \& Wright (2005), Kim and Orphanides (2005), Piazzesi, Salomao \& Schneider (2015), Chernov \& Mueller (2012)


## Roadmap

Data

The Term Structure of Expectations

Results

Conclusion

Appendix

## Data

- Use all U.S. professional forecasts (SPF, BCEI, BCFF, CE, SPD, ...)
- Real GDP growth, CPI inflation and 3-month T-Bill
- Total of 602 different survey-horizon pairs in our sample
- Do not observe the entire term structure of expectations at each point in time, for some horizons observe multiple forecasts across different surveys.
- Extract common information across all available surveys, and provide consistent proxies for missing observations, especially nominal short rate for which fewer longer-term forecasts available.
$\rightarrow$ Use a simple three-variable VAR with shifting endpoints to fit all survey data: "sophisticated interpolation" of missing horizons.


## Model Fit: Examples

Real GDP (Q2)
2nd month of quarter


CPI (Q3)
2nd month of quarter


TBILL (Q2)
2nd month of quarter


Real GDP (Y2)
10th, 11th month of quarter


CPI (Y2)


TBILL (Y2)
10th, 11th month of quarter


Real GDP (Long-run) Assorted horizons


CPI (Long-run)
Assorted horizons


TBILL (Long-run)
Assorted horizons


## Term Structure of Expectations: Nominal Short Rate



- Expected short rate paths vary substantially over time: steeper at beginning and flatter at end of tightening cycles.
- Since 2009 consistent with perceived zero lower bound on interest rates.
- Long-term forecasts gradually decline in the post-Volker period; down to $\approx 3.5 \%$ since end of 2014


## The Term Structure of Expectations: Inflation



- Expected paths of inflation vary far less, $\approx$ RW starting from current level.
- Consistent with perceived nominal rigidities.
- Long-run expected inflation stable around $2.5 \%$ since late 1990 s.


## The Term Structure of Expectations: Real Short Rate



- Movements in expected real rates closely follow expected nominal rates.
- Long-run forecasts of the real short rate $\left(r_{t}^{*}\right)$ rangebound around $2 \%$ for 30 years; down to less than $1 \%$ since late 2014.


## Decomposing the Yield Curve

With expected short rate paths at hand, we can decompose yields into expectations hypothesis component and term premiums.

- Focus on forward rates: current yield of an $n$-year bond maturing in $n+m$ years.

$$
f_{t}(n, m)=\frac{1}{m}\left[(n+m) y_{t}^{(n+m)}-n y_{t}^{(n)}\right]
$$

- Obtain forward term premium as difference between $f_{t}(n, m)$ and the consensus expected short-term rate over the $m$ years $n$ years hence:

$$
\begin{aligned}
t p_{t}^{f w d}(n, m) & =f_{t}(n, m)-\frac{1}{m} E_{t} \sum_{i=n+1}^{n+m} i_{t+i} \\
& =f_{t}(n, m)-\frac{1}{n} \sum_{i=m+1}^{n+m} \mathbb{E}_{t}\left[r_{t+i}+\pi_{t+i+1}\right]
\end{aligned}
$$

## Forward Rate Decompositions

1Y1Y Forward



## What Drives Time-Series Variation in Bond Yields?

We decompose the variance of forward rates into

$$
\frac{\operatorname{Cov}\left(f_{t}(n, m), t p_{t}^{f w d}(n, m)\right)}{\operatorname{Var}\left(f_{t}(n, m)\right)}+\frac{\operatorname{Cov}\left(f_{t}(n, m), \frac{1}{n} \sum_{i=m+1}^{n+m} \mathbb{E}_{t}\left[r_{t+i}\right]\right)}{\operatorname{Var}\left(f_{t}(n, m)\right)}+\frac{\operatorname{Cov}\left(f_{t}(n, m), \frac{1}{n} \sum_{i=m+1}^{n+m} \mathbb{E}_{t}\left[\pi_{t+i+1}\right]\right)}{\operatorname{Var}\left(f_{t}(n, m)\right)}
$$

and similarly for monthly differences:

|  | 1Y | 1 Y 1 Y | 2Y1Y | 3 Y 1 Y | 4Y1Y | 5Y1Y | 6Y1Y | 7Y1Y | 8Y1Y | 9Y1Y |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Levels |  |  |  |  |  |  |  |  |  |  |
| Avg Exp Real Rate | 0.58 | 0.43 | 0.29 | 0.20 | 0.15 | 0.12 | 0.11 | 0.11 | 0.12 | 0.12 |
| Avg Exp Inflation | 0.31 | 0.30 | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 |
| Fwd Term Premium | 0.13 | 0.29 | 0.42 | 0.50 | 0.55 | 0.56 | 0.56 | 0.55 | 0.54 | 0.53 |
| Differences |  |  |  |  |  |  |  |  |  |  |
| Avg Exp Real Rate | 0.49 | 0.18 | 0.07 | 0.03 | 0.01 | 0.01 | 0.01 | 0.00 | 0.00 | -0.00 |
| Avg Exp Inflation | 0.11 | 0.07 | 0.06 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.06 | 0.06 |
| Fwd Term Premium | 0.41 | 0.75 | 0.88 | 0.92 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 | 0.94 |

- Pre-crisis results tell the same story


## What Drives the Co-movement among Bond Yields?

- So far have looked at the time series comovement of the components of yields for a given maturity. Now will look at the cross-sectional comovement of each component across maturities.
- What are the underlying forces leading bond yields to move together?

$\rightarrow$ Forward expectations are only modestly correlated (Corr $\approx 0.2$ )


## What Drives Co-movement among Bond Yields? (cont'd)



- Forward premiums strongly correlated across maturities (Corr $\approx 0.6$ )
- Not imposed in any way in our approach, but consistent with common factor in expected returns (Cochrane \& Piazzesi 2005)
- Correlation breaks down in the crisis


## The Yield Curve and Economic Activity

- Yield curve can be informative about economic activity: low term spreads predict recessions (Estrella \& Hardouvelis, 1991, ...). Which component of term spreads predict GDP growth?
- We decompose the ten year three month spread into the expected future short rate and term premium components and run predictive regressions

$$
\Delta g_{t+h}=\beta_{0}+\beta_{1} \cdot t p_{t}(120)+\beta_{2} \cdot\left(\frac{1}{n} \sum_{i=1}^{120} \mathbb{E}_{t}\left[i_{t+i}\right]-i_{t}\right)+\zeta_{t+h}^{g}
$$

Full Sample: 1983Q1-2016Q3

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Term Spread | $\begin{gathered} \hline 0.156 \\ (0.230) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.228 \\ (0.211) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.299 \\ (0.223) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.397 \\ (0.248) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 0.480^{*} \\ & (0.285) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 0.522 \\ (0.326) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline 0.609^{*} \\ & (0.354) \\ & \hline \end{aligned}$ | $\begin{gathered} \hline 0.537 \\ (0.360) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.509 \\ (0.334) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.383 \\ (0.322) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.223 \\ (0.313) \\ \hline \end{gathered}$ | $\begin{gathered} \hline 0.155 \\ (0.304) \\ \hline \end{gathered}$ |
| $R^{2}$ | 0.005 | 0.011 | 0.019 | 0.035 | 0.053 | 0.062 | 0.085 | 0.066 | 0.059 | 0.034 | 0.012 | 0.006 |
| Expected Term Spread | $\begin{aligned} & \hline-0.027 \\ & (0.220) \end{aligned}$ | $\begin{gathered} \hline 0.080 \\ (0.224) \end{gathered}$ | $\begin{gathered} 0.172 \\ (0.244) \end{gathered}$ | $\begin{gathered} \hline 0.280 \\ (0.265) \end{gathered}$ | $\begin{gathered} \hline 0.378 \\ (0.289) \end{gathered}$ | $\begin{gathered} \hline 0.438 \\ (0.334) \end{gathered}$ | $\begin{gathered} \hline 0.539 \\ (0.357) \end{gathered}$ | $\begin{gathered} \hline 0.465 \\ (0.353) \end{gathered}$ | $\begin{gathered} \hline 0.421 \\ (0.312) \end{gathered}$ | $\begin{gathered} \hline 0.288 \\ (0.289) \end{gathered}$ | $\begin{gathered} \hline 0.123 \\ (0.269) \end{gathered}$ | $\begin{gathered} \hline 0.046 \\ (0.248) \end{gathered}$ |
| 10-Year Term Premium | $\begin{gathered} 0.690^{* * *} \\ (0.166) \\ \hline \end{gathered}$ | $\begin{gathered} 0.684^{* * *} \\ (0.172) \\ \hline \end{gathered}$ | $\begin{gathered} 0.703^{* * *} \\ (0.179) \\ \hline \end{gathered}$ | $\begin{gathered} 0.773^{* * *} \\ (0.255) \\ \hline \end{gathered}$ | $\begin{gathered} 0.808^{* * *} \\ (0.285) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.788^{* *} \\ & (0.316) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.835^{* *} \\ & (0.347) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.766^{*} \\ & (0.390) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.783^{* *} \\ & (0.368) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.674^{*} \\ & (0.374) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.520 \\ (0.381) \\ \hline \end{gathered}$ | $\begin{gathered} 0.472 \\ (0.391) \\ \hline \end{gathered}$ |
| $R^{2}$ | 0.122 | 0.095 | 0.086 | 0.094 | 0.098 | 0.092 | 0.106 | 0.087 | 0.089 | 0.069 | 0.048 | 0.047 |

## What We Have Learned So Far?

(1) Term premiums are the main driver of bond yields, accounting for bulk of variation in levels and nearly all in differences.
(2) Term premiums, not expected rates, are the dominant source of co-movement across rates.
(3) Term premiums, not expected rates, predict future output growth.
$\rightarrow$ If central bank aims at stabilizing economy via long-term rates, our results suggest can primarily do so by affecting term premiums.

This raises the following questions:

- Do term premiums respond to policy shocks? What other macroeconomic shocks drive term premiums?


## Response to Macroeconomic Shocks

- We assess how term premiums respond to previously identified structural shocks. Follow Romer-Romer $(2004,2010)$ and estimate

$$
t p_{t}=a+\sum_{i=0}^{N} b_{i} \epsilon_{t-i}+\sum_{j=1}^{M} c_{j} t p_{t-j}+e_{t}
$$

where $\epsilon_{t}$ the structural shock and $e_{t}$ an unobserved disturbance. Lag lengths $N$ and $M$ chosen via BIC. Similarly for expected short rates.

- Consider three categories of macro shocks:

Policy: Kuttner (2001) monetary, Mertens-Ravn (2012) fiscal shock
Demand: Gilchrist-Zakrajsek (2012) and Del Negro et al. (2012) financial shocks, Jo-Sekkel (2016) and Basu-Bundick (2015) uncertainty shocks
Supply: Hamilton $(1996,2003)$ oil price shock, Kilian (2008) oil quantity shock, Barsky-Sims (2011) news shock

## Policy Shocks

Kuttner（2001）：1988m11－2007m12
Expected Rates（1Y1Y）





Mertens and Ravn（2012）：1983Q1－2006Q4
Expected Rates（1Y1Y）





## Term Premiums and Macroeconomic Shocks: Takeaways

- "Policy" and "Demand" shocks have significant long-lasting effects on term premiums.
- Consistent with Piazzesi \& Schneider (2007) and Rudebusch \& Swanson (2012): term premium proportional to negative of covariance between consumption growth and inflation
- MP shocks drive output/inflation in same direction, lower term premium.
- Similar for financial (spread) shocks: induce positive co-movement between output and inflation and reduce term premiums
- Less clear evidence about uncertainty and supply shocks.


## Conclusion

- Long literature has used models to decompose yields into expected future short rates and term premiums, treating both as unobserved, and often reaching very different conclusions.
- We obtain term premiums as difference between observable yields and observable survey measures of expected future short rates.
- Find they are the dominant source of variation and co-movement in term structure of interest rates.
- Show they are economically important.
- If one would like to have any hope of explaining the variation in bond yields in a model, have to incorporate time varying term premiums.


## Appendix Slides

## Data

- We use all available U.S. surveys of professional forecasters:

BCEI Blue Chip Economic Indicators
BCFF Blue Chip Financial Forecasts
CE Consensus Economics
DMP Decision Makers' Poll
EF Economic Forecasts
GN Goldsmith-Nagan
Liv. Livingston Survey

SPD Survey of Primary Dealers
SPF Survey of Professional Forecasters

- Collect monthly forecasts for Real GNP/GDP growth, CPI inflation, and three-month T-bill
- All available forecast horizons from very short term (e.g. current quarter) up to very long term (e.g., "long-run" or 7-11 years ahead).
- Total of 602 survey-horizon pairs for 1955-2016 sample.


## Data (cont'd)

|  | BCFF | BCEI | CE | DMP | EF | GN | Liv. | SPD | SPF |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Survey Sample (full) |  |  |  |  |  |  |  |  |  |
| Frequency | Monthly | Monthly | Monthly | Irregular | Monthly | Quarterly | Biannually | 8 per year | Quarterly |
| RGDP: | 1984-present | 1978-present | 1989-present | n /a | 1984-1995 | $\mathrm{n} / \mathrm{a}$ | 1971-present | 2011-present | 1968-present |
| CPI: | 1984-present | 1980-present | 1989-present | 1978-1987 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 1946-present | 2011-present | 1981-present |
| TBILL: | 1982-present | 1982-present | 1989-present | $\mathrm{n} / \mathrm{a}$ | 1984-1995 | 1969-1986 | 1992-present | $\mathrm{n} / \mathrm{a}$ | 1981-present |
| Survey Sample (LT) |  |  |  |  |  |  |  |  |  |
| Frequency | Biannually | Biannually | Quarterly | $n / a$ | $n / a$ | $n / a$ | $n / a$ | 8 per year | Quarterly |
| RGDP: | 1984-present | 1979-present | 1989-present | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 1990-present | 2012-present | 1992-present |
| CPI: | 1984-present | 1984-present | 1989-present | 1978-1987 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 1990-present | 2011-present | 1991-present |
| TBILL: | 1983-present | 1983-present | 1998-present | n /a | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | 1992-Present |
| Horizons (NT) |  |  |  |  |  |  |  |  |  |
| RGDP: | $\begin{aligned} & \mathrm{Q} 0-\mathrm{Q} 6, \mathrm{Y} 2 \\ & \mathrm{Y} 1-5, \mathrm{Y} 2-6 \end{aligned}$ | $\begin{aligned} & \text { Q0-Q7, Y2, Y0-4, } \\ & \text { Y1-5, Y2-6, Y1-10 } \end{aligned}$ | Q0-Q8, Y1, Y2 | n / a | Q1-Q4 | $\mathrm{n} / \mathrm{a}$ | $\begin{aligned} & \text { Q1-2, Q3-4, Y2, } \\ & \text { Y0-9 } \end{aligned}$ | Q0-Q2, Y1, Y2 | Q0-Q4, Y2, Y0-9 |
| CPI: | $\begin{aligned} & \mathrm{Q} 0-\mathrm{Q} 6, \mathrm{Y} 2 \\ & \mathrm{Y} 1-5, \mathrm{Y} 2-6 \end{aligned}$ | $\begin{aligned} & \text { Q1-Q7, Y2 } \\ & \mathrm{Y} 1-5, \mathrm{Y} 2-6 \end{aligned}$ | Q2-Q8, Y1, Y2 | Y1-10 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | Q3-4, Y2, Y0-9 | $\mathrm{n} / \mathrm{a}$ | $\begin{gathered} \mathrm{Q} 2-\mathrm{Q} 4, \mathrm{Y} 1, \mathrm{Y} 2 \\ \mathrm{Y} 0-4, \mathrm{Y} 0-9 \end{gathered}$ |
| TBILL: | $\begin{aligned} & \text { Q0-Q6, Y1, Y2 } \\ & \text { Y1-5, Y2-6 } \end{aligned}$ | $\begin{aligned} & \text { Q1-Q7, Y1, Y2, } \\ & \text { Y1-5, Y2-6 } \end{aligned}$ | $\mathrm{M} 3, \mathrm{M} 12, \mathrm{Y} 1, \mathrm{Y} 2$ | $\mathrm{n} / \mathrm{a}$ | $\begin{aligned} & \text { Q1-Q4 } \\ & \mathrm{Y} 0, \mathrm{Y} 1 \end{aligned}$ | M3 | $\begin{aligned} & \mathrm{Q} 0, \mathrm{Q} 2, \mathrm{Q} 4, \\ & \mathrm{Y} 1, \mathrm{Y} 2 \end{aligned}$ | $\mathrm{n} / \mathrm{a}$ | $\begin{aligned} & \text { Q1-Q4, Y1, Y2, } \\ & \text { Y0-9 } \end{aligned}$ |
| Horizons (LT) |  |  |  |  |  |  |  |  |  |
| RGDP: | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6 \\ & \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6, \\ & \mathrm{Y} 5-9, \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | Y3-Y10 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | Y0-9 | Y3, LR | Y3, Y0-9 |
| CPI: | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6 \\ & \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6, \\ & \mathrm{Y} 5-9, \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | Y3-Y10 | Y1-10 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | Y0-9 | Y5-10 | Y0-4, Y0-9 |
| TBILL: | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6 \\ & \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | $\begin{aligned} & \mathrm{Y} 3, \mathrm{Y} 4, \mathrm{Y} 5, \mathrm{Y} 6, \\ & \mathrm{Y} 6-10, \mathrm{Y} 7-11 \end{aligned}$ | Y3-Y10 | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | $\mathrm{n} / \mathrm{a}$ | Y3, Y0-9 |

## Fitting the Data

- Use VAR with time-varying mean
- Model $z_{t}=\left(g_{t}, \pi_{t}, i_{t}\right)^{\prime}$, monthly real output growth, inflation and the short-term interest rate as,

$$
\begin{aligned}
z_{t}-\bar{z}_{t} & =\Phi\left(z_{t-1}-\bar{z}_{t-1}\right)+\nu_{t} \\
\binom{\bar{g}_{t}}{\bar{\pi}_{t}} & =\binom{\bar{g}_{t-1}}{\bar{\pi}_{t-1}}+\eta_{t}
\end{aligned}
$$

where $\bar{z}_{t}=\left(\bar{g}_{t}, \bar{\pi}_{t}, \bar{i}_{t}\right)^{\prime}$ are the stacked drifts and:

- Long-term rate given by Fisher equation: $\bar{i}_{t}=\psi \cdot \bar{g}_{t}+\bar{\pi}_{t}+\bar{\zeta}_{t}$
- $\nu_{t} \sim \mathcal{N}\left(0, \Sigma^{\nu}\right)$ and $\eta_{t} \sim \mathcal{N}\left(0, \Sigma^{\eta}\right)$ with $\Sigma^{\eta}$ diagonal
- $\Delta \bar{\zeta}_{t} \sim \mathcal{N}\left(0, \sigma^{2}\right)$ captures e.g., changes in preferences


## Discussion of Model Features

(1) Shifting endpoints

- Capture structural change and learning. Agents filter from observed data slow-moving drift in fundamentals and temporary factors.
- Consistent with forecaster commentary and observed long-term survey forecasts.
(2) Multivariate setup
- Agents take into account the dynamic interactions across variables when forming expectations.
- Important for explaining stylized facts about forecast dispersion, see Andrade, Crump, Eusepi, Moench (2016).


## MLE Estimation

- Sample: 1955M1-2016M9
- Observation equation

$$
\binom{y_{t}^{A}}{y_{t}^{S}}=\left[\begin{array}{c}
H_{t}^{A} \\
H_{t}^{S}
\end{array}\right] Z_{t}+\binom{\varepsilon_{t}^{y, A}}{\varepsilon_{t}^{y, S}}
$$

where $Z_{t}=\left(z_{t}, z_{t-1}, z_{t-2}, z_{t-3}, z_{t-4}, \bar{z}_{t}\right)^{\prime}$.

- Measurement error $\varepsilon_{t}^{y, S}$ variances differ by variable and forecast horizon, not by survey
- Group by: very short term, short term, medium term, long term
- Nominal long-run short rate equation: $\bar{i}_{t}=\bar{\pi}_{t}+0.92 \cdot \bar{g}_{t}+\bar{\zeta}_{t}$


## MLE Estimation (cont'd)

- Matrices $H_{t}^{A}$ and $H_{t}^{S}$ are constructed using linearized growth rates
- Allows to map quarterly and annual growth rates to underlying monthly growth rates
- Can also use $H_{t}^{S}$ to map explicitly "longer-run" forecasts (e.g., SPD) to appropriate element of $\bar{z}_{t}$


## How Do Constituent Components Co-move?



## Do Expectations Behave Like We Would Expect?



- Strong positive correlation between short-term real rate and medium-term real rate expectations $\rightarrow$ Consistent with standard New-Keynesian monetary transmission mechanism.


## What Drives Co-movement among Bond Yields?



- Short-horizon and long-horizon forwards broadly co-move (Corr $\approx 0.5$ )


## Comparing Survey and Model-Based Term Premia

- Decompose yields using statistical models:
(1) yields only (level, slope and curvature);
(2) adding macroeconomic variables;
(3) estimated over different samples.
- Statistical models imply very different expectations paths for the short term rate. Suggests results are highly model-dependent.
- Term premiums tend to co-move broadly but differ significantly.


## Expected Rates: Short-Term




## Expected Rates: Medium-Term




## Expected Rates: Long-Term




## Term Premiums: Short-Term




## Term Premiums: Medium-Term




## Term Premiums: Long-Term




## Short-Term Interest Rate Forecasts and Forwards

Relative Vol. of Expectations wrt Forwards


Corr. b/t Expectations \& Forwards


- Changes in short-rate expectations are relatively volatile even at longer horizons
- But: correlation between changes in expectations and forward rates declines sharply across maturities


## Short-Term Interest Rate Forecasts and Forwards (cont'd)

## Beta of Expectations wrt Forwards



Corr b/t Expectations \& Term Prem.


- Hence: Share of variance of rate expectations and forwards declines sharply with maturity
- Correlation between expected rates and term premiums is small at shorter maturities and modestly negative at longer maturities


## Financial Shocks

Gilchrist and Zakrajšek (2012): 1983Q1-2010Q3

Expected Rates (4Y1Y)


Del Negro et al. (2013): 1983Q1-2014Q4


Expected Rates (1Y1Y)



Term Premium (4Y1Y)


## Supply Shocks

Kilian (2008): 1983Q1-2004Q3


Barsky and Sims (2011): 1983Q1-2007Q3
Expected Rates (1Y1Y)


Term Premium (1Y1Y)


Term Premium (4Y1Y)


