Estimating yield curves in the wake of the financial crisis

Yield curves capture the relationship between bond maturities and bond yields. They provide a whole range of information, such as insights into market participants' growth and inflation expectations, and are therefore also relevant to monetary policy. Since the onset of the financial, banking and sovereign debt crisis, however, it has become more difficult to interpret yield curves, as factors such as liquidity risk or default risk are now having an increasing impact on yields. Against this backdrop, the following article comments on the results of methods that can be used to isolate a wide array of yield curve determinants. These include growth and inflation expectations and term premiums which change over time, as well as influences stemming from the market structure, such as liquidity haircuts. These methods range from simply determining the differences between two yield curves to identify premiums which are contained in only one of the two curves to estimating affine term structure models with macroeconomic factors. The article finds that it is not always possible to clearly identify changes in inflation expectations or changes driven by liquidity or creditworthiness. However, the analytical tools it presents help to shed some light on developments in yield curves and their determinants. Yield curve models are therefore also a valuable point of departure for gaining a better understanding of monetary policy transmission.

Definition, determinants and monetary policy significance of yield curves

Yield curves describe the relationship between bond yields and bond maturities

The bonds of an issuer with varying maturities are generally remunerated with different rates of interest. A yield curve is a graphical representation of yields on bonds in relation to their residual maturity. Depending on the type of bond, the level and shape of a yield curve can be influenced by the rate of interest paid on alternative investments, expectations regarding future interest rates for shorter maturities as well as by the premiums that are paid to offset the risk of holding a bond over a certain period of time. Risk premiums to hedge against an unexpected change in future short-term interest rates and unexpected inflation developments, but also the risk of a payment default and the possibility, or not, of being able to sell a bond at any time without the danger of influencing its market price,[1] constitute maturity-specific components of bond yields. Furthermore, the interest rates for different maturities of a particular type of bond are also tied together in an arbitrage relationship, under which efficiently functioning markets can be expected to ensure that the relationship between interest rates across different maturities does not offer any scope for investors to pursue trading strategies designed to earn risk-free profits (arbitrage).

Zero-coupon bond yield as a standardised indicator of the total return

For benchmark securities, such as Federal bonds (Bunds), the Bundesbank (in addition to private information service providers) provides estimates of zero-coupon yields for a broad maturity spectrum. To determine these yields, a representative market-traded bond is first selected for each maturity and its zero-coupon yield is calculated. The zero-coupon yield represents a bond's total return, assuming that a one-off inpayment is made at the start of the bond's term and a one-off outpayment at the end of its term. Any payments made during the maturity period (floating or fixed coupon payments) have to be factored into the difference between the purchase price and the redemption price. The zero-coupon yield therefore represents a standardised indicator of a bond's total return – irrespective of the existence, amount and frequency of a coupon. On the basis of a statistical financial model, these representative zero-coupon yields can be used to estimate the relationship between bond yields and bond maturities.[2] The resulting estimated continuous yield curve can then be used to determine the zero-coupon yield for any given maturity.[3]

Bund yield curves generally slope upwards, …

The relationship between bond yields and residual maturities illustrated by the yield curve varies over time. Historically, this curve has tended to slope upwards. On average, the yield curve for Federal securities was a steadily rising curve in the period from January 1999 to May 2013, for example (see chart on page 35). The premium demanded for holding longer-term bonds compensates investors for the maturity risk they incur.

… but can also be flat or slope downwards

But yield curves sometimes have a different shape altogether. From 2006 up until the outbreak of the financial crisis, the yield curve was extremely flat, possibly because future short-term interest rates were expected to fall and because of an environment of low term premiums. Back then, expectations that interest rates would be cut offset the term risk premium included in longer-term bond yields. Market participants' expectations with regard to future interest rate developments are not the only insights that can be gained from the yield curve. The literature also looks into the issue of dividing interest rate expectations into inflation and growth expectations as well as the information

1 See Y Amihud, H Mendelson and L H Pedersen (2005), Liquidity and Asset Prices, Foundations and Trends in Finance, Vol 1, No 4, pp 269-364.
2 The Bundesbank uses the Svensson parametric model. See Capital market statistics, June 2013, Statistical Supplement 2 to the Monthly Report, p 6 ff, and L Svensson (1994), Estimating and Interpreting Forward Interest Rates: Sweden 1992-1994, NBER Working Papers 4871.
3 Data should not be extrapolated either at the short or long end beyond the residual maturities of the bonds that were originally used for the estimate, see R S Gürkaynak, B Sack and J H Wright (2007), The U. S. Treasury yield curve: 1961 to the present, Journal of Monetary Economics, Vol 54, No 8, p 2291 ff.

content of the slope and changes in the slope of the yield curve for future macroeconomic developments (see section 2).

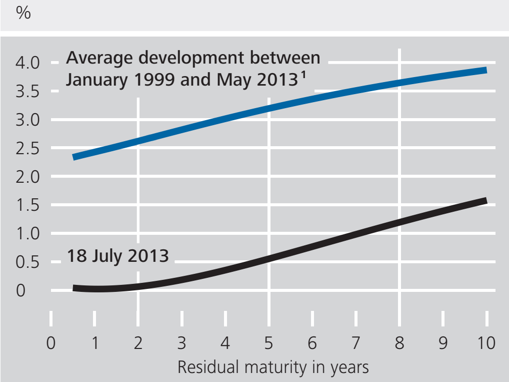
Calculating risk premiums for different bond types

A yield curve can be used to depict more than just government bond yields. Other issuers also offer bonds with different maturities. Yield curves can be estimated for Pfandbriefe or KfW agency bonds, for example, but in these cases, the relationship between yields and maturities differs from that of Federal bonds primarily in terms of the credit default and/or liquidity premiums. Given that Federal securities are deemed to be virtually default-proof and traded in a highly liquid market, the yield curve for Federal bonds lends itself as a suitable pricing benchmark for other securities and investments for which discount rates across different maturities are required in order for them to be evaluated. A comparison of the yield curves of various bonds consequently allows risk premiums to be approximated. Comparing the yield curves of Federal securities and KfW bonds offers insights into liquidity premiums for different maturities, for instance. Meanwhile, implied inflation expectations and inflation risk premiums can be derived from the spread between nominal and inflation-indexed Federal bond yields.

Different approaches in the literature for estimating yield curves

A number of different approaches for modelling yield curves are put forward in the literature.[4] Statistical financial models can also be used in cases where the estimation precision and the forecasting performance need to be as good as possible.[5] If, however, it is the fundamental, economic determinants of interest rate developments that are of interest, empirical models, which lie at the crossroads between the macroeconomic and finance literature, provide more suitable information about the relationship between term structure dynamics and macroeconomic developments. In addition to statistical factors, these models consequently also draw on macroeconomic variables in order to estimate the term structure.[6] Applied versions of this class are also presented in the models discussed in section 3.

Yield curves of German Federal securities*



* Interest rates for (hypothetical) zero-coupon bonds (Svensson method) based on listed Federal securities. **1** Average since the launch of monetary union, calculated on the basis of month-end data.

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Relationship between short and long-term interest rates important for monetary policy transmission

From a central bank's perspective, the yield curve is an important benchmark as the interest rates across the entire maturity spectrum are key determinants for future economic and price developments. Its level plays a crucial role in intertemporal decisions made by the various agents – in the corporate sector for instance – about whether an investment project is profitable or not. In this context, the monetary policy short-term interest rate has traditionally been the lever used by central banks to exert an influence on macroeconomic activity and price developments. Yet, impulses from the short-term end are not transmitted one to one to the long-term end of the yield curve. Long-term

4 An overview of this literature can be found, inter alia, in R S Gürkaynak and J H Wright (2012), Macroeconomics and the Term Structure, Journal of Economic Literature, Vol 50, No 2, p 331 ff and G D Rudebusch (2010), Macro-Finance Models of Interest Rates and the Economy, The Manchester School Supplement, p 25 ff. See also Deutsche Bundesbank, Monthly Report, April 2006, p 15 ff.
5 See C R Nelson and A F Siegel (1987), Parsimonious modeling of yield curves, Journal of Business, Vol 60, p 473 ff; L Svensson (1994), op cit, as well as F X Diebold and C Li (2006), Forecasting the term structure of government bond yields, Journal of Econometrics, Vol 130, No 2, p 337 ff.
6 See A Ang and M Piazzesi (2003), A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables, Journal of Monetary Economics, Vol 50, p 745 ff; J H Wright (2011), Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset, American Economic Review, Vol 101, p 1514 ff and S Joslin, M Priebsch and K J Singleton (2009), Risk Premiums in Dynamic Term Structure Models with Unspanned Macro Risks, Working Paper.

interest rates declined in the years prior to the financial crisis, for example, despite the fact that the Federal Reserve, for instance, gradually raised its (short-term) interest rates. This seemingly contradictory response by long-term interest rates, which can be explained by risk premiums moving in opposite directions, clearly shows that monetary policy measures do not perfectly determine long-term interest rates.[7]

Quantitative easing aims to flatten the yield curve at the long end

With interest rates nearing the zero lower bound in the short-term segment, a number of central banks have in recent years tried to apply non-standard monetary policy measures in the form of quantitative easing to directly influence long-term interest rates by making substantial purchases of long-dated government bonds in the secondary market. Under no-arbitrage conditions, such a policy can serve to drive down interest rates if it can convince market participants more effectively than purely verbal communication that the central bank is prepared to keep short-term interest rates low for an extended period of time. An alternative transmission channel would exist if the bond market were divided into different maturity segments and bonds and central bank liquidity did not constitute perfect substitutes for investors. Yields would then result from the respective maturity-specific bond supply and demand. Ultimately, however, one must assume that the link between interest rates for different maturities in the deep and liquid markets for benchmark bonds cannot be permanently disrupted in this manner. Nevertheless, the relationship between bond yields and bond maturities has attracted increasing attention as a result of the non-standard monetary policy measures.

The slope of the yield curve as a leading indicator

Negative yield spreads as a leading indicator for recessions …

Market participants' forward-looking expectations regarding growth and notably inflation contained in the nominal yield curves are a matter of particular interest for monetary policymakers. A large number of econometric studies filter this information out of the data. For example, they find a positive empirical relationship between the slope of the yield curve and future growth and output variables.[8] Moreover, negative yield spreads between long and short-term interest rates, ie an inverse yield curve, appear to precede a recession.[9] At the Bundesbank, this is done using a model based on financial market variables for determining the likelihood of a recession in Germany.[10] It is shown that the difference between ten-year and three-month interest rates can be used to capture relatively accurately the periods of weak cyclical growth since 1977 with a lead time of five months.

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7 For a more detailed description of the seemingly contradictory interest rate developments in the USA in 2005, see Deutsche Bundesbank, Monthly Report, October 2007, p 27 ff.

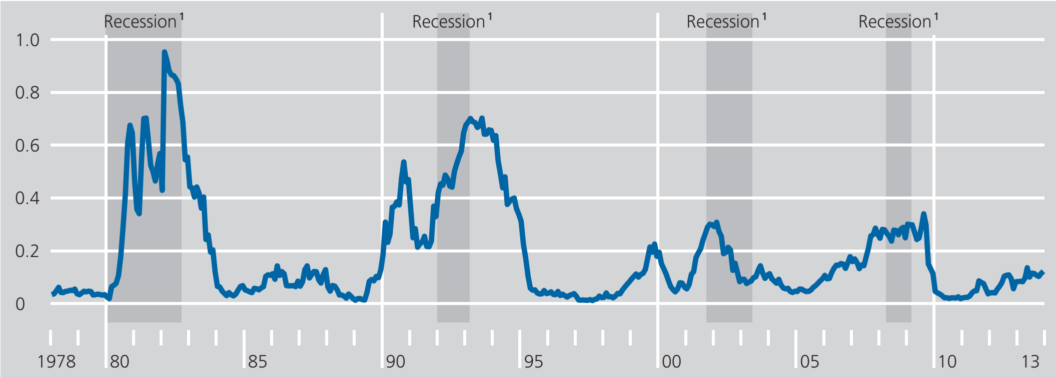
8 An early study was conducted by A Estrella and G Hardouvelis (1991), The term structure as a predictor of real economic activity, Journal of Finance, p 555 ff.

9 Here, the inverse term structure is the explanatory variable used in a probit model, which can be used to estimate the probability of a recession. See, for instance, A Estrella and F S Mishkin (1998), Predicting U.S. recessions: Financial variables as leading indicators, Review of Economics and Statistics, p 45 ff; A Estrella (2005), Why does the yield curve predict output and inflation? The Economic Journal, p 722 ff, as well as A Estrella, A O Rodrigues and S Schich (2003), How stable is the predictive power of the yield curve? Evidence from Germany and the United States, Review of Economics and Statistics, p 629 ff.

10 The results of a comparable model published by the Federal Reserve Bank of New York can be found at www.newyorkfed.org/research/current_issues/ci12-5.pdf. The probit estimation equation of the German model applies the formula: $Recession_{t+5} = F(\alpha+\beta(10J-3M)_t)$, with F denoting the cumulative normal distribution function and the parameters α and β denoting the values -0.50 and -0.49 respectively. The estimation was carried out using month-end data from the term structure estimates of the Bundesbank and the BIS. In only one instance, in the estimation using the spread between long-term and short-term interest rates, did no downturn ensue after the 50% threshold value had been exceeded (type 1 error). This was in the year of German reunification in 1990. The recession between the years 2001 to 2003 is shown with a probability of less than 50% in both estimates (type 2 error). In Germany, there is no fixed definition of recession periods like there is in the USA, which is decided on by the NBER Business Cycles Dating Committee. In line with the classification of cycles used in the research on business cycles, downturns – ie periods with declining output and decreasing capacity utilisation – were applied. In contrast to the technical definition of a recession usually cited in the press – a decline in seasonally adjusted GDP over two consecutive quarters – the number of recessions is consequently reduced. The chosen classification has a greater focus on the turning points of an economic cycle and disregards any short-term fluctuations.

Estimated recession probability using a probit model based on interest rate differentials*

Monthly data



* Yield spread of German ten-year Bunds and 3-month money market rates. **1** A recession is defined as a period of economic downturn with declining output and decreasing capacity utilisation. In contrast to the technical definition of a recession – a decline in seasonally adjusted GDP over two successive quarters – this approach has a greater focus on the turning points of an economic cycle and disregards any short-term fluctuations.

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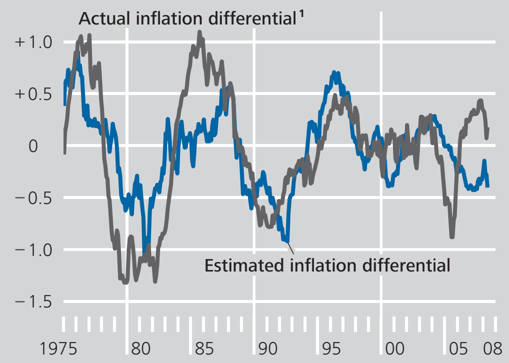
… and as an indicator of future inflation rates

A further possible application is to estimate the empirical explanatory power of the slope of the yield curve for future inflation rate changes.[11] If there is a change in the spread between interest rates for different maturities – ie the slope of the yield curve is altered – this also reflects a shift in expectations for the inflation and growth path over the maturity periods under review. From a monetary policy perspective, expectations of changes in future inflation rates are particularly relevant. The starting point here is that the nominal interest rate for a period can be broken down into the expected inflation rate and the *ex-ante* real interest rate for the same period (Fisher equation). If liquidity, inflation or other risk premiums remain constant over time and if the real yield curve also exhibits a relatively stable yield to maturity relationship over time, changes in the slope of the yield curve can potentially deliver information about changing inflation expectations.[12] Given the above assumptions, however, the outcome of using monthly German term structure data over a period from 1975 to 2013 is that the term structure for differences in maturities of below one year to three years has no more than minimal explanatory power for future inflation rate changes. It is, however, possible that the assumptions used to derive the estimation equation are also too restrictive in the case of short maturities. In this maturity segment, in particular, it would appear that the time-varying term premiums and a high degree of variance in short-term real interest rates largely determine the fluctuations in the nominal term structure.[13] By contrast, the mid-maturity segment of the term structure is rather informative in terms of future inflation differences and at least can be used to explain more than a third of their variance. Against the backdrop of a stability-oriented monetary policy, investors are likely to perceive (short-term) inflation shocks as nothing more than a temporary phenomenon. A comparison of the actually observed inflation differences and those estimated using the term structure for the three to five-year

11 See F S Mishkin (1990), What does the term structure tell us about future inflation?, Journal of Monetary Economics 25, p 77 ff and for German data: S Schich (1999), The information content of the German term structure regarding inflation, in: Applied Financial Economics 9, pp 385-395 and S Gerlach (1997), The Information Content of the Term Structure: Evidence for Germany, in: Empirical Economics, 22(2), pp 161-179.
12 This implies that the market participants have rational expectations, ie that they do not make any systematic projection errors, and that the inflation expectations at any given point in time are the best estimator for future realised inflation.
13 See, for example, F S Mishkin (1990), The information in the longer maturity term structure about future inflation, in: The Quarterly Journal of Economics 105, p 815 ff. Nevertheless, this effect appears to be particularly pronounced in the case of maturities of less than one year, and less so for yield spreads between two and three years. See E F Fama and R R Bliss (1987), The information in long maturity forward rates, American Economic Review 77, p 680 ff.

Comparison of estimated and realised inflation differentials*

%, monthly data



Source: Eurostat and Bundesbank calculations. * Inflation rate in five years' time minus the inflation rate in three years' time. **1** Moved forward by five years.

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horizon does indeed exhibit a relatively high degree of correlation. A break in the co-movement between the estimated and the actually observed inflation differences has, however, been observed over the past few years, which could be linked to the growing importance of risk premiums as a determining factor for yield curves.

Development of forward inflation rate shows changes in long-term inflation expectations

As an alternative to the Fisher equation, inflation-indexed bonds can be used as a means of extracting inflation expectations. An inflation-indexed bond is an instrument which, in addition to a coupon, also pays an amount, mostly annually, to offset the realised inflation rate in the period in question. The compensation included in the yield on a nominal bond for expected inflation rates and an additional inflation risk premium to cover the uncertainty about whether the expected inflation rate will materialise are thus dispensed with (see chart on page 39). The break-even inflation rates (BEIR) derived from the spread between the nominal and real yield curves indicate how market participants expect inflation rates to develop, on average, until maturity. Since the anchoring of long-term inflation expectations has a particular bearing on assessments of monetary policy credibility and effectiveness, long-term forward inflation rates are used for observation purposes. These disregard the volatile short-term break-even inflation rates and derive long-term inflation expectations from the slope of the BEIR curve. Calculating spreads for a second time additionally eliminates term premiums, which remain constant over the maturities. The five to ten-year forward inflation rate for the European harmonised index of consumer prices (HICP) moved unevenly during the crisis years from 2008 to 2013 without giving any indication of a robust trend towards increasing or decreasing inflation expectations. If we interpret the anchoring of inflation expectations as being a long-term stable mean value, we can therefore speak of inflation expectations remaining anchored. The other components making up the definition of anchoring, namely uncertainty or dispersion and the low level of responsiveness to temporary changes in the macroeconomic environment will not be discussed here. Studies based on surveys of households and professional forecasters, and data on inflation options, which depict higher moments of the probability distribution of expected inflation rates, have, however, revealed that shifts have most certainly taken place here.[14]

Forward inflation rate influenced by economic developments and effects of the crisis

Fluctuations in the long-term forward inflation rate (see chart on page 40) likewise need to be interpreted. Some movements can be explained by changes in the economic situation in the euro area which, in a downturn, could ease price pressure or, if the growth outlook brightens, might lead to increased capacity utilisation, thus potentially driving prices higher. This was evident *inter alia* in the economic slump at the end of 2008, the slowdown in economic activity in mid-2011 and the recovery in 2010. On the other hand, some movements can be attributed to (technical) market effects, which can only be removed from the data to a limited extent. Investors thus regard Federal bonds as a safe haven for their capital in times of crisis. Not only is the risk of default extremely remote,

14 See, for instance, Deutsche Bundesbank, Monthly Report, November 2012, pp 44-45 or J Menz and P Poppitz (2013), Household's disagreement on inflation expectations and socioeconomic media exposure in Germany, Deutsche Bundesbank Discussion Paper, forthcoming.

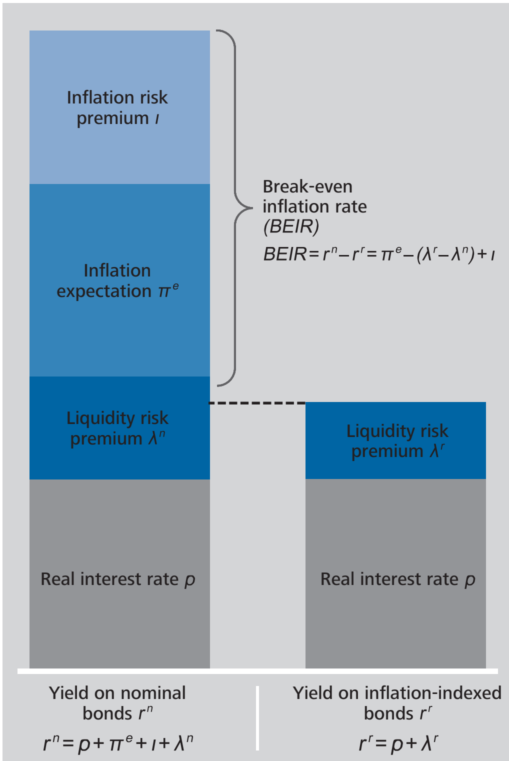
the market for nominal Federal bonds, in particular, is also very liquid. It has likewise been observed that the liquidity premium for shorter maturities is higher than that for longer maturities during critical spells. This is chiefly the case for nominal bonds owing to the higher outstanding volume of this paper, the existence of hedging derivatives and the livelier trading activity at a variety of trading venues. In times of stress, such as those which have punctuated the landscape since the outbreak of the financial crisis, many investors tend to shift their investments, notably those in the short to medium-term maturity segment, out of inflation-indexed bonds into nominal paper. This depressed nominal yields, boosted real yields and caused the BEIR to decline, particularly so for short to medium-term maturities, thereby sending the forward inflation rate higher.

Liquidity premiums not directly observable, corrections thus prone to error

However, much like other premiums, reliably adjusting yields and curves for liquidity effects is a tall order since liquidity premiums likewise cannot be observed directly and must therefore be derived either by making model-based assumptions on the basis of total returns – which are prone to specification and estimation uncertainties – or by calculating spreads relative to other bonds. The spread between KfW agency bonds and Federal bonds indicates just how significant this liquidity premium is for investors. Since KfW bonds have been guaranteed by the Federal government against default, they have the same credit standing as Federal bonds but are traded on a much smaller scale.[15] Accordingly, the spread between KfW and Federal bonds ought to reflect the premium which investors are prepared to pay for holding a particularly liquid security. This premium varies significantly not just over time but also from one maturity segment to the next (see chart on page 41). Yield spreads between KfW and Federal bonds widened considerably when the dotcom bubble on the equity markets burst in 2000, when the US investment bank Lehman Brothers filed for bankruptcy in September 2008, and when, in 2011, the scale and magnitude of the European sovereign debt crisis caused widespread unrest among investors on the capital markets.

Demand for safe liquid securities such as Federal bonds in crisis periods

Calculating break-even inflation rates



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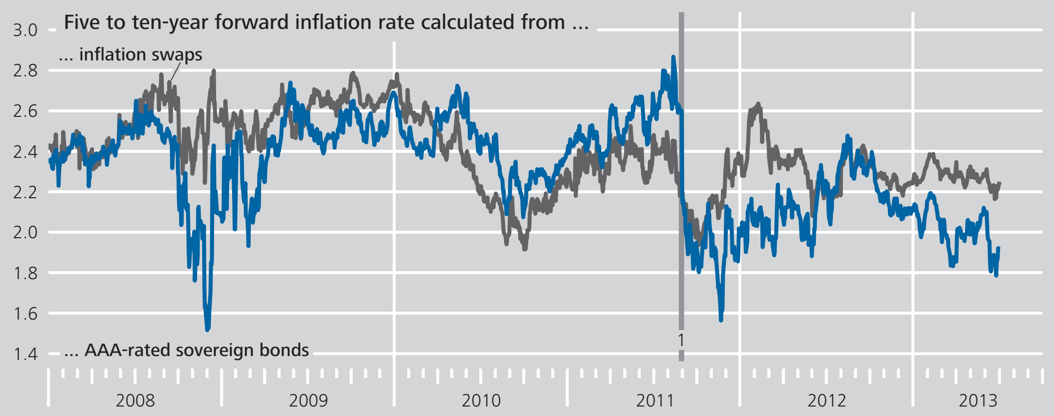
There are different ways of curbing the impact of such liquidity-related effects in the analysis. The US Treasury Department, for instance, moved to only estimating its real yield curves on the basis of more liquid "on the run" bonds.[16] However, this would not be possible for the euro area owing to a lack of ongoing issuances. To minimise estimation errors resulting from differing credit standing and liquidity effects between German and French bonds

15 In June 2013, the volume of KfW bonds in circulation amounted to €372 billion. At the same time, the Bundesbank included Federal bonds with an outstanding volume of €1,291 billion in its yield curve estimate for Federal bonds.

16 The changeover occurred as at 1 December 2008; "on the run" bonds are the most recently issued bonds in a particular maturity range. These are replaced by the most recently issued bond whenever new paper is issued. For further information, see www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=realyield.

Break-even inflation rates for the euro-area HICP derived from yield curve data

%, daily data



Sources: Bloomberg, EuroMTS and Bundesbank calculations. **1** Change from a joint estimation based on all AAA-rated sovereign bonds to a separate estimation based on German and French bonds, respectively, with subsequent aggregation of data with GDP weights.
Deutsche Bundesbank

across different maturities, the Bundesbank switched its calculation of real and nominal yield curves to country-specific data, which were subsequently aggregated with GDP weights.[17]

Shifting to a BEIR calculated from derivatives (in the case of inflation swaps, the fixed leg of a swap agreement to exchange fixed annual payments for the payment of the annual compensation for inflation) as a measure of market participants' inflation expectations, likewise, would not constitute a clearly superior alternative for calculating inflation expectations either. On the one hand, inflation swaps which are not fully secured always reflect *inter alia* the time-varying default risk of the counterparties (primarily banks). On the other hand, a study based on high frequency data has shown that it is sovereign bonds, (reputedly) the most secure form of investment, which dominate price formation in the swap market, particularly so during times of crisis.[18]

Despite the aforementioned problems, break-even inflation rates and forward inflation rate curves estimated using financial market data generally represent an important point of departure for monetary policymakers looking to quickly capture market participants' inflation expectations. The insights these data provide should, however, be taken with a pinch of salt given that risk and liquidity premiums have frequently given rise to sizeable distortions, notably during the financial crisis, as mentioned earlier in this article. This should continue to act as an incentive to improve model estimates which, in turn, would facilitate more robust interpretations of the data.

Liquidity stress, risk premiums and addressing structural changes in affine term structure models – their significance in the financial crisis

Study examines influence of liquidity on yield curve

The influence of the aforementioned liquidity premium on the Federal bond term structure has also been examined in greater detail in an affine multifactor term structure model (see box on pages 43 and 44).[19] Such models use

17 The European Central Bank uses a similar method. See ECB Monthly Bulletin, December 2011, Box 5.
18 See A Schulz und J Stapf (2013), Price discovery on traded inflation expectations: Does the financial crisis matter?, European Journal of Finance, forthcoming.
19 The results cited here refer to A Halberstadt und J Stapf (2012), An Affine Multifactor Model with Macroeconomic Factors for the German Term Structure: Changing Results during the Recent Crises, Deutsche Bundesbank Discussion Paper No 25/2012. For a good overview of affine term structure models, see M Piazzesi (2010), Affine term structure models, in: J Heckman and E Leamer (eds) Handbook of Econometrics, Ch 12.

factors to provide a linear depiction of the impact of different variables. The model specification used here deploys three latent – that is, not directly observable – factors as well as one real activity factor and one "liquidity stress factor". The term "liquidity stress" refers to a situation in which market participants have an extraordinarily strong preference for liquidity, ie they prefer liquid instruments that can be readily traded. They are prepared to forgo yields in favour of liquid financial instruments. Both macroeconomic factors (ie the real activity factor and the liquidity factor) are obtained using a principal component analysis.[20] This approach thus draws on the findings in the literature on macroeconomic factor models, which allow the common dynamics in extensive datasets to be conflated in a small number of factors. Owing to the reduced dimensions, the factors thus obtained mean that the information content of large datasets can be condensed in a small number of variables and harnessed for model estimation purposes.[21]

Investors see Federal bonds as safe haven in times of crisis

Since liquidity stress is directly included in the term structure estimation as an explanatory factor, it is possible to examine the effects of a liquidity shock, ie of an increased preference for liquidity, in an impulse-response analysis. A one standard deviation increase in liquidity stress triggers a decrease in Federal bond yields. While long-term yields such as those on ten-year bonds diminish only marginally, those on one-year bonds respond with an immediate and considerable decline. The deviation from the original yield lasts for almost two years.[22]

Interest rate responses to macroeconomic developments

Similarly, the real activity factor is extracted using a principal component analysis and conflates the dynamics of multiple real activity indicators. An unexpected positive impulse of real activity measured in this way increases interest rates across the entire term. Responses to shocks in real activity are more pronounced than those to liquidity shocks. Moreover, yields for shorter maturities are more strongly influenced by unexpected macroeconomic developments than those on long-term bonds. The impact on interest rates, however, is only a short-lived phenomenon which disappears again after about two years. The model specification with three latent factors, one real activity factor and the liquidity stress factor does indeed deliver estimates that are equally accurate as those provided by the model conventionally used in the literature, in which the short-term interest rate depends on the output gap and inflation as in a Taylor-rule setting.

Yield spread between KfW and Federal bonds for different maturities

Basis points, monthly data

Source: Bloomberg and Bundesbank calculations.
Deutsche Bundesbank

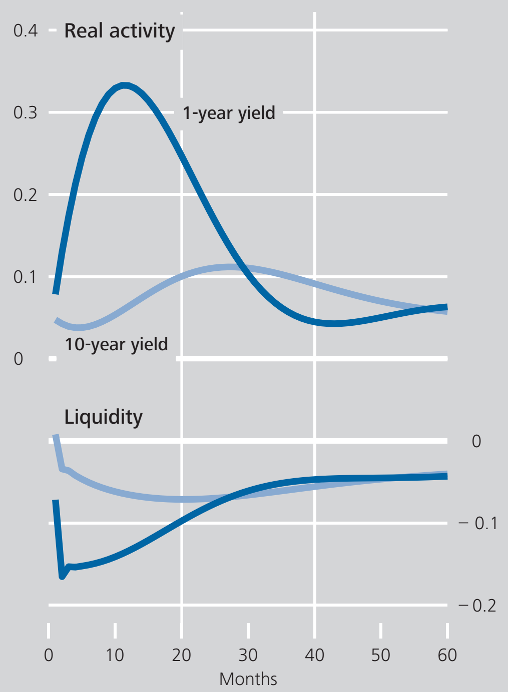
Derivation of risk premiums

Affine term structure models are based on the no-arbitrage assumption, ie the yield curve offers no scope for risk-free profits. The actual risk associated with holding a bond over a given period of time is isolated here through estimates under two probability measures (see also page 44). The model allows yields to be calculated which reflect the average risk-neutral short-term interest rate expectation across a set period of time. Under a different probability

20 See A Ang und M Piazzesi (2003), op cit.
21 The liquidity stress factor represents the second principal component in the yield spread between KfW bonds and Federal bonds and of other liquidity measures such as bid/ask spreads. It captures the maturity-specific liquidity effect and is directly included in the term structure model estimate as a state variable.
22 For more information on the problems entailed in clearly identifying parameters and macroeconomic shocks in affine term structure models see, for instance, J Hamilton and J Wu (2012), Identification and estimation of Gaussian affine term structure models, Journal of Econometrics, 131, pp 405-444.

Impulse responses* of one-year and ten-year yields

%



* Yield response to a positive real economic shock or to a positive one standard deviation shock in terms of liquidity stress.
Deutsche Bundesbank

measure, however, it is also possible to calculate those yields which additionally contain the risk premium for holding a bond. Thus, the risk premium can be calculated as the spread between yields estimated using the two alternative probability measures. The derived (term) risk premiums are higher for long terms than for short ones and also vary considerably over time. Correspondingly, premiums for all maturities were low during the calm economic period around 2005 (see chart on page 44).[23] Interest rate expectations were considered to be relatively stable, which meant that investors demanded a smaller risk premium for providing capital over longer periods of time. A decomposition of the yield on ten-year Federal bonds into the risk premium and interest rate expectations also illustrates that these two yield components were roughly equal between 2005 and 2007,[24] indicating that compensation for risk was relatively insignificant in terms of the total return.

Structural changes determine time-variable estimates

When estimating interest rates over extended periods of time, analysts should be aware that the structural relationships assumed in the estimation approach are subject to change over time. For instance, compared with the calmer economic period between 2002 and 2007, the financial and sovereign debt crisis in recent years has altered the potential for macroeconomic indicators to influence the term structure. Crisis-induced developments now have a greater bearing on the yield curve than conventional determinants, such as real activity and inflation expectations. The reunification of Germany and the establishment of the European monetary union were also events which caused significant breaks in continuity. Such breaks can be accounted for using a time-variable estimate.

Learning: agents assumed to be boundedly rational

Literature on the learning behaviour of economic agents (learning) provides an intuitive approach to accounting for gradual structural changes in modelling.[25] The various expectations under the learning approach are derived from the assumption that market participants are only boundedly rational. Instead of placing equal emphasis on all the information available to them for assessing the economic situation, they tend to focus on the most recent developments. By contrast, information obtained from earlier observations of macroeconomic time series are given a lower weighting in their estimates. The results of this model approach are compared with those obtained using a stand-

23 The risk premium estimates are based on a model specification with a price factor and a real activity factor. See A Halberstadt and J Stapf (2012), op cit, p 8 ff.
24 The results of separating interest rate expectations and risk premiums vary according to the model specification chosen. For a comparison of risk premiums in different models, see G D Rudebusch, B P Sack und E T Swanson (2007), Macroeconomic implications of changes in the term premium, Federal Reserve Bank of St. Louis Review, July 2007, pp 241-265.
25 See T Laubach, R J Tetlow und J C Williams, Learning and the Role of Macroeconomic Factors in the Term Structure of Interest Rates, 2007 meeting papers, Society for Economic Dynamics. An internal research paper which draws on US data demonstrates how the learning approach can be used to derive investors' alternative interest rate expectations. See A Halberstadt (2013), The Term Structure of Interest Rates and the Macroeconomy: Learning about Economic Dynamics from a FAVAR, mimeo.

Gaussian affine multifactor term structure models

Gaussian affine multifactor term structure models are frequently used to examine the yield developments of securities with differing maturities. They combine the arbitrage-free model (holding portfolios with identical cash outflows but different bond positions allows no room for risk-free profits) that is anchored in financial market theory with macroeconomic determinants of the yield curve. Besides the short-term interest rate which reflects the monetary policy stance, the variables contained in multifactor models include unobservable factors and developments in the real economy that can potentially affect the level or the steepness of the yield curve. Term structure models are described as being affine if the (logarithmic) bond prices they contain depict a linear function (including a constant) of the determinants. Specification as a Gaussian model implies a joint multivariate normal distribution of bond yields and of the factors with constant conditional variances. Ultimately, specifying the dynamics of the determinants in this way ensures that the stochastic discount factor (SDF) is, in turn, an affine function of the determinants. This SDF guarantees that yields on bonds with differing maturities are arbitrage-free by clearly defining the relationship between current bond prices and expected future bond prices for all maturities.

1) $P_t^n = E_t(M_{t+1} P_{t+1}^{n-1})$,

at all points in time t and for all maturities n where P_t^n denotes the price of a zero-coupon bond with guaranteed payment of a monetary unit at point in time $(t+n)$ and E_t is the expected value at point in time t, conditional on the information available at that juncture. The positive random variable M_t is the SDF. The determinants X_t can take the form of observable macroeconomic variables or of statistical (latent) factors derived from the yield curve. The determinants follow a vector autoregressive process:

2) $X_t = \mu + \Theta X_{t-1} + \Sigma \upsilon_t$,

with υ_t serving as the Gaussian error term ($\upsilon_t \sim N(0, I_d)$), μ as the constant and Θ and Σ as parameter matrices whereby the number of factors (d) provides the dimension. The short-term interest rate y_t^1 is an affine function of the determinants:

3) $y_t^1 = \delta_0 + \delta_1' X_t$.

The aforementioned SDF is defined as:

4) $M_{t+1} = \exp(-y_t^1 - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\upsilon_{t+1})$,

where $\lambda_t = \lambda_0 + \lambda_1 X_t$, and λ_t is also denoted as the market price of risk. This, too, is an affine function of the determinants. If risk-neutrality is assumed, investors assess the potential for good or bad investment outcomes in equal measure, in which case $\lambda_0 = \lambda_1 = 0$, rendering the SDF M_{t+1} in equation 4) dependent only on the short-term interest rate y_t^1. By contrast, the more realistic assumption of an aversion to risk on the part of market participants delivers a positive market price of risk λ_t.

At the same time, the prices of all bond maturities are decided by the determinants via the discount factor. Equations 1), 2), 3) and 4) imply the price of a zero-coupon bond with a maturity of n:

5) $P_t^n = \exp(A_n + B_n' X_t)$,

where A_n and B_n act as functions of the model parameters, such as the variance of the determinants or of the risk parameters.[1]

Term structure models can provide information on the size of (maturity) risk premiums demanded by risk-averse investors, provided two different probability measures are assumed for the development of the state variables in equation 2). In addition to being estimated using the physical probability measure (which generates the actually observable variation) as in equation 2), the process is also examined using the risk-neutral probability measure:

6) $X_t = \mu^* + \Theta^* X_{t-1} + \Sigma \upsilon_t$.

The risk premiums which an investor demands for holding a long-term bond ultimately arise from the difference between the levels implicit in the model and are calculated using the coefficients estimated on the basis of the risk-neutral probability measure and the physical probability measure.

1 $A_{n+1} = -\delta_0 + A_n + B_n'(\mu - \Sigma\lambda_0) + \frac{1}{2} B_n'\Sigma\Sigma' B_n$ and $B_{n+1} = (\Theta - \Sigma\lambda_1)' B_n - \delta_1$. For a derivation, see A Ang and M Piazzesi (2003), op cit.

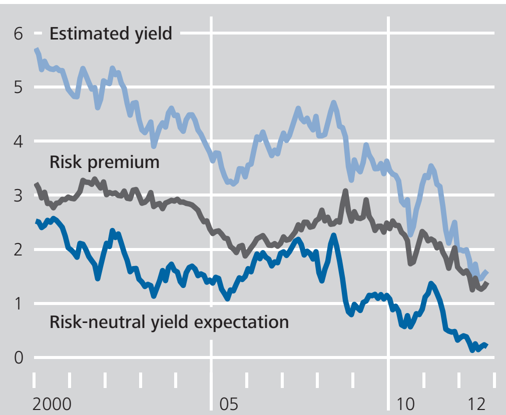
ard approach, that is, one in which the agents are assumed to act entirely rationally.[26]

Limiting the information set: a quasi real-time approach

Both approaches are implemented by ascertaining the dataset which was actually available to investors at each point in time during the observation period. The interest rate estimate is based on this information and the individual estimates for each point in time are then strung together in a series. This approach ultimately provides a quasi real-time estimate of macroeconomic dynamics and interest rates,[27] and also allows a comparison to be made of the uncertainty expressed in the time-specific variances of the residuals regarding the development of macroeconomic factors and the short-term interest rate as calculated under the standard approach and the learning approach. The general decline in uncertainty prior to 2007 reflects the moderation of macroeconomic indicators since the 1980s as described in the lit-

Decomposition of ten-year Federal bond yield

%, monthly data



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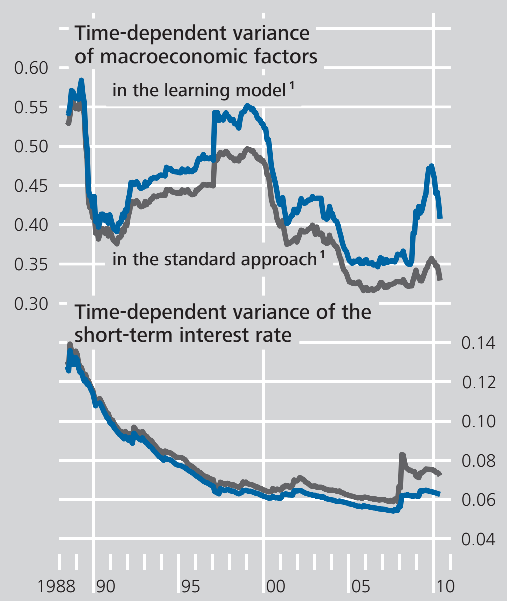
26 See E Moench (2008), Forecasting the yield curve in a data-rich environment: A no-arbitrage factor-augmented VAR approach, Journal of Econometrics, 146, pp 26-43.
27 The procedure is not a complete real-time approach as neither real-time data were used (ie data which were not subsequently revised) nor were delays in the publication of data taken into account. Macroeconomic data are not generally available at the end of the month in question, but rather are published with a time lag of a few weeks.

erature.[28] The focus on more recent developments can be seen in the variance response, notably so at the outset of the financial and sovereign debt crisis in 2008. While the variances calculated using the standard approach hardly respond to the upheaval, the learning approach quickly exhibits a discernible increase.

Time-dependent variances of macroeconomic factors and the short-term interest rate

Monthly data



1 The learning model is estimated based on the assumption that market participants attach greater importance to more recent data than to earlier information. By contrast, the standard approach assumes that market participants give an equal weighting to older and more recent data.

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Conclusion

Term structure models as a means of understanding transmission processes

The models presented in this article are a means of gaining greater insight into the relation of short and long-term interest rates. They can be used to isolate risk premiums and expectation components and thus allow market participants' expectations to be viewed separately from technical factors such as liquidity. They therefore offer a clearer picture of the factors behind the transmission of short-term interest rates, which are driven by monetary policy, through to long-term yields, which tend to be determined more by real activity.

Analytical toolkit needs to be adapted to suit the effects of the financial crisis

The critical developments which have punctuated the landscape in recent years have caused investors to shift the composition of their portfolios and rethink their risk perception, besides sparking numerous regulatory initiatives, bringing about significant structural changes in the financial markets. Taking heed of these developments and steadily refining or adapting the analytical toolkit used for assessing the term structure are important steps towards placing statements on the implications of term structure dynamics and on the efficiency and credibility of monetary policy on a more stable footing.

28 See, for instance, J H Stock and M W Watson (2003), Has the Business Cycle Changed and Why?, NBER Macroeconomics Annual 2002, Vol 17, pp 159-230.