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Time series properties of a rating system based on financial ratios

Ulrich Krüger
(Deutsche Bundesbank)

Martin Stötzel
(Universität Karlsruhe)

Stefan Trück
(Universität Karlsruhe)

Discussion Paper
Series 2: Banking and Financial Studies
No 14/2005

Deutsche Bundesbank, Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main,
Postfach 10 06 02, 60006 Frankfurt am Main

Tel +49 69 9566-1
Telex within Germany 41227, telex from abroad 414431, fax +49 69 5601071

Please address all orders in writing to: Deutsche Bundesbank,
Press and Public Relations Division, at the above address or via fax +49 69 9566-3077

ISBN 3–86558–102–1

Abstract

This paper provides an overview on classical and new methods for testing time series properties of migration matrices.
It is well known that due to cyclical behavior of the economy transition matrices for many credit portfolios cannot be considered to be constant through time. Further, transition matrices are dependent on the used rating methodology. We investigate the changes in migrations of an extensive rating system based on financial ratios. Our findings are time-inhomogeneity, second-order Markov behavior, a tendency for "rating equalization" and vast effects of migration behavior on risk figures like expected shortfall and VaR. We further illustrate how changes in migration matrices can be related to macroeconomic factors.

Keywords Reduced Form Models, Rating Transitions, Markov Property, Internal Rating Systems, Time Homogeneity, Matrix Norms

JEL classification C13, G20, G33

Non-Technical Summary

Migration or transition matrices are major inputs for risk management and Value-at-Risk calculations in credit portfolios. Migration matrices are matrices reporting the probabilities of migrating from a given rating to another rating over a chosen time period.

Due to cyclical behavior of the economy migration matrices for many credit portfolios cannot be considered to be constant through time. Further, transition matrices are clearly dependent on the used rating methodology and will therefore show completely different properties. This paper provides an overview on classical and new methods for testing time series properties of migration matrices.

In an empirical analysis we provide results on an extensive rating system based on financial ratios.

We further review several distance measures for matrices in order to analyze effects of shifts of probability mass inside the matrices on risk figures of credit portfolios like Value-at-Risk and Expected Shortfall. The risk figures are obtained by a continuous-time simulation procedure using generator matrices. Finally, we investigate to what extent the suggested distance measures reflect correlations between the risk figures of credit portfolios and macroeconomic variables.

The results can be summarized as follows:

- The observed rating system shows Markov behavior of higher order. Thus, not only the current rating state influences the rating in the next period but also the rating history.
- Instead of the so-called *rating drift* often stated in the literature we find a tendency that upgrades in the system are more likely to be followed by a downgrade in the next period and vice versa.
- A refinement of the rating grid from 7 to 18 classes yields additional information for determining future rating distributions.
- Time-homogeneity has to be rejected for the considered rating system.
- There are strong effects of differences in the migration matrices on the Value-at-Risk figures of an exemplary credit portfolio.
- Using adequate distance measures cyclical changes in the economy can be directly linked to changes in transition matrices.

Based on these findings, we strongly recommend that each bank investigates the individual rating system and migration behavior of its credit portfolio through time to avoid extreme misspecifications of credit risk.

Nichttechnische Zusammenfassung

Migrations- oder Übergangsmatrizen sind eine wesentliche Information im Risikomanagement beziehungsweise zur Berechnung des Value-at-Risk von Kreditportfolios. Die Elemente von Migrationsmatrizen sind die Wahrscheinlichkeiten des Überganges von einem bestimmten Ratingszustand zu einem anderen bezüglich eines bestimmten Zeithorizontes.
Aufgrund von Schwankungen im Konjunkturzyklus können solche Matrizen zumeist nicht als konstant angenommen werden. Weiterhin sind die Eigenschaften von Übergangsmatrizen stark von der angewendeten Rating-Methodologie abhängig. Dieses Arbeitspapier gibt einen Überblick über klassische und neuere Methoden zum Test der Zeitreihen-Eigenschaften von Migrationsmatrizen.
Im Rahmen einer empirischen Analyse untersuchen wir verschiedene Eigenschaften eines auf Bilanzkennzahlen basierenden Ratingsystems.
Weiterhin werden verschiedene Abstandsmaße für Matrizen betrachtet, um Effekte von Änderungen innerhalb der Matrizen auf Risikomaße wie zum Beispiel Value-at-Risk und Expected Shortfall für ein Kreditportfolio zu analysieren. Die Risikokennzahlen ergeben sich durch Simulation in stetiger Zeit. Schließlich untersuchen wir, inwiefern die vorgeschlagenen Abstandsmaße geeignet sind, Korrelationen zwischen den Risikokennzahlen der Kreditportfolios und makroökonomischen Variablen wiederzugeben.
Die Resultate können wie folgt zusammengefasst werden:

- Das untersuchte Ratingsystem zeigt Markov-Eigenschaften höherer Ordnung. Das heißt, dass für die Verteilung der Ratings in der kommenden Periode nicht nur die aktuellen Ratings, sondern auch die Ratinghistorien relevant sind.

- Anstatt eines häufig in der Literatur zitierten *rating drift* zeigen die Ratings die Tendenz, sich nach einer Ratingverbesserung in der nächsten Periode wieder zu verschlechtern und umgekehrt.

- Eine Verfeinerung des Ratingschemas von sieben auf 18 Klassen liefert zusätzliche Informationen für die Ratingverteilung in der nächsten Periode.

- Die Zeithomogenität der Übergangsmatrizen ist für das betrachtete Ratingsystem nicht gegeben.

- Die Unterschiede in den Übergangsmatrizen haben gravierende Auswirkungen auf den Value-at-Risk für ein exemplarisches Kreditportfolio.

- Unter Verwendung geeigneter Abstandsmaße können zyklische Änderungen in der Makroökonomie direkt mit Änderungen in den Übergangsmatrizen in Verbindung gebracht werden.

Die Resultate machen deutlich, dass Banken für ihre internen Ratingsysteme und Kreditportfolien grundsätzlich Ratingübergänge und das Verhalten von Übergangsmatrizen im Zeitablauf untersuchen sollten. Ansonsten kann eine deutliche Fehleinschätzung des Kreditrisikos nicht ausgeschlossen werden.

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Time Series Properties of a Rating System based on Financial Ratios[1]

1 Introduction

The last decade has seen a rapidly increasing interest in ratings-based systems for managing and evaluating Credit Portfolio Risk. Credit risk managers in banks put much attention on the development of bank-internal rating systems in order to efficiently calculate economic capital charges which should cover losses arising from the deterioration of the creditworthiness of their borrowers. On the other side, this development was equally encouraged by the Basel Committee of Banking Supervision and by the national supervisory authorities of the G10-group of countries who decided to completely revise the 1988 Basel I Capital Accord. All approaches of assessing credit risk available under the Basel II framework, the standardized approach as well as the internal ratings based approaches use rating information of their borrowers as a crucial input for the calculation of regulatory capital charges.[2]

Sophisticated credit risk models have been developed or demanded by banks to assess the risk of their credit portfolio better by recognizing the different underlying sources of risk. Following Altman et al [1] credit pricing models can be divided into the two main categories "Structural Form Models" and "Reduced Form Models". In contrast to structural form models reduced form models do not condition default on the value of the firm, and parameters related to the firm's value need not to be estimated to implement them.

Default probabilities for the different rating categories but also the probabilities for moving from one rating state to another are important issues in these models. Following Jarrow et al [18], not only the "worst case" event of default has impact on the price of a bond, but also possible changes in the rating of a company or an issued bond. Therefore, several models using transition matrices as the driver of credit risk were introduced in theory and practise, among them the discrete-time Markovian model by Jarrow, Lando, and Turnbull [18] or the CreditMetrics approach to determining the VaR of a credit portfolio.

[1] The authors would like to thank Thilo Liebig, Klaus Düllmann, Christian Schmieder, Til Schürmann and Peter Raupach for their insightful comments and Meik Eckhardt for his assistance.

[2] For detailed information on the New Basel Capital Accord for example see [6].

Due to cyclical behavior of the economy, migration matrices for many credit portfolios cannot be considered to be constant through time. Further the first-order Markov property which is often used as an assumption for credit modelling may not be satisfied. Nickell et al [23] show that there is quite a big difference between transition matrices during an expansion of the economy and a recession. For example, Bangia et al [3] and Lando and Skodeberg [20] showed the presence of non-Markovian behavior such as rating drift. However, each rating system behaves differently and results obtained for Moody's or Standard&Poor's migration matrices cannot be expected to be true for migration matrices which result from other rating methodologies, for example the internal rating systems of banks. Therefore, each bank is supposed to investigate the behavior of migration matrices through time for its credit portfolio.

This paper provides an overview on methods for testing time series properties of migration matrices. In an empirical analysis a rating system based on balance-sheet data of Deutsche Bundesbank is tested for time-homogeneity and Markov properties. In a second step the sensitivity to the business cycle is examined more closely and the question to what extent inhomogeneity can be used in order to describe relations between the migration matrices and the economic cycle is addressed.

We would like to point out that in contrast to other studies we investigate rating transitions which are based on changes in credit scores only. Soft factors or personal judgements that might be included in the rating procedure by the major rating agencies or bank's internal rating systems are not considered. Thus, our study could be interpreted as an investigation of "real" changes in credit quality as far as they can be described by scoring models which are based on financial ratios.

The paper is organized as follows: In the remainder of Section 1 we provide some further motivation by describing the importance of transition matrices for purposes of stress testing of credit portfolios. The next subsection includes a brief description of types of rating systems as the rating methodology may have a substantial impact on the transition matrices. Section 2 gives an overview of the considered data and the rating system based on financial ratios. Section 3 summarizes different classic techniques on testing for time series properties of migration matrices and provides empirical results for the considered rating system. Section 4 implements advanced methods for the comparison of migration matrices including continuous-time analysis and matrix norms or distance measures for comparing transitions through time. Section 5 concludes.

1.1 Time Series properties of migration matrices

The investigation of time-series properties of rating systems will be especially important for purposes of carrying out stress tests. By the rules of the Basel II Capital Framework banks are explicitly required to perform a credit risk stress test to assess the effect of certain specific conditions on their regulatory requirements. Banks have to consider the effect of at least mild recession scenarios.[3] These stress tests should be designed to make sure that the minimum capital requirements will be also satisfied under adverse macroeconomic conditions and will be subject to the Supervisory Review Process (Pillar II), see the rules text [4, paragraph 765].

As aforementioned, transition matrices could be used as an input for estimating portfolio loss distributions and Value-at-Risk figures. Based on the distributions of ratings of a bank's credit portfolio transition matrices can be used for simulations of the portfolio loss distribution. If additional exogenous factors are included, the conditional transition matrix can also be used for stress testing. Therefore, to calculate Value-at-Risk figures for a portfolio based on internal ratings it is a key issue to have an adequate transition matrix for the bonds or loans.

1.2 A few words on types of rating systems

Even if this paper does not intend to classify types of rating-systems at least a few remarks are necessary because the rating methodology may have a substantial impact on calculated transition matrices.
First of all, we have to decide whether a rating system is an obligor-specific one. Usually, the borrowers who share a similar risk-profile are assigned to the same rating grade. Afterwards a probability of default (PD) is assigned. Very often the same PD is assigned to all borrowers of the same rating grade. For such a rating methodology the PDs do not discriminate between better and lower creditworthiness inside one rating grade. Consequently, the probability to migrate to a certain other rating grade is the same for all borrowers having the same rating.

[3] For example two consecutive quarters of zero growth to assess the effects of stress scenarios on the estimated risk parameters like probabilities of defaults (PDs) or loss rates (LGDs), see the rules text [4, paragraph 435]. As one possible source of information to assess the possible effects of an economic downturn rating migrations of the exposures of a bank are explicitly mentioned in paragraph 436.

An important classification of rating-systems is the decision whether a rating system is point-in-time (PIT) or through-the-cycle (TTC). A PIT-PD describes the actual creditworthiness within a certain time horizon whereas TTC-PDs take account of possible changes in the macroeconomic conditions. A TTC-PD is not affected when the change of the creditworthiness is only caused by a change of macroeconomic variables which more or less describe the state of the economy and which more or less affect the creditworthiness of all borrowers in a similar way. These two types have to be considered as extreme types of possible rating methodologies. Most rating-systems are somewhere in between these two methods and are neither PIT nor TTC in a pure fashion. As a more detailed reference to these concepts for example see [5]. The question whether a rating system is of the type TTC or PIT is important since obviously we would expect that a TTC-rating method would show fewer rating migrations as the assignment of an upper and lower threshold for the PDs may be adjusted because the state of economy is taken into consideration.
Very often expert judgments override a rating-assignment which originally resulted from a rating-algorithm. We would like to point out here that the rating-system we will consider is based on a logit-model which is estimated on the basis of balance-sheet data only. No expert judgments were considered at all. Consequently, we would expect that the ratings we consider will change more often than the ratings produced by rating agencies. Usually the rating agencies take time to upgrade or downgrade companies whose default risk has changed (for example see [8, p. 85]).

2 The Data

In this section we introduce further notation and give a brief overview of the concepts *Markov property* and *time-homogeneity*. We describe the Bundesbank pool for balance-sheet data which was used to estimate a logistic regression model. In a separate section we explain our rating-score and the balance sheet ratios which were used as input in more detail.

2.1 Discrete time, time-homogenous Markov chains

Throughout this paper we follow the approach of Jarrow, Lando and Turnbull (cp. [18]) of modelling transition probabilities. They considered a discrete time, time-homogenous Markov chain on a finite state space S={1,......,K}. The state space S represents the different rating classes. While 1 denotes the best credit rating, K represents the default case. Hence, the (KxK) one-period transition matrix can be denoted as

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{12} & \cdots & p_{2K} \\ \cdots & \cdots & \cdots & \cdots \\ p_{K-1,1} & p_{K-1,2} & \cdots & p_{K-1,K} \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

where $p_{ij} \geq 0$ for all i, j, i $\neq$ j, and $p_{ii} \equiv 1 - \sum_{\substack{j=1 \\ j \neq i}}^{K} p_{ij}$ for all i. The variable p_{ij} represents the actual probability of going to state j from state i within the next year. The default state K is an absorbing one.

The precise mathematical definitions of the concepts Markov property and time-homogeneity are given in the Appendix A. The definition of the Markov property essentially states that the conditional distribution of X_t on past states is a function of X_{t-1} alone, and does not depend on previous states $X_{t-2}, X_{t-3}, \ldots, X_0$.

For further analysis all data records have been classified into rating categories. The rating we have chosen consists of 18 classes and is based on historically observed class limits following [9].

For most of the performed empirical analyses we found 8 rating categories to be sufficient: From AAA as the best rating category to CCC as the worst for non-defaulted companies. The rating class D includes all defaulted companies of the respective year. This notation follows *Standard & Poor's* labeling standards.

For two ratings $R, Q \in \{AAA, AA, ..., D\}$ we will write $R \leq Q$ if the rating R is worse or equal than the rating Q and, similarly, $R < Q$ if R is worse than Q.

2.2 Balance sheet data for corporates at Deutsche Bundesbank

The data used for the analysis cover a period of 16 years from 1988 to 2003 with totally 500,441 entries of 101,571 different companies of which 2784 companies went into default.

On average balance-sheet data of 5 subsequent years per company are included.[4]

[4] The data of this report are taken from a database of annual balance sheet infor-

18 classes	7 classes	lower PD	upper PD
AAA	AAA	0.00%	0.025%
AA+		0.025%	0.035%
AA	AA	0.035%	0.045%
AA-		0.045%	0.055%
A+		0.055%	0.07%
A	A	0.07%	0.095%
A-		0.095%	0.135%
BBB+		0.135%	0.205%
BBB	BBB	0.205%	0.325%
BBB-		0.325%	0.5125%
BB+		0.5125%	0.77%
BB	BB	0.77%	1.12%
BB-		1.12%	1.635%
B+		1.635%	2.905%
B	B	2.905%	5.785%
B-		5.785%	11.345%
CCC+		11.345%	17.495
CCC	CCC	17.495%	-

Table 1: Rating categories

In order to create a homogeneous evaluation basis, the following restrictions were imposed on the original data-set:

- Listed companies, co-operatives, foundations, and associations were excluded.
- Records from East German companies were deleted.

Each record includes the two optional fields month and year of bankruptcy and an additional indicator variable K with possible values 0 or 1 indicating whether the company went into bankruptcy within the two following years.

mation of non-financial enterprises. The primary purpose of this database is to classify trade-bills of companies as eligible collateral for the use in refinancing operations. See <http://www.bundesbank.de/aufgaben/aufgaben_aufgaben.php>. Further information on balance-sheet-based corporate analysis which is carried out at Deutsche Bundesbank can also be found in the monthly report [10].

2.3 The default model

Based on the data set described above, following Engelmann, Hayden and Tasche [11] we estimate a logit model for the default probabilities (*PDs*) and used the balance sheet ratios X_i which are shown in Table 2 as regressors. All

Variables	Balance sheet ratio
$X1$	Liabilities/Assets
$X2$	Bank Debt/Assets
$X3$	Cash Flow/(Liabilities – Advances)
$X4$	Current Liabilities/Assets
$X5$	Current Assets/Net Sales
$X6$	Cash/Current Liabilities
$X7$	Accounts Payable/Net Sales
$X8$	(Net Sales – Material Costs)/Personnel Costs
$X9$	Net Sales/Assets
$X10$	Ordinary Business Income/Operating Income
$X11$	Net Sales/Net Sales Last Year (linearised)

Table 2: Input factors for Logistic Regression

computations were done using *STATA*-software. By the coefficients obtained from the regression we define the score

$$Z = 6.015 - 2.08X_1 - 0.89X_2 + 1.74X_3 - 0.89X_4 + 1.72X_5 - 0.97X_6 - 4.61X_7 + 0.24X_8 + 0.11X_9 + 5.89X_{10} - 0.58X_{11}$$

and assign a default probability to each record in the usual way by $PD = \frac{e^Z}{1+e^Z}$.

The maximal PD for the given data set is 18.6%. For most records, the calculated PD is much lower. The 99% quantile is $PD = 4.12\%$. Model valuation has been performed with different approaches, and results approve a good model quality. The area under the *ROC-Curve*[5] is 82.24%. The calculation of the *Accuracy Ratio* led to a similar good result confirming the quality of the model. Pairwise comparison of defaulted and non-defaulted records resulted in the ratio of concordant and discordant pairs (*Somers' D*) of 64.86%.

The rating score defined above is almost the same as the score Engelmann et al investigated in [11]. The difference is that their scoring model was

[5] Receiver operating characteristic

calibrated on data from 1987 – 1993. The authors further applied their score to the balance sheets of 325.000 small and medium-size enterprises and performed an out-of-sample and out-of-time validation.

3 Results from the empirical study

3.1 Characteristics of the rating system

Mapping the records with its *PD* to rating classes results in annual rating distributions. We perform most of our analyses on the basis of the average rating distribution and average rating transitions.

Compared to published external ratings as *Standard & Poor's* [24] we observe a lot of records with very small *PD*'s (*AAA* rating), and very few records with higher *PD*'s (*CCC* rating). This fact is not surprising as – by its purpose – the Bundesbank database of balance-sheets of non-financial enterprises contains much entries of companies which have a high rating.

Our main focus in this paper is the analysis of transition matrices. Hence, we count the number of transitions from one year to the following. We are interested in transitions to any of the rating categories, but especially in transitions to the *Default* state.

	AAA	AA	A	BBB	BB	B	CCC	D	Σ	Portion
AAA	3249	679	479	263	61	13	0	2	4744	14,73%
AA	686	721	744	400	71	12	0	1	2635	8,18%
A	431	744	1805	1648	259	32	0	4	4923	15,29%
BBB	218	368	1552	6609	2288	259	0	31	11325	35,17%
BB	45	56	192	2034	3672	864	1	82	6946	21,57%
B	8	6	22	180	748	762	3	71	1800	5,59%
CCC	0	0	0	0	1	3	0	0	4	0,02%

Table 3: Average rounded 1-year transitions

Tables 3 and 4 show the average number of annual transitions and the corresponding transition probabilities.

Transition probabilities are calculated by Maximum Likelihood estimation as:

$$\hat{p}_{ij} = \frac{N_{ij}}{N_i}$$

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	68.48%	14.30%	10.10%	5.54%	1.29%	0.27%	0.00%	0.03%
AA	26.01%	27.37%	28.24%	15.19%	2.69%	0.46%	0.00%	0.04%
A	8.76%	15.12%	36.66%	33.46%	5.26%	0.65%	0.00%	0.08 %
BBB	1.93%	3.25%	13.70%	58.36%	20.20%	2.29%	0.00%	0.27%
BB	0.65%	0.80%	2.76%	29.29%	52.86%	12.43%	0.02%	1.18%
B	0.45%	0.35%	1.24%	10.00%	41.54%	42.33%	0.15%	3.94%
CCC	0.00%	2.91%	0.97%	5.83%	19.90%	66.02%	4.37%	0.00%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Table 4: Average transitions probabilities

where N_{ij} denotes the number of transitions from rating i to j and N_i denotes the total number of transitions from rating i. Hence, one property of the transition (probability) matrices is that row sums add to 1:

$$\sum_j p_{ij} = 1.$$

Comparison to *Standard & Poor's* annual transition matrices reveals that for the given rating system diagonal entries are rather small. Taking into account that the process of assigning rating grades described here does not involve any expert judgements this observation becomes quite plausible.

Rank ordering in transition probabilities could be approved for almost the entire matrix. Following [14, p. 75], rank ordering is associated with the following 3 properties:

- Better ratings should never have a higher chance of default.
- The chance of migration should become less as the migration distance (in rating notches) becomes greater.
- The chance of migrating to a given rating should be greater for more closely adjacent rating categories.

Monotonicity of the default frequencies is violated for the rating CCC. Except for the transition $AA \longrightarrow A$ (where the transition probability (28.24%) is higher than the probability of remaining in rating class AA (27.37%)), all other transition probabilities satisfy these monotonicity conditions.

We also analyzed the *Coefficient of Variation* for each entry p_{ij} of the transition matrix. The coefficient of variation is defined as the ratio of the standard deviation divided by the mean. Thus, it compares the standard deviation for a rating migration from i to j to its mean and gives information on the uncertainty of the transition probability over time.

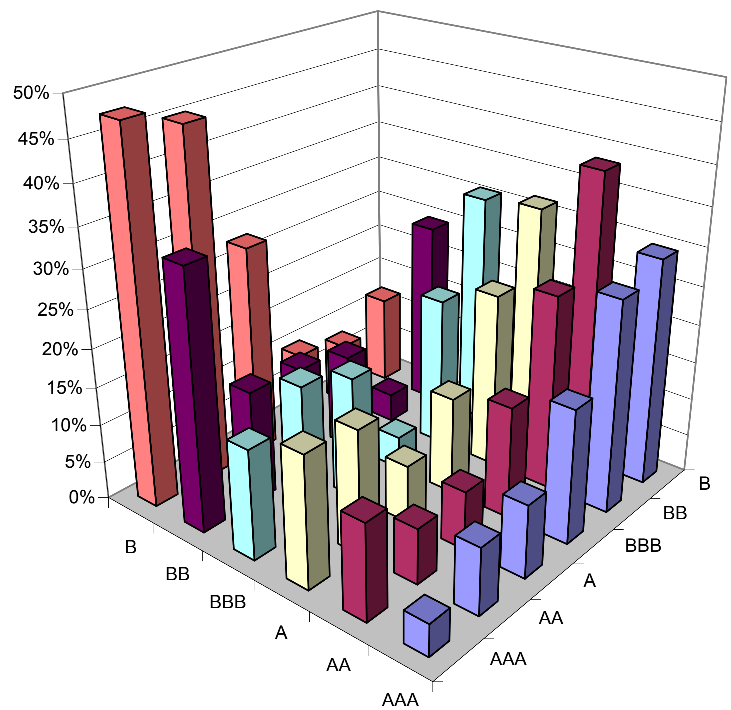


Figure 1: Coefficients of variation for the considered rating system (1988-2003).

We found that the variation of the transition probabilities is less for diagonal entries or entries close to the diagonal. The further away from the diagonal, the higher is the variation coefficient. This means that far migrations are subject to larger variations through time and more uncertain. The result is similar to [3], where higher parameter uncertainty was also observed for entries far off the diagonal.

3.2 Markovian behavior

In this subsection we test to what extent the transition probability matrices for the process X_t of rating distributions can be predicted using information on the distribution of previous years. For this purpose we will proceed in three steps: At first we will check whether the rating transitions from year t to year $t+1$ depend on how the rating in the year t was achieved. We will distinguish whether the current rating (year t) was achieved through an upgrade or a downgrade or whether the rating of year $t-1$ was maintained. In two further steps we will additionally check whether the ratings of the years $t-2$ and $t-3$ have predictive power for the rating of the year $t+1$.

3.2.1 Likelihood Ratio Test

Our first approach to test the Markov property is based on a *Likelihood Ratio Test*. The test compares the likelihood functions L_A and L_B of two models A and B with a different number of parameters, where the model with less parameters is the more restricted one. The Likelihood Ratio Test which is applied in this subsection is described in detail in Bickenbach and Bode [7]. We first consider the hypothesis that the rating distribution in year t does not depend on the rating distributions of the previous years. This hypothesis is denoted by (ND) and is tested against the hypothesis of a *first-order Markov chain* (MK_1). The likelihood functions are given by

$$L_{ND} = \prod_{j=1}^{n} p_j^{n_j},$$

$$L_{MK_1} = \prod_{i=1}^{n} \prod_{j=1}^{n} p_{ij}^{n_{ij}}.$$

In these functions p_j is the probability that a company belongs to the rating grade j, p_{ij} denotes the element with indices i and j of the transition matrix as usually, and n_j, n_{ij} are the numbers of the respective observations.
In order to test on the same data basis, we included in L_{ND} only those records, which have at least one period of history. Using L_{ND} and L_{MK_1} we construct the *Likelihood Ratio*:

$$LR = 2 \ln \left(\frac{\prod_{i=1}^{n} \prod_{j=1}^{n} p_{ij}^{n_{ij}}}{\prod_{j=1}^{n} p_j^{n_j}} \right) = 2 \left(\sum_{i=1}^{n} \sum_{j=1}^{n} n_{ij} \ln p_{ij} - \sum_{j=1}^{n} n_j \ln p_j \right).$$

The likelihood ratio is supposed to be χ^2-distributed with Δm degrees of freedom, where Δm is the difference of estimated parameters in both models. Notice that Δm depends on the number of combinations i, j of rating grades for which at least one rating change from i to j was observed.

We next test *first-order Markov property* (MK_1) against the hypothesis of *second-order Markov property* (MK_2), i.e. we extend the rating "history" by one year and take the rating distributions of the two years $t-1$, $t-2$ into consideration. The likelihood functions are given by

$$L_{MK_1} = \prod_{i=1}^{n} \prod_{j=1}^{n} p_{ij}^{n_{ij}} \quad \text{and} \quad L_{MK_2} = \prod_{i=1}^{n} \prod_{j=1}^{n} \prod_{k=1}^{n} p_{ijk}^{n_{ijk}},$$

and the *Likelihood-Ratio* is calculated as

$$LR = 2\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{k=1}^{n} n_{ijk} \ln p_{ijk} - \sum_{i=1}^{n} \sum_{j=1}^{n} n_{ij} \ln p_{ij} \right),$$

and is again supposedly χ^2-distributed with Δm degrees of freedom.

The results of these two tests show that the hypothesis of second-order Markov property is confirmed, and thus, rating history of two periods is necessary to estimate the actual state distribution as precise as possible. We therefore check with a third test whether the hypothesis of *third-order Markov property* (MK_3) leads to even better results. The testing procedure works theoretically in the same way. We find that the hypothesis of third-order Markov property can be rejected against the hypothesis of second-order Markov property.

H_0	H_1	# Companies	Likelihood-Ratio	Δm	$\chi^2(f)$
ND	$\mathbf{MK_1}$	343436	**26426**	39	54.57
MK_1	$\mathbf{MK_2}$	307033	**1852**	192	225.33
$\mathbf{MK_2}$	MK_3	242214	800	783	**849.21**

Table 5: Results from the LR-Tests

Summarizing our results, we found that our rating system and the rating state distribution depend on two periods of history. All test results are shown in table 5.[6] Significant hypotheses are highlighted in bold.

3.2.2 Rating Drift

In several publications, see for example Bangia et al [3] or Lando and Skødeberg [20], first-order Markov property has been rejected by testing for *Rating Drift* or *Path Dependence*. Rating changes from the previous period have continued in the actual period in most cases. In a first-order Markov chain the rating distibution of the next period is only dependent on the present

[6] The number of companies included is the lowest for the MK_2-MK_3 test, because we can only take into account records with a three-period history.

state and not on any developments in the past. If there is a so-called rating drift or path dependence, then it is assumed that loans that have been downgraded before are less frequently upgraded in the next period, while loans that have experienced prior upgrading are prone to further upgrading. Therefore, two-period changes like 'Down-Down'[7] or 'Up-Up' are generally considered to be more probable than alternating rating changes like 'Down-Up' or 'Up-Down' - the former is the so-called *rating drift*.

In order to investigate if such a rating drift exists in our data we rely on the matrix M which includes the total number of transitions from one rating grade to another, i.e. $\{M(t)\}_{ij}$ gives the number of transitions from rating grade i at time t to rating grade j at time $t+1$. The matrix M is split into the sum of 3 matrices, called *Up-Momentum-Matrix*, *Maintain-Momentum-Matrix*, and *Down-Momentum-Matrix*.[8] These 3 matrices are defined element-by-element in the following way:

$$\begin{aligned}
\{M_{\text{Up}}(t)\}_{ij} :=\ & \text{number of transitions from } i \text{ to } j \text{ of companies} \\
& \text{which were upgraded during the year } t-1 \text{ to } t, \\
\{M_{\text{Maintain}}(t)\}_{ij} :=\ & \text{number of transitions from } i \text{ to } j \text{ of companies} \\
& \text{which had no rating change during the year } t-1 \text{ to } t, \\
\{M_{\text{Down}}(t)\}_{ij} :=\ & \text{number of transitions from i to j of companies} \\
& \text{which had no rating change during the year } t-1 \text{ to } t.
\end{aligned}$$

By construction we have

$$M(t) = M_{\text{Up}}(t) + M_{\text{Maintain}}(t) + M_{\text{Down}}(t).$$

The average transition probabilities we obtained based on M_{Up}, M_{Maintain} and M_{Down} for the the years 1990 until 2003 can be found in Table 6, Table 7 and Table 8. Note that due to the very small number of observations in the CCC rating category we excluded the category from the analysis.
We found an interesting result for our rating system: Companies in a rating category that were upgraded in the previous period are more likely to be downgraded than companies in the same rating category that were downgraded in the previous period. Considering transitions probabilities obtained from the Up-Momentum-Matrix, we find that upgrades (elements at the left side of the diagonal) have *lower* probabilities than downgrades (elements at the right side of the diagonal). In the Down-Momentum-Matrix we see that upgrades have *higher* probabilities than downgrades. To illustrate this

[7] For example a series of subsequent downgrades like AAA $\longrightarrow$ AA $\longrightarrow$ A.
[8] This procedure is the same as in [3].

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	49.37%	21.30%	16.67%	9.80%	2.35%	0.47%	0.00%	0.05%
AA	**20.40%**	24.83%	**32.05%**	18.64%	3.43%	0.59%	0.00%	0.06%
A	6.15%	11.15%	33.57%	40.93%	7.13%	0.93%	0.01%	0.12%
BBB	1.27%	1.86%	7.42%	51.81%	32.88%	4.31%	0.00%	0.46%
BB	0.35%	0.74%	1.90%	17.68%	52.76%	24.28%	0.06%	2.24%
B	0.00%	1.96%	0.00%	7.84%	0.00%	78.43%	11.8%	0.00%

Table 6: Average Transition Probabilities obtained from the Up-Momentum-Matrix

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	75.87%	12.07%	7.42%	3.66%	0.80%	0.15%	0.00%	0.03%
AA	**24.42%**	32.48%	**28.71%**	12.39%	1.80%	0.19%	0.00%	0.01%
A	6.55%	14.98%	41.60%	32.55%	3.95%	0.32%	0.00%	0.06%
BBB	1.12%	2.16%	11.90%	63.58%	19.25%	1.75%	0.00%	0.24%
BB	0.29%	0.47%	1.58%	23.66%	59.54%	13.17%	0.01%	1.27%
B	0.39%	0.26%	0.71%	5.64%	34.22%	54.06%	0.17%	4.54%

Table 7: Average Transition Probabilities obtained from the Maintain-Momentum-Matrix

behavior, in tables 6, 7, and 8 some transition probabilities for upgrades and downgrades of AA rated records have been highlighted in bold. More formally, our observations concerning conditional upgrade and downgrade probablities of a rating process X can be written as

$$P(X_{t+1} > X_t \mid X_t < X_{t-1}) > P(X_{t+1} > X_t \mid X_t > X_{t-1}),$$
$$P(X_{t+1} < X_t \mid X_t > X_{t-1}) > P(X_{t+1} < X_t \mid X_t < X_{t-1}).$$

To investigate whether the differences are significant for single states (rows) and for the entire matrices we used Pearson's χ^2 test. We considered the values of the Maintain-Momentum-Matrix as *expected events* and transitions of the Up-Momentum-Matrix (Down-Momentum-Matrix) as *observed events*. The result both for row-wise comparison and matrix-wise comparison confirms that the matrices are significantly different.
Summarizing the results of this subsection we find that for the considered rating system rating transitions tend to compensate previous-period rating

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-	-	-	-	-	-	-	-
AA	**34,50%**	26,89%	**23,37%**	12,70%	2,20%	0,28%	0,00%	0,04%
A	13,88%	18,95%	35,23%	27,04%	4,26%	0,55%	0,00%	0,10%
BBB	4,01%	6,44%	22,34%	51,97%	13,35%	1,66%	0,00%	0,21%
BB	0,93%	1,11%	4,10%	38,60%	45,77%	8,57%	0,02%	0,88%
B	0,38%	0,40%	1,45%	12,21%	45,89%	35,69%	0,08%	3,90%

Table 8: Average Transition Probabilities obtained from the Down-Momentum-Matrix

changes. These results are quite different to the so-called *rating drift* observed in previous studies, for example Bangia et al [3] and Lando and Skødeberg [20]. The authors found a tendency that companies in a certain rating category which have been downgraded in the previous period are more likely to be downgraded in the next period than other companies in the same rating category which have been upgraded in the previous period. An analogous statement was found for upgrades. Our results may be a consequence of the fact that, in contrast to other studies, we investigate rating transitions which are based on changes in credit scores only. Personal judgements or so-called soft factors included in the rating procedure by the major rating agencies that might induce effects like a rating drift were not considered.

3.3 Refining the Rating Categories

The credit quality of a counterparty is usually being assessed by an extensive credit analysis. One should expect that obligors with the same rating are supposed to have similar estimated parameters of risk, and that rating classes are a sufficient indicator for credit risk estimation. Thus, the rating should contain enough information, in order to be able to explain the probabilities for rating changes. A study by Miller [22] investigated the issue whether in one and the same rating class a finer grid or additional information could provide better information on future rating transitions and defaults.

Following this idea, we investigated whether additional information could be obtained from a finer rating grid. Therefore, based on the obtained default probabilities and the suggested rating class limits in table 1 a finer rating grid was applied. In table 1 the class AA has been split up into $AA+$, AA, and

AA-, and so forth.[9] The key question is if transition probabilities for records at the top of the rating class (and respectively at the bottom) are different from those records which have average default probabilities. To illustrate the examination an excerpt of the result of our analysis (for rating AA) is shown in table 9.

	AAA	AA	A	BBB	BB	B	CCC	D	Σ
	Observations								
AA+	319	279	230	124	24	5	0	1	982
AA	215	242	254	135	24	4	0	0	875
AA-	152	200	260	142	22	4	0	0	779
Σ	686	722	744	401	70	12	0	1	2637
p_{ij}	26,0%	27,4%	28,2%	15,2%	2,7%	0,5%	0,0%	0,04%	100%
	Expected distribution								
AA+	256	269	277	150	26	5	0	0	982
AA-	203	213	220	119	21	4	0	0	779

Table 9: Transitions from rating class AA and from finer rating classes AA-, AA and $AA+$.

Transitions of the other rating classes have been calculated in the same way. We test for significance within the rating classes using the Pearson χ^2-Test. The analysis proofs that differences are highly significant for all rating classes at a 95% confidence level. Even at a 99% confidence level, almost all transitions within the rating classes (and thus its probabilities) showed significant deviations. Therefore, using a refined rating scheme according to table 1 could be helpful to get more accurate forecasts for future transitions of the loans. We conclude that for the given rating system, subdividing the rating classes, and analyzing the rating transitions based on 18 classes provides significantly different results from a scheme with seven rating categories. This result indicates that the choice of an appropriate number of rating buckets is always a tradeoff between accuracy of results and availability of sufficient observations.

[9] Records in $AA+$ have lower PD's, and records in AA- have higher PD's than records in AA.

3.4 Testing time homogeneity

Another assumption frequently stated for rating systems is that rating migration is time-homogenous. The property of time-homogeneity is frequently referred to in the literature related to credit risk modelling. Nevertheless, this property includes some element of idealization. The reasons, why this property cannot be "perfectly" satisfied by observed data is well described in Jafry, Schuermann [17, p. 6]. Due to the fact that transition matrices are stochastic matrices, which means that the sum of the elements in each row is 1, the biggest Eigenvalue is 1. Starting from an initial rating distribution at year s and applying a fixed transition matrix t-times in order to obtain the rating distribution after t years yields a "steady-state"-solution x_∞ as t approaches ∞. In the case that there is an absorbing default state, what we always assume in this paper, the steady-state solution is the unity-vector which corresponds to the default state. Consequently, for long transition periods time homogeneity ultimately implies that all companies default in a certain year in the future. According to the Frobenius-Perron-Theorem, which is well-known from the theory of Markov Chains, the rate at which the system decays towards x_∞ corresponds to the second largest eigenvalue of the transition probability matrix.

For simplicity, let us denote the transition probability matrix for two subsequent years by P. The property time-homogeneity offers the nice feature that the state vector x_u at any future date u can be calculated in terms of the initial state vector x_s by $x_u = P^t x_s$, where P^t denotes the t-th power of the matrix P.

3.4.1 Eigenvalues and Eigenvectors

A first test of time-homogeneity is obtained by considering the Eigenvalues of the transition probability matrix $P = P_1$. Decomposing P into the diagonal-matrix of Eigenvalues $\text{diag}(\Theta_1)$ and the basis-transformation matrix $\Phi = \{\phi_1, \ldots \phi_n\}$ of Eigenvectors we obtain

$$P_1 = \Phi \, \text{diag}(\Theta_1) \, \Phi^{-1}, \tag{1}$$

where the diagonal elements are given by $\theta_{11}, \theta_{12}, \ldots, \theta_{1n}$. Without loss of generality we assume that the column-indices are ordered such that $|\theta_{11}| \geq |\theta_{12}| \geq \ldots \geq |\theta_{1n}|$. [10] Using this decomposition the t-th power of P can be

[10] In our notation the first index is used for the time horizon in years in the homogeneous case whereas the second index is the the number of the Eigenvalue. If the transition

expressed as

$$P_t = \Phi \ \mathrm{diag}(\Theta_1)^t \ \Phi^{-1},$$

with

$$(\mathrm{diag}(\Theta_1))^t = \begin{pmatrix} {\theta_{11}}^t & 0 & \dots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \dots & 0 & {\theta_{1n}}^t \end{pmatrix}.$$

Obviously, the Eigenvalues of P_t are given by $(\theta_i)^t$.
Because of $\ln {\theta_{1i}}^t = t \ln \theta_{1i}$ the sequence θ_{1i}, θ_{2i}, θ_{3i} and θ_{4i} of the i-th Eigenvalues of the matrices P_1, P_2, P_3, P_4 is a log-linear function of t. For example, $\ln \theta_{42} = 4 \ln \theta_{12}$.

Based on these facts from Linear Algebra we check the property time homogeneity as follows: We consider the average transition matrices[11] $\bar{P}_1, \bar{P}_2, \bar{P}_3$ and $\bar{P}_4$ for the time horizons of 1, 2, 3 and 4 years from our data-set and plot the logarithms $\ln(\bar{\theta}_2(t))$, $\ln(\bar{\theta}_2(t))$, $\ln(\bar{\theta}_3(t))$, $\ln(\bar{\theta}_5(t))$ of the four biggest Eigenvalues smaller than 1 of these 4 matrices (see figure 2). Under the assumption of time-homogeneity we would expect that each sequence $\ln \bar{\theta}_j(t)$ ($j \in \{2, \dots, 5\}$) of Eigenvalues would fit in a line. Two observations can be made immediately in the figure: First, the fourth and fifth Eigenvalues do not show log-linear behavior, as the plotted lines are not really straight. And second, the log-Eigenvalues for periods of 2 or more years are not t-multiples of the log-Eigenvalue for 1 year.
As is can be seen in figure 2 the property of time-homogeneity cannot be confirmed by the Eigenvalue analysis described above.

Another way of approaching time-homogeneity is analyzing the Eigenvectors of P. It can be easily checked that the matrices P and any arbitrary power P^t of P have the same set of Eigenvectors.
Plotting the i^{th} Eigenvectors[12] for different time horizons t should always yield approximately the same result, independently of t. We computed the second Eigenvector for $t = 1, \dots, 4$ years and assigned the components of the 4 Eigenvectors to the corresponding rating grades. As figure 3 shows the Eigenvectors are not equal. The curve is getting less steeper as the time

matrices are not necessarily time-independent, for example, if they are empirical matrices that were obtained from the data-set, we use the time-horizon as an argument and not as an index.

[11] Each element of the average transition matrix $\bar{P}_i$ is the arithmetic mean of all transition matrices P_i with a time horizon of i years between 1998 and 2003.

[12] The Eigenvector to the i^{th} largest Eigenvalue is referred to as the i^{th} Eigenvector.

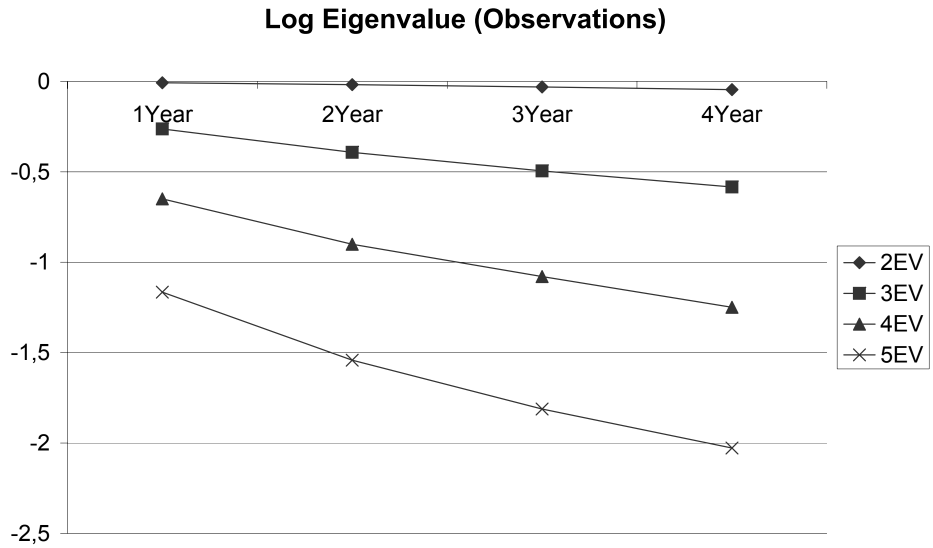


Figure 2: Log-Eigenvalues of 4 years

horizon increases. Based on figure 3 the hypothesis that the process of rating migrations is a homogeneous one should be rejected.

4 VaR Simulation and Distance Measures for Migration Matrices

4.1 Continuous-time analysis

Lando and Skødeberg [20] consider the estimation of credit rating transitions based on continuous-time observations. In reality rating events may happen several times within a one year period and agencies or banks often have access to exact dates of rating events. Following [20, p. 2], the advantage of the continuous-time approach is that transition probabilities and especially default probabilities can be estimated more accurately.

We make use of this approach in order to test the effects of changes in migration behavior on credit VaR or capital requirements. In the continuous-time approach there exists a simple representation of the transition matrices P for a time interval of length t:

$$P(t) = e^{t\Lambda}.$$

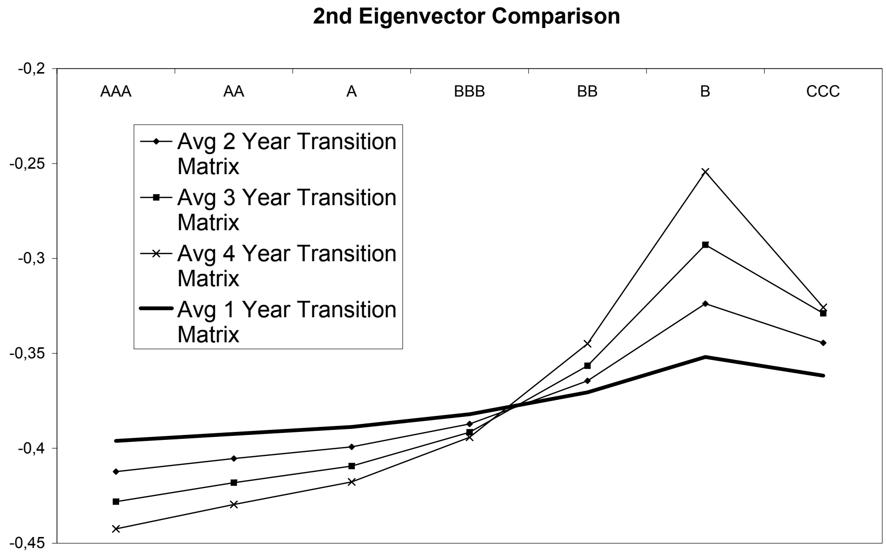


Figure 3: Second Eigenvector for different time horizons

The transition matrix P is calculated as matrix exponential[13] of the *Generator* matrix Λ multiplied by the time horizon t. The generator as $(K \times K)$-Matrix is specified by

$$\Lambda = \begin{pmatrix} -\lambda_1 & \lambda_{1,2} & \cdots & \lambda_{1,K-1} & \lambda_{1,K} \\ \lambda_{2,1} & -\lambda_2 & \cdots & \lambda_{2,K-1} & \lambda_{2,K} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{K-1,1} & \lambda_{K-1,2} & \cdots & -\lambda_{K-1} & \lambda_{K-1,K} \\ 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

There exist several algorithms for the estimation of the Generator. The most cited approach in the literature is the Maximum-Likelihood estimator that is used in the model of Jarrow, Lando and Turnbull and is referred to by Lando & Skødeberg [20]: For $i \neq j$ the (off-diagonal) entries are calculated as the ratio of transitions from rating i to rating j within time t, divided by the total number of entries in rating i during time t:

$$\widehat{\lambda}_{ij} = \frac{N_{ij}(T)}{\int_0^T Y_i(t)dt}.$$

[13] A description of how to calculate a matrix exponential is given in [20] in the appendix. Detailed results on the existence and uniqueness of generators are given in Israel, Rosenthal, Wei [15].

The diagonal entries $\widehat{\lambda}_i$ are specified by:

$$\widehat{\lambda}_i = \sum_{j=1}^{K} \lambda_{i,j}.$$

The concept of using continuous-time transition matrices has some advantages, see [20]. Using generator matrices one can obtain transition matrices for arbitrary time horizons. Due to the fact that within a time-period t multiple rating changes are allowed one can get non-zero estimates for probabilities of rare events which the discrete transition approach estimates to be zero. Further, the continuous framework permits generating confidence sets for default probabilities, the dependence on covariates can be tested and business cycles effects can be quantified. Trück and Rachev [28] use the continuous time approach to determine Value-at-Risk figures for credit portfolios. Based on a continuous time simulation using the generator matrices for a portfolio especially the time series properties of transition matrices from a risk perspective are considered. Therefore, for a hypothetical or real portfolio VaR figures can be simulated for an arbitrary time horizon using the determined generator matrices. The findings in [28] are that VaR figures for the same portfolio vary to a quite large extend. For bad economy scenarios and the corresponding transition matrices VaR figures were up to eight times higher for the same portfolio than for business cycle peaks.

To illustrate the concept of simulating Value-at-Risk figures based on our rating system we consider two different time intervals of 3 years length (1996-1999 and 1999-2002) and calculate the transition matrices and corresponding generators for the two time periods. Tables 10 and 11 show the results for the obtained generators. The periods were chosen since they reflect a period of average macroeconomic conditions for 1996-1999 in comparison to the second period which includes two years of financial distress with a high number of defaults in the years 2000-2001 and 2001-2002. This should result in different estimates for the simulated portfolio risk as well.

In a second step, following the same methodology as in [28], we use a continuous-time simulation approach to derive Value-at-Risk figures for the same hypothetical portfolio for both periods.

As the waiting time for leaving state i has an exponential distribution with the mean $\frac{1}{\lambda_i}$ we draw an exponentially-distributed random variable t_1 with the density function

$$f(t_1) = \lambda_{ii} e^{-\lambda_i t_1}$$

for each company with initial rating i. Depending on the considered time horizon T for $t_1 > T$, the company stays in its current class during the entire

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-0.4331	0.3257	0.0760	0.0210	0.0082	0.0020	0.0000	0.0003
AA	0.6260	-1.6109	0.9070	0.0542	0.0188	0.0049	0.0000	0.0000
A	0.0354	0.5177	-1.2726	0.7104	0.0000	0.0088	0.0000	0.0003
BBB	0.0112	0.0019	0.3209	-0.6983	0.3629	0.0000	0.0000	0.0014
BB	0.0053	0.0120	0.0000	0.5841	-0.8553	0.2355	0.0004	0.0180
B	0.0014	0.0000	0.0347	0.0000	0.9482	-1.0747	0.0027	0.0878
C	0.0000	0.0047	0.0000	0.0882	0.0000	1.1495	-1.2408	0.0000
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 10: ML-estimator for the generator 1996-1999

	AAA	AA	A	BBB	BB	B	CCC	D
AAA	-0.4850	0.3680	0.0748	0.0292	0.0065	0.0049	0.0000	0.0016
AA	0.5404	-1.5901	0.9909	0.0432	0.0131	0.0010	0.0000	0.0015
A	0.0668	0.4978	-1.3050	0.7278	0.0000	0.0114	0.0000	0.0012
BBB	0.0079	0.0035	0.3211	-0.7083	0.3690	0.0000	0.0004	0.0065
BB	0.0086	0.0119	0.0000	0.5982	-0.9374	0.2677	0.0000	0.0510
B	0.0122	0.0000	0.0526	0.0000	1.1056	-1.3760	0.0029	0.2027
C	0.0000	0.0020	0.0000	0.0000	0.0000	1.2451	-1.2471	0.0000
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 11: ML-estimator for the generator 1999-2002

period T. If we get $t_1 < T$, we have to determine which rating class the company migrates to. For this, the interval [0,1] is divided into sub-intervals according to the migration intensities calculated via $\frac{\lambda_{ij}}{\lambda_i}$. Then a uniform distributed random variable between 0 and 1 is drawn. Depending on which sub-interval the random variable lies in, we determine the new rating class j the company migrates to. Then we draw again from an exponentially-distributed random variable t_2 – this time with parameter λ_j from the generator matrix. If we find that $t_1 + t_2 > T$, the considered company stays in the new rating class and the simulation is completed for this firm. If $t_1 + t_2 < T$ we have to determine the new rating class. The procedure is repeated until we get $\sum t_k > T$ or the company migrates to the default state. The default state is considered to be an absorbing state.

This simulation procedure is conducted for every company in the portfolio. Based on the simulation we determine the number and size of losses for the considered portfolio. We assume the number of companies in each rating to the average allocation of ratings for the whole time period from 1988-2002, see table 12. For simplification we assumed for each loan an average exposure of 1 mln Euro and a recovery rate of 0.55.

Rating	AAA	AA	A	BBB	BB	B	CCC
No. of firms	4744	2635	4923	11325	6946	1800	5

Table 12: Distribution of Ratings for the considered average portfolio.

For both time periods 1000 simulations for a 1-year time horizon were run, the obtained loss distribution are displayed in figure 4. Comparing the two loss distributions, the results show a clear difference in the risk figures, for example in the figures for the expected loss, Value-at-Risk and expected shortfall. For the second period from 1999-2002 the simulated mean and Value-at-Risk are more than twice than for the previous period from 1996-1999, see table 13. Even the estimated expected shortfall for the period 1996-1999 is still lower than the mean of the loss distribution for the period of financial distress. The loss distributions for the considered periods are displayed in figure 4. We conclude that differences in credit migration matrices lead to substantial changes also in necessary risk capital and should not be ignored by using an average migration matrix as a basis for credit VaR estimation.

	Mean	$VAR_{0.95}$	$VAR_{0.99}$	$ES_{0.95}$	$ES_{0.99}$
Gen_{96-99}	236.50	277.75	299.75	290.84	309.10
Gen_{99-02}	588.50	654.50	680.62	670.07	689.15

Table 13: Simulated one-year Value-at-Risk and Expected Shortfall for the considered time periods in mln Euro

4.2 Using Distance Measures for analyzing Migration Matrices

In the previous chapter classical tests and evaluation methods for examining time series properties of transition matrices were described. The main issues were the often assumed properties of time homogeneity and Markov

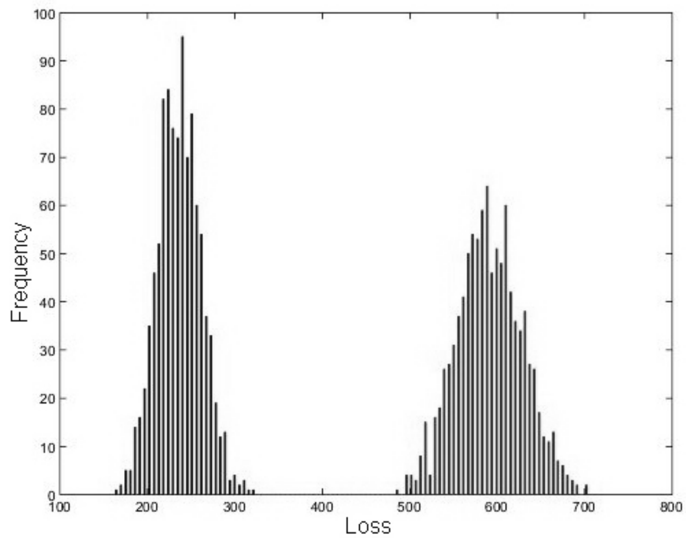


Figure 4: Comparison of Loss Distributions for Considered Portfolios for the two time periods.

behavior. We found that for the considered rating system the assumptions have to be rejected. In the last section we found that different transition matrices lead to quite different capital requirements even for the same portfolio. Therefore, from a Risk Management perspective it is also interesting to measure the grade of deviances and whether the differences in transition matrices are related to Value-at-Risk or capital requirements of a portfolio. In the literature one can find several measures based on cell-by-cell distances, Eigenvalues [13], Eigenvectors [2], singular-values [17] or distance measures including the VaR perspective [26]. We will first give a selection of the measures described in the literature so far and then conduct an empirical analysis on the changes for the considered rating system and transition matrices.

4.2.1 Classical Matrix Norms

The first group to be mentioned are the classical cell-by-cell distance measures. Probably the most intuitive and prominent among this class of measures are the L^1, L^2 metric. They are defined by

$$\begin{aligned} D_{L^1}(P,Q) &= \sum_{i=1}^{n}\sum_{j=1}^{n}|p_{ij}-q_{ij}|, \\ D_{L^2}(P,Q) &= \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(p_{ij}-q_{ij})^2}, \end{aligned}$$

where n is the number of columns and rows as migration matrices are square matrices. The L^1 metric is used for example in Israel et al [15] for comparing migration matrices while Bangia et al [3] suggest the L^2 metric as a distance measure. The literature provides several variations and extensions of the L^1 and L^2 metric. Some of them were used solving optimization problems, e.g. in input-output analysis [16]. Most of them can be represented by a category of distance measures of the form

$$D_{weight}(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n}p_{ij}^{k}|p_{ij}-q_{ij}|^{p}. \tag{2}$$

with k varying from -1 to 1 and p varying from 1 to infinity . For k less than 0, the elements p_{ij} cannot be zero, or the fraction will be undefined.
Lahr [19] suggests a so-called weighted absolute difference (WAD) measure for input-output analysis. The measure is expressed as:

$$D_{WAD}(P,Q) = \sum_{i=1}^{n}\sum_{j=1}^{n}p_{ij}\cdot|p_{ij}-q_{ij}|. \tag{3}$$

Obviously $D_{WAD}(P,Q) \neq D_{WAD}(Q,P)$, so D_{WAD} does not satisfy the symmetry condition. This could be guaranteed for example by defining a distance measure $D_{WAD^{symm}}(P,Q) = 0.5\cdot(D_{WAD}(P,Q) + D_{WAD}(Q,P))$ or $D_{WAD^{symm}} = \max((D_{WAD}(P,Q), D_{WAD}(Q,P))$. Even if the property "symmetry" of a difference index seems to be a desirable property because we would expect it from intuition, it is not very important from a numeric point of view when we will use the indices to describe correlations between the state of the economy and the migration matrices later.
Matuszewski et al [21] suggest a different version of the absolute differences using normalized absolute differences (NAD). In this formulation, differences in large coefficients will contribute less to the value of distance than will equally sized differences in small coefficients. Clearly, this imposes a greater penalty on changes in small coefficients:

$$D_{NAD}(P,Q) = \sum_{i=1}^{n}\sum_{j=1,p_{ij}\neq 0}^{n}\frac{|p_{ij}-q_{ij}|}{p_{ij}}.$$

Since $D_{NAD}(P,Q) \neq D_{NAD}(Q,P)$, the symmetry condition is not satisfied as well but could be ensured by using the same procedure as it was suggested for D_{WAD}. Similar expressions for the L^2 metric are straightforward. The measures obtained are then called weighted squared differences (WSD) and normalized squared differences (NSD).
Trück [26] shows that from a "risk perspective" the surveyed cell-by-cell distance measures are not optimal to measure changes in transition matrices. They do not distinguish between differences in default or non-default transitions. Also there is no distinction between differences that appear in cells to the left (upgrades) or right (downgrades) of the diagonal.

4.2.2 A Norm measuring the mobility of a matrix

Another approach is provided by Jafry and Schuermann [17]. They develop a scalar metric which captures the overall dynamic size of given matrices and contains sufficient information to facilitate meaningful comparisons between different credit migration matrices. Primarily the so-called mobility matrix $\tilde{P}$ is calculated by subtraction of the identity matrix I from the original transition matrix P. Obviously, the identity matrix can be considered as a static migration matrix. Subtracting the identity matrix the authors conclude that only the dynamic part of the original matrix remains.
Further, following Strang [25], the mobility of a matrix can be captured by its so-called "amplifying power" on a state vector x. In [25], therefore it is suggested to use the largest singular value of a matrix as a mobility norm. However, Jafry and Schuermann [17] conclude that the maximally-amplified vector x for a migration matrix is not representative of a feasible state vector. Thus, it is proposed to use the average of all singular values of $\tilde{P}$ to capture the general characteristics of P. The metric is defined as the average of the singular values of the mobility matrix:[14]

$$\mathbf{M}_{SVD}(P) = \frac{\sum_{i=1}^{n} \sqrt{\lambda_i(\tilde{P}'\tilde{P})}}{n}.$$

Therefore, to measure the difference between two migration matrices according to the mobility metric by Jafry and Schuerman one has to calculate

$$D_{SVD}(P,Q) = M_{SVD}(P) - M_{SVD}(Q). \tag{4}$$

The authors show that this metric captures the so-called "amplification factor" or the dynamic part of the migration matrix. Therefore, it approximates

[14] The singular values of $\tilde{P}$ are equal to the square root of the eigenvalues of $\tilde{P}'\tilde{P}$.

the average probability of migration which can be considered as a meaningful magnitude calibration for a metric.

4.2.3 Risk-adjusted difference indices

In the previous section we found that changes in transition matrices have substantial impact on capital requirements for a credit portfolio. Therefore, Trück [26] suggests to consider changes in transition matrices from the angle of risk management. The following issues should be considered in the definition of an appropriate distance measure:

The direction of the transition (DIR): If we would like to introduce a risk-sensitive measure for differences between migration matrices, the direction of the shift in probability mass matters. If more mass is shifted to upgrades there will be less defaults to expect and a shift of the probability mass to downgrades will end in a higher risk for the credit portfolio.

Capturing transitions to the default state (TD): Since defaults can be considered to be the major risk for the companies in the portfolio, a risk-sensitive distance measure has to separate the default columns from the others. Changes in this column should receive a higher weight. In [26] it is suggested to use multipliers depending on the dimension n of the transition matrix P. Possible multipliers could be n, $2n$, n^2 or $exp(n)$.

Capturing the probability mass in a cell (PM): To capture the probability of transition in an individual cell, weighted difference indices NAD, NSD, WAD and WSD can be used. Thus, the probability mass in the cell of the original matrix is considered as a weight either by multiplying or dividing by p_{ij}.

Capturing the migration distance (MD): To capture the difference between close and far migrations a coefficient $(i - j)$ for measuring the distance between the two rating states is used.
Among the so-called directed difference indices suggested in [26] in our study we will consider

$$d_1(i,j) = (i - j) \cdot \frac{(p_{ij} - q_{ij})}{p_{ij}}$$

and

$$d_2(i,j) = (i - j) \cdot sign(p_{ij} - q_{ij}) \cdot (p_{ij} - q_{ij})^2.$$

Both indices include the coefficient for the migration distance: (i-j) will be positive for differences in migrations to the left of the diagonal and negative on the right hand side of the diagonal, so DIR is also captured. Due to the

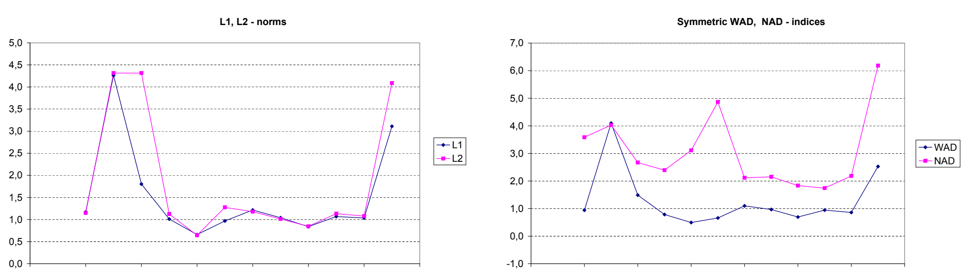
weighting coefficient $\frac{1}{p_{ij}}$ also the PM criteria is captured. Obviously, in the first expression normalized absolute differences are used. Taking into account that default probabilities are rather small while wrong estimates for these probabilities have significant impact on VaR estimates, the use of normalized differences could be a promising approach. In the second expression the squared differences are used without a weighting coefficient. An index based on these weights provided particular good results in [26].
In a second step multipliers for the default column n are added, so the difference indices are of the following form:

$$
\begin{aligned}
D_1(P,Q) &= \sum_{i=1}^{n}\sum_{j=1}^{n-1} d_1(i,j) + \sum_{i=1}^{n} n \cdot d_1(i,n), \\
D_2(P,Q) &= \sum_{i=1}^{n}\sum_{j=1}^{n-1} d_2(i,j) + \sum_{i=1}^{n} n \cdot d_2(i,n).
\end{aligned}
$$

In Trück and Rachev [27] it is shown that the distance measures $D_1(P,Q)$, $D_2(P,Q)$ and $D_2(P,Q)$ are highly correlated to changes in Value-at-Risk figures due to shifts in probability mass of the transition matrices. In the next section we will calculate the introduced difference indices for the considered rating system and investigate their changes through time.

4.2.4 Changes of the metrics through time

In this section we consider the changes of the considered distance indices for migration matrices. In a first step we calculated for the time period from 1990 to 2001 the distance of the migration matrices from the average migration matrix. The results sorted by groups of difference indices are displayed in the figures below. To make the figures more comparable all distances were standardized to have variance of $\sigma^2 = 1$.



(a) L1 and L2 Metric

(b) WAD and NAD distance Measures

Figure 5: Distances from average matrix for the period 1990-2001

The plotted figures illustrate the advantage of the SVD metric and the risk-sensitive difference indices $D1$, $D2$. Measures like the $L1$ or $L2$ metric but also NAD, WAD etc do not give a direction for the differences according to the criteria giving information on the risk inherent in a migration matrix. Whether the probability mass is shifted to the upper right or left hand side of the diagonal cannot be determined. So all that can be concluded from these distance measures is that there are high or low deviances from the average migration matrix of the period 1990-2001. For example we can see that the migration matrices changed considerably in the years 1991 and 1992. However, these deviances do not give much information about changes in the Value-at-Risk for a portfolio.

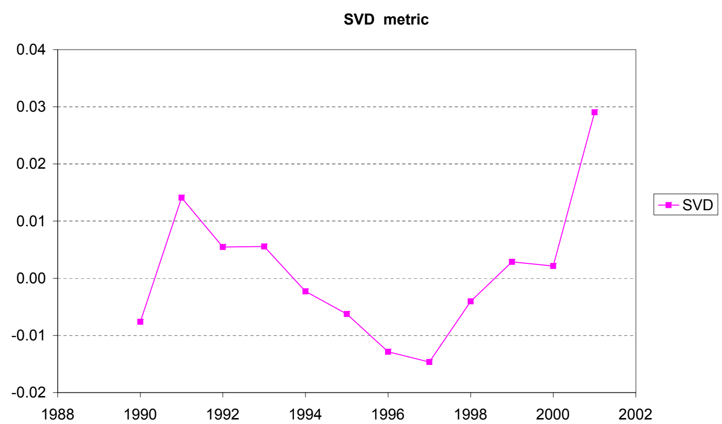


Figure 6: Distance from average matrix for the period 1990-2001 for the risk sensitive SVD metric

The SVD metric (figure 6) and the risk-sensitive difference indices $D1$ and $D2$ (figure 7) are measures that can also be negative. Measuring changes in "amplifying power" of a migration matrix the SVD metric includes information on the average probability of migration. These numbers are higher for periods of economic changes like 1991-1993 and 1999-2001. Additionally, difference indices like $D1$ and $D2$ discriminate between changes on the right hand side of the diagonal from those on the left hand side and give information on potential risk figures. While in the expansion years 1995 and 1996 the measure provides clearly positive values especially for the transition matrix 2000 to 2001 and 2001 to 2002 the highest negative deviation is obtained. This corresponds with high default probabilities due to macroeconomic recession. The information content of mobility measures like the SVD metric and of directed difference indices will more closely be examined in the next section, where relations between these measures with a macroeconomic variable will be discussed.

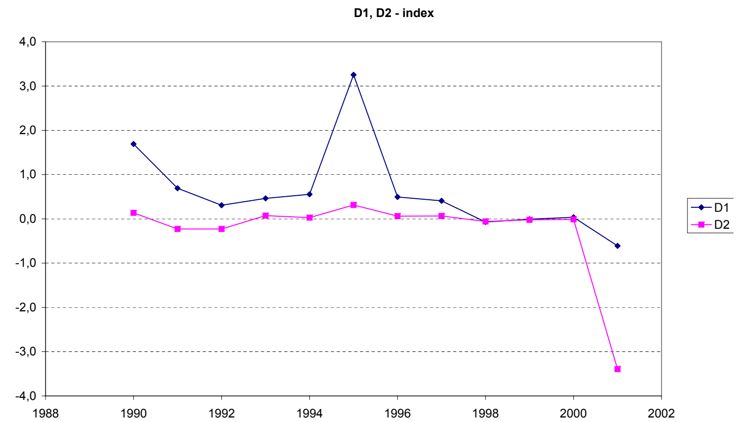


Figure 7: Distances from average matrix for the period 1990-2001 for the risk sensitive D1 and D2 measures

4.3 Matrix norms and Cyclical Behavior

Following Trück and Rachev [27] we will use the matrix norms to measure how changes in the transition matrices are correlated with changes in the macroeconomy for the period 1996-2001. As an indicator for the state of the economy we use the EuroCOIN-index for the same period. EuroCOIN is published monthly and, according to the Centre of Economic Policy Research [12], is the leading coincident indicator of the Euro area business cycle available in real time.[15] The indicator is supposed to provide an estimate of the monthly growth of Euro area GDP. It is adjusted for measurement errors, seasonal and other short-run fluctuations.

The period was chosen since it comprises the subperiods January 1996 - December 1997 and 1999 that are generally considered as economic expansions and the subsequently following years 2000 - 2001 that can be considered as a recession with economic distress. We calculated deviations from yearly migrations to the average transition matrix of that period according to the three different types of difference indices introduced in the previous section: according to the $L1/L2$-norms, to the SVD metric and to the $D1, D2$-distance measures.

[15] See also <http://www.cepr.org/data/eurocoin> for a description of the methodology.

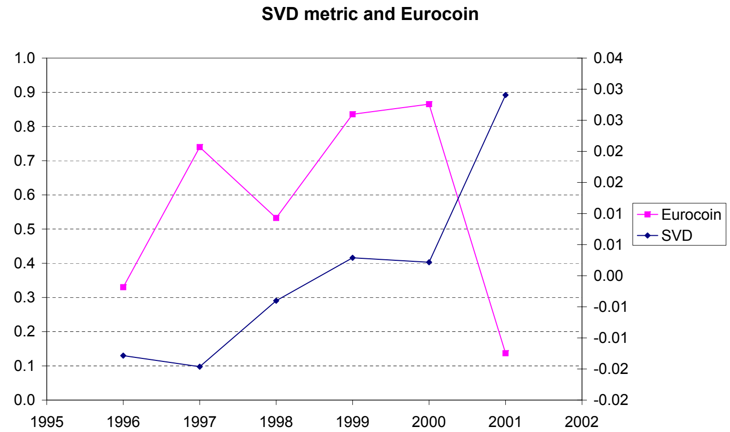


Figure 8: EuroCOIN index and SVD distance from the average transition matrix for the period 1996-2001

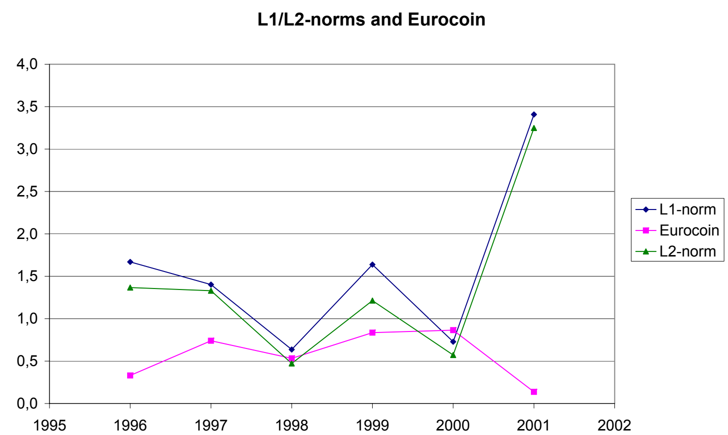


Figure 9: L1 and L2 metric

As expected, the cell-by-cell distance measures like the $L1$- and $L2$-norms are not able to reflect macroeconomic conditions expressed by the Eurocoin-index. While for example in the period $2000-2001$ a high number of defaults could be observed due to recession, the distance from the average transition matrix is rather small (see figure 9). We get a negative correlation of approximately -0.7 both for the $L1$- and $L2$-norms with the Eurocoin-index. These high negative correlations seem to be mainly caused by the high values of the $L1/L2$-norms in 2001.
The SVD metric and the results for the risk-sensitive difference indices are better indicators of the state of the economy. However, both difference indices have to be interpreted differently as already laid open in the previous

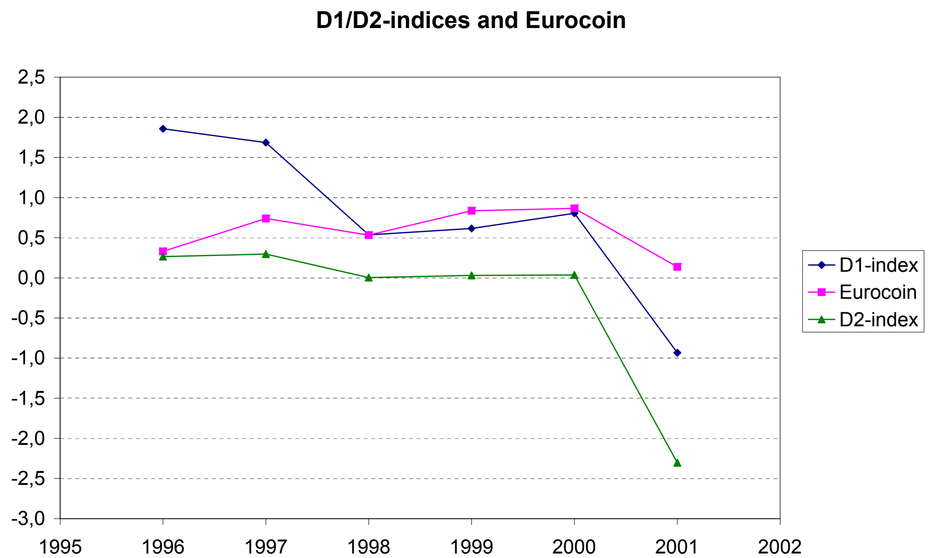


Figure 10: EuroCOIN and distance indices from average matrix for the period 1996-2000

subsection: For the SVD metric we would expect that it takes on big values during periods of big changes of the economy, regardless whether these changes are economic downturns or expansions. In both cases we expect a "high mobility" of the ratings. In contrast, we would expect risk-sensitive measures to be correlated with the variable which indicates the state of the economy. If the economy is growing, a risk-sensitive distance measure should take on positive values, otherwise, in the case of an economic downturn, it should be negative.

Considering the difference of yearly transition matrices from the average transition matrix for 1996 - 2001 we find that especially for the last two years 1999 - 2000 and 2000 - 2001 quite large positive deviations from the average matrix could be observed (see figure 8). This corresponds with the stock market bubble and its burst by mid 2000. Correlation is -0.45 and significantly different from zero. Between 2000 and 2001 the economic downturn continued. The rapid deterioration of the macroeconomic situation was accompanied by an increase of the SVD metric from the absolute value of 0.002 to 0.029 which is equal to approximately 1.7 standard deviations of the SVD metric for the whole period 1996 - 2001.

For the risk-sensitive difference indices $D1$ and $D2$ we find that during the expansion period $1996 - 1998$ we get positive deviances from the average transition matrix while especially for the year $2000 - 2001$ we obtain a large

negative deviation indicating higher risk figures (see figure 10). We obtain correlation coefficients of approximately 0.41 and 0.69 indicating the interrelation between the EuroCOIN index and the changes in the migration matrices. According to the CEPR-document [12] EuroCOIN-values above 0.8 – such values were determined throughout the whole year 2000 – are associated with a strong expansion of the economy. Conversely, the sharp fall of the index between 2000 and 2001 could also be observed for the $D1/D2$-difference indices which showed a considerable decrease during the same period, figure 10 illustrates these observations for the $D2$-distance.
Comparing the time-series we obtained for the SVD metric and the $D1/D2$-difference indices we see that both measures are related to the state of the economy in different ways: Whereas a decrease of the $D1/D2$-difference indices is to be associated with the decrease of the EuroCOIN-index, it is not clear whether an decrease/increase of a mobility-measure like the SVD metric should be associated with a period of recession/expansion. Rather mobility-measures should be understood as an indicator of change. The different relationship of the two types of difference indices with the economic cycle can well be seen in the figures 8 and 10 for the years 2000 and 2001. The same fact – the economic downturn between 2000 and 2001 – causes a decrease of the $D1/D2$-distances, whereas the SVD metric falls sharply.
We conclude that using mobility indicating or risk sensitive measures can be interrelated to the macroeconomic situation. However, the results also show that it is important to use the right measures because explanations in terms of "direction" of the change can hardly be given on the basis of standard cell-by-cell matrix distance measures.

5 Conclusion

The aim of our analyses was to investigate a rating system which is based on logit-scores and financial ratios obtained from balance-sheet data of German non-financial enterprises. We examined whether these ratings could be modelled as time-homogenous Markov chains.

We examined conditional transition matrices[16] and also considered Eigenvalues and Eigenvectors of transition matrices. Based on these techniques we found that for our rating system the first-order Markov property does not

[16] Conditional means that a transition matrix is split into an Up-, Down- and Maintain-Momentum-Matrix where these matrices refer to the sub-sample of companies who were upgraded, downgraded or kept the same rating as in the last year.

hold as rating changes from the previous period have a significant impact on actual transitions. Second-order Markov property could be approved using a Likelihood Ratio test. Testing for the second-order Markov property, we found that actual transition probabilities are correlated with the last rating movement. However, the transitions matrices based on our considered rating-system exhibit a behavior which is in contrast to the observations of other authors: Our ratings tend to compensate previous-period rating changes. Not a *Rating Drift* but a tendency for *Rating Equalization* has been observed, i.e., there is a tendency that corporates receive a rating which they already received 2 or 3 years ago before they were up- or downgraded. We conclude that it is incorrect to assume a similar behavior of different rating systems with regard to Markov properties. Markov properties should be examined separately for each individual rating system.

Looking at the transition probabilities inside the rating classes, we observed that a rating grid with eight classes contained insufficient information for the determination of the probability of a rating change as the transition probabilities vary systematically inside each rating class: Upgrade probabilities are higher for records at the top end of the rating class (lower PD), and downgrade probabilities are higher for records at the bottom (higher PD). This suggests the use of finer rating grids for analyzing and forecasting rating migration behavior.

Finally, we analyzed the assumption of time-homogeneity, and conclude that this property (often preconditioned for rating systems) is not satisfied by the rating grades we obtained from the logit-scores. Transitions probabilities and therefore the entire matrices vary over time, which could be approved by Eigenvalue and Eigenvector comparison.

We further investigated the effect of changes in migration matrices on capital requirements for credit portfolios using a bootstrapping approach which was applied to determine Value-at-Risk figures. Considering two different time periods from 1996-1999 and 1999-2002 we observed substantial differences between the portfolio loss distribution and Value-at-Risk or expected shortfall. This is an indication that ignoring business cycle effects by using an average migration matrix as a basis for credit VaR estimation can lead to a clear underestimation of actual credit portfolio risk.

We further used classical and recently introduced difference indices to examine the changes in migration matrices with respect to the credit risk of a portfolio. We found that the classical cell-by-cell distance measures were not able to capture these changes while mobility measures like the SVD metric and risk sensitive directed difference indices gave much better results. In a further study changes in the considered difference measures were related to changes in the macroeconomic situation. We found that both SVD and the risk-sensitive difference measures showed significant correlations between the business cycle and changes in migration matrices for a considered period from 1996-2001.

A Markov property and time-homogeneity

The goal of this appendix is to give the precise definitions of the first-order Markov property and time-homogeneity.

Definition A.1 (first-order Markov property). A stochastic process X_t based on the finite state space $S = \{x_1 \dots x_n\}$ satisfies the *first-order Markov property* if the following holds:

$$P(X_t = x_t \mid X_{t-1} = x_{t-1}, X_{t-2} = x_{t-2}, \dots, X_0 = x_0) = \\ P(X_t = x_t \mid X_{t-1} = x_{t-1}).$$

According to this definition the conditional distribution of X_t on past states is a function of X_{t-1} alone, and does not depend on previous states $X_{t-2}, X_{t-3}, \dots, X_0$. Usually, the first-order Markov property is simply referred to as Markov property. It is straightforward to define the Markov property of second- and third-order.[17]

For the definition of time-homogeneity let us consider the two years s and u and the corresponding state vectors X_s, X_u. Let us denote the transition matrix which transforms X_s into X_u by $P_t(s)$, where $t := u - s$ denotes the time horizon in years.

Definition A.2 (time-homogeneity). A Markov chain is *time-homogenous*, if the property $P(X_s = x_s \mid X_{s-1} = x_{s-1}) = P(X_u = x_u \mid X_{u-1} = x_{u-1})$ holds for the state vectors X_s and X_u at two different dates s and u, where s and u are arbitrary.

As an immediate consequence of this definition we have $X_u = P_{u-s}X_s = P_tX_s$, where P_t does not depend on the initial date s but only on the difference t between the initial date s and the date u.

In the non-homogenous case the transition probability matrix would depend on the initial date s as well as on the distance t between the dates s and u, i.e. we would have $X_u = P_t(s)X_s$ instead, whereas in the homogenous case the transition probability matrix is a function of the distance between dates and not the dates themselves.[18]

[17] For this purpose we would need to include the states X_{t-2} and X_{t-3} in the right side of the equality in Definition A.1.

[18] See Lando [20, p. 441].

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