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Inflation expectations, disagreement, and monetary policy

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Non-technical summary

Research Question

Many central banks have adopted a price stability objective in recent decades. To achieve this objective central banks spend a vast amount of resources to closely track inflation expectations as well as disagreement about inflation expectations. Despite substantial efforts to increase central bank transparency, two stylized facts from survey data stand out: (i) inflation expectations from the private sector are not fully aligned with official central bank forecasts and (ii) market participants disagree about the future inflation path. Against this background, this paper addresses the question: what are the macroeconomic implications if the central bank responds to private sector expectations as opposed to their own forecasts?

Contribution

We develop a New Keynesian model where price setting firms have dispersed information and disagree about the state of the economy. In particular, firms observe idiosyncratic signals about the supply and demand conditions in the economy. They also obtain an endogenous public signal, the central banks nominal interest rate, which conveys additional information about the state of the economy. We account for the fact that inflation expectations are an important component of interest rate decisions and contrast the case where the central bank either responds to private sector inflation expectations or their own, better informed forecasts.

Results

We show that our dispersed information model matches the two stylized facts from survey data: (i) private sector and central bank forecasts are not fully aligned and (ii) private sector forecasters disagree about inflation expectations. Furthermore, we find that under dispersed information the transmission of shocks is sensitive to whether the central bank applies a policy rule that conditions on their optimal forecasts as opposed to private sector forecasts. In particular, in contrast to the full information equilibrium, expansionary monetary policy can lead to lower inflation and inflation expectations as well as higher output, but less so when the central bank responds to private sector expectations. This finding also translates into higher output volatility when the central bank responds to private sector expectations, whereas inflation is less volatile.

Nicht-technische Zusammenfassung

Fragestellung

Viele Zentralbanken haben in den letzten Jahrzehnten ein Preisstabilitätsziel eingeführt. Zur Erreichung dieses Ziels setzen die Zentralbanken umfangreiche Ressourcen ein, um die Inflationserwartungen und deren Dispersion genau zu beobachten. Trotz erheblicher Bemühungen zur Steigerung der Transparenz von Zentralbanken lassen sich aus Umfragedaten zwei stilisierte Fakten hervorheben: a) die Inflationserwartungen des privaten Sektors stimmen nicht vollständig mit den offiziellen Prognosen der Zentralbanken überein, und b) Marktteilnehmer haben unterschiedliche Inflationserwartungen. Vor diesem Hintergrund wird der folgenden Frage nachgegangen: Welche makroökonomischen Auswirkungen ergeben sich, wenn die Zentralbank auf die Inflationserwartungen des privaten Sektors reagiert statt auf ihre eigenen Prognosen?

Beitrag

Es wird ein neukeynesianisches Modell entwickelt, in dem preissetzende Unternehmen über heterogene Informationen verfügen und dadurch unterschiedliche Auffassungen hinsichtlich der Wirtschaftslage haben. Dabei beobachten die Unternehmen insbesondere idiosynkratische Signale zu den Angebots- und Nachfrageveränderungen in der Wirtschaft. Darüber hinaus leiten sie aus einem endogenen öffentlichen Signal - dem Nominalzins der Zentralbank - zusätzliche Informationen zur Wirtschaftslage ab. Wir berücksichtigen, dass Inflationserwartungen eine wichtige Rolle für Zinsentscheidungen spielen und vergleichen den Effekt, wenn die Zentralbank auf die Inflationserwartungen des privaten Sektors statt auf ihre eigenen Prognosen reagiert.

Ergebnisse

Das Modell mit heterogenen Informationen repliziert die beiden stilisierten Fakten aus den Umfragedaten: a) die Prognosen des privaten Sektors und der Zentralbank stimmen nicht vollständig überein, und b) die Prognostiker des privaten Sektors haben unterschiedliche Inflationserwartungen. Darüber hinaus wird aufgezeigt, dass sich die Transmission von Schocks im Modell mit heterogenen Informationen ändert, wenn die Zentralbank bei geldpolitischen Beschlüssen ihren eigenen optimalen Vorausschätzungen zugrunde legt im Vergleich zu Prognosen des privaten Sektors. Im Gegensatz zu einer Situation vollständiger Information, führt eine expansive Geldpolitik zu einer niedrigeren Inflation und geringeren Inflationserwartungen sowie zu einem höheren Produktionsniveau; der Einfluss ist aber weniger stark, wenn die Zentralbank auf die Erwartungshaltung des privaten Sektors reagiert. Daraus resultiert auch, dass das Bruttoinlandsprodukt volatiler wird, wenn die Zentralbank ihre Entscheidungen auf die Erwartungen des privaten Sektors basiert, während die Inflationsvolatilität geringer ausfällt.

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Inflation expectations, disagreement, and monetary policy*

Mathias Hoffmann† Patrick Hürtgen‡

Abstract

Survey data on inflation expectations show that: (i) private sector forecasts and central bank forecasts are not fully aligned and (ii) private sector forecasters disagree about inflation expectations. To reconcile these two facts we introduce dispersed information in a New Keynesian model, where as a result, inflation expectations differ between the private sector and the central bank. We show that output and inflation responses change markedly when the central bank responds to private sector inflation expectations rather than to their own.

Keywords: business cycles, survey data, learning, disagreement, monetary policy

JEL classification: E52, E31, D83.

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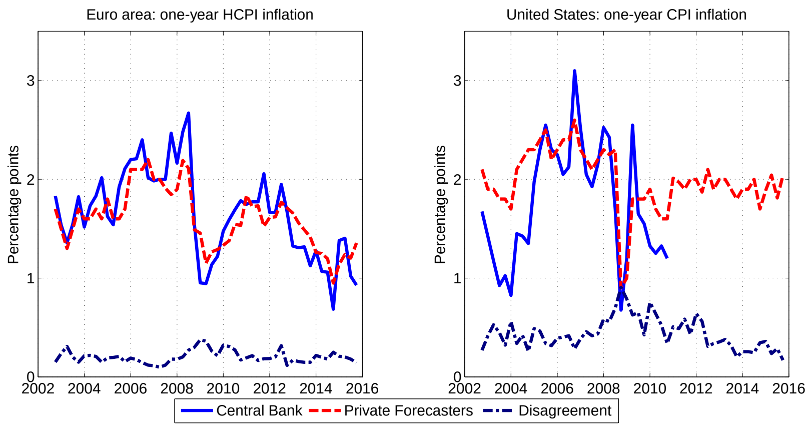
1 Introduction

Many central banks in advanced economies have adopted a price stability objective in recent decades. To achieve this objective central banks spend a vast amount of resources to closely track inflation expectations as well as disagreement about inflation expectations.[1] Specifically, the ECB conducts the Survey of Professional Forecasters (SPF) for the euro area and, likewise, the Federal Reserve Bank of Philadelphia collects comparable survey data for the US. Figure 1 shows the evolution of one-year ahead inflation expectations for the euro area and the US according to which official central bank forecasts differ markedly from professional private sector forecasters. The correlation between these two sectors is 0.86 in the euro area and 0.74 in the US. In addition, average disagreement about one-year inflation expectations among professional forecasters is 0.20 percentage points for the euro area and 0.44 percentage points for the US. Thus, despite substantial efforts to increase central bank transparency, survey data indicate that inflation expectations from the private sector are not fully aligned with official central bank forecasts and market participants disagree about the future inflation path.

Figure 1: Central bank and private forecasters inflation expectations in EA and US



Notes: Inflation is measured as the percentage change in the quarterly average CPI index four quarters ahead and the respective current quarter. Private forecasts are from Consensus Economics (CE). Disagreement: standard deviation across forecasters, calculated from the monthly CE surveys following Knüppel and Vladu (2016). Using the same method, ECB staff forecasts are transformed from calendar year forecasts into fixed horizon forecasts. Fed forecasts are generated from Greenbook quarter-on-quarter growth rates. *Source:* ECB, Consensus Economics, and Philadelphia Fed.

Importantly, under the assumptions of full information and rational expectations, structural DSGE models are inconsistent with the two stylized facts from Figure 1 as these models imply that (i) inflation expectations are identical for all agents/sectors and

[1]The ECB MB (May 2009) states: *"Monetary policy involves anticipating future developments, monitoring and managing private sector expectations over the cycle [...] the ECB needs to monitor private sector perceptions of economic prospects and to preserve its ability to steer expectations over the medium term."*.

(ii) agents unanimously agree about any future macroeconomic variable.[2] Against this background we reconcile these two facts using a dispersed information New Keynesian model where, in contrast to the central bank, firms are imperfectly informed. We also account for the fact that inflation expectations are an important component of interest rate decisions as documented by Romer and Romer (2004) and Cloyne and Hürtgen (ming). Therefore, we extend the dispersed information model of Melosi (2014) with a Taylor rule that either responds to private sector inflation expectations or their own forecasts.

Our contribution is twofold: we show that (i) our dispersed information model matches the two stylized facts in Figure 1 and (ii) under dispersed information the transmission of shocks is sensitive to whether the central bank applies a Taylor rule that conditions on their optimal forecasts as opposed to private sector forecasts. In particular expansionary monetary policy can lead to lower inflation (and inflation expectations) and higher output, but less so when the central bank responds to private sector expectations.

In contrast to the dispersed information models of Nimark (2008, 2014); Lorenzoni (2009); Mackowiak and Wiederholt (2009); Angeletos and La'O (2009, 2013) our model contains an endogenous public signal, the nominal interest rate, that conveys additional information of the central bank about the economy.[3] This is a plausible assumption as nominal interest rates are publically announced and widely reported by news media.

2 Model

The monetary authority sets the interest rate under full information, whereas firms set prices under dispersed information taking into account the action of the monetary authority. In particular, firms observe private idiosyncratic signals about productivity and demand shocks as well as an endogenous public signal, i.e. the policy rate set by the monetary authority. We examine two Taylor rule specifications: We refer to Case 1 when the central bank responds to their own inflation and output gap expectations and, to Case 2, when the central bank responds to the expectations formed by imperfectly informed firms.

[2]For a detailed empirical study on further empirical facts on disagreement see Andrade, Crump, Eusepi, and Moench (2014).

[3]Rondina and Walker (2014) assess the effects of endogenous signals in a dispersed information asset pricing model and propose a novel solution method.

2.1 Households

Households have perfect information and maximize the following utility function

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s D_{t+s} \left[\frac{C_{t+s}^{1-\gamma}}{1-\gamma} - \frac{N_{t+s}^{1+\varphi}}{1+\varphi} \right] , \tag{1}$$

where E_t is the households' full information rational expectations operator, $\gamma > 0$ is the relative risk aversion parameter, and $\varphi \geq 0$ is the inverse Frisch elasticity. Households are subject to a demand shock D_t, which follows an AR(1) process $d_t = \rho_d d_{t-1} + \epsilon_t^d$, $\epsilon_t^d \sim N(0, \sigma_d^2)$ and with $x = \log X$. Households optimally choose consumption C_t, labor N_t and bond holdings B_t subject to:

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t - T_t \ . \tag{2}$$

2.2 Firms

A representative intermediate firm j produces output based on a linear production technology

$$Y_t(j) = A_t(j) N_t(j) \ , \tag{3}$$

where $N_t(j)$ is the amount of labour used by a firm j. Firm-specific productivity $A_t(j)$ is the combination of aggregate productivity A_t and a white-noise firm-specific component $\eta_t^a(j)$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \ , \qquad \epsilon_t^a \sim N(0, \sigma_a^2) \tag{4}$$

$$a_t(j) = a_t + \eta_t^a(j) \ , \qquad \eta_t^a \sim N(0, \tilde{\sigma}_a^2). \tag{5}$$

Each firm j also observes a private signal about demand conditions

$$d_t(j) = d_t + \eta_t^d(j) \ , \qquad \eta_t^d \sim N(0, \tilde{\sigma}_d^2) \tag{6}$$

Moreover, firms also observe the interest rate R_t decided by the central bank and the prices set up to period t. Firm j's information set $\mathbb{I}_{j,t}$ in period t is

$$\mathbb{I}_{j,t} = \{\log A_\tau(j), \log D_\tau(j), R_\tau, P_\tau(j) : \tau \leq t\} \ . \tag{7}$$

Based on their information set firms set staggered prices (see Calvo, 1983). To set prices firms find it optimal to forecast the forecasts of other firms as emphasized by Townsend

(1983). A representative firm j that is allowed to change its price $P_t(j)$ maximizes its profits conditional on its information set $\mathbb{I}_{j,t}$:

$$\underbrace{\max}_{P_t(j)} \qquad E_t(j)\left[\sum_{s=0}^{\infty}(\beta\theta)^s\left(\frac{C_{t+s}}{C_t}\right)^{-\gamma}\frac{D_{t+s}}{D_t}\frac{P_t}{P_{t+s}}\left(P_t(j)-MC^n_{t+s}(j)\right)Y_{t+s}(j)\right] \tag{8}$$

subject to the firm's production technology and $Y_t(j) = C_t(j)$, where $C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\nu} C_t$.

In equilibrium aggregate inflation is a function of firms' dynamic higher-order expectations

$$\widehat{\pi}_t = (1-\theta)(1-\theta\beta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{mc}^{(k)}_{t|t} + \theta\beta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}^{(k)}_{t+1|t}\ , \tag{9}$$

where real marginal costs $\widehat{mc}^{(k)}_{t|t} = (\gamma+\varphi)\hat{y}^{(k)}_{t|t} + \varphi\hat{a}^{(k)}_{t|t} - \hat{a}^{(k-1)}_{t|t}, k > 1$ are a function of average higher-order beliefs and $\hat{x} = \log(x_t) - \log(\bar{x})$. Appendix B.2 provides a detailed derivation. To fix notation, $\widehat{\pi}^{(k)}_{t+1|t}$ is the average k-th order expectation about the next period's inflation rate $\widehat{\pi}^{(k)}_{t+1|t} = \underbrace{\int E^j_t \ldots \int E^j_t \ldots \int E^j_t}_{k} \widehat{\pi}_{t+1} dj \ldots dj.$

Survey data on next period's inflation expectations are by definition first-order expectations, i.e. $\widehat{\pi}^{(1)}_{t+1|t}$ is the average expectation of all firms about next period's inflation rate. Later we explore the macroeconomic consequences of monetary policy when the central bank responds to these first-order expectations. Below we also refer to firms' first-order expectations as private sector expectations, which differ from fully informed households' expectations.

2.3 The monetary and fiscal authorities

The central bank sets the interest rate following a Taylor-type reaction function:

$$\hat{r}_t = \phi_\pi E_t\hat{\pi}_{t+1} + \phi_y E_t\hat{y}^{gap}_{t+1} + \hat{m}_t \tag{10}$$

$$\hat{m}_t = \rho_m\hat{m}_{t-1} + \epsilon^m_t\ , \qquad \epsilon^m_t \sim N(0,\sigma^2_m) \tag{11}$$

The central bank observes the true shocks and internalizes that firms set prices under dispersed information. Therefore, under dispersed information, the expectations about the inflation and output gap profile differ between the (better informed) central bank and firms. In setting interest rates the central bank can convey non-redundant information to the private sector. We investigate two cases of the Taylor rule where we use (i) central bank full information rational expectations: $E_t\hat{\pi}_{t+1}$ and $E_t\hat{y}^{gap}_{t+1}$; and (ii) firms' first-order expectations: $\hat{\pi}^{(1)}_{t+1|t}$ and $\hat{y}^{gap,(1)}_{t+1|t}$ as defined in Section 2.2.

To close the model the fiscal authority collects lump-sum taxes (T_t) to satisfy the government budget constraint $R_{t-1}B_{t-1} - B_t = T_t$.

2.4 Calibration and solution method

We employ parameter values commonly used for the New Keynesian model which are summarized in Table 1. The two parameters $\tilde{\sigma}_a$ and $\tilde{\sigma}_d$ determine the signal-to-noise ratios and, therefore, the extent to which information is dispersed in the economy. We set the signal-to-noise ratios to match average disagreement (the standard deviation) about one-year inflation expectations from Consensus Economics, which equals 0.2 percentage points from 2001Q4 to 2015Q4.

Table 1: Calibration

Parameter	Description	Value
β	Discount factor	0.99
γ	Relative risk aversion	1.1
φ	Inverse Frisch elasticity	0
θ	Calvo pricing	0.75
ϑ	Elasticity of substitution of goods	10
ϕ_π	Taylor rule coefficient	1.5
ϕ_y	Taylor rule coefficient	0.05
ρ_m	Autocorr. monetary policy	0.5
ρ_a	Autocorr. TFP	0.9
ρ_d	Autocorr. demand	0.9
$100\sigma_m$	Std. dev. MP	0.3
$100\sigma_a$	Std. dev. TFP	1.0
$100\sigma_d$	Std. dev. demand	1.0
$100\tilde{\sigma}_a$	Std. dev. idiosyncratic TFP noise	1.0
$100\tilde{\sigma}_d$	Std. dev. idiosyncratic demand noise	2.0

The solution steps follow those outlined in Nimark (2011) and Melosi (2014). We provide a detailed description in Appendix B.

3 Results

3.1 Stylized facts of the model

We show that our dispersed information model is consistent with the stylized facts from Figure 1. We report results from a prior predictive analysis where we vary the signal to noise ratios and Taylor rule coefficients. We draw the idiosyncratic standard deviation of shocks, $100\tilde{\sigma}_a$ and $100\tilde{\sigma}_d$, from a normal distribution with mean 1.0 and 2.0, respectively and a standard deviation of 0.5. We draw the Taylor rule coefficient on inflation expectations from a normal distribution with mean 1.5 and standard deviation 0.25. For the Taylor rule coefficient on output gap expectations we use a normal distribution with mean 0.15/4 and a standard deviation of 0.05. In Table 2 we report the mean, the 10th and the

90th percentile of our statistics of interest based on 20000 draws and model simulations of 150 periods with 1000 repetitions.

Table 2: Model statistics from simulations of prior distributions

		Case 1:		Case 2:	
Statistics	Data	Mean	10-90th Percentile	Mean	10-90th Percentile
$\sigma(\pi^{(1)}_{t+4\|t})$	0.20	0.16	[0.13, 0.21]	0.24	[0.16, 0.34]
$autocorr(\pi^{(1)}_{t+4\|t})$	0.83	0.88	[0.87, 0.90]	0.89	[0.87, 0.91]
$corr(\pi^{(1)}_{t+4\|t}, E_t\pi_{t+4})$	0.86	0.97	[0.96, 0.99]	0.97	[0.96, 0.98]

Notes: $\pi_{t+4} = P_{t+4}/P_t$ refers to one year-ahead inflation expectations. Data is from 2002Q1 to 2015Q4. *Sources:* Consensus Economics and ECB.

The prior predictive analysis shows that the model generates a realistic amount of disagreement about one-year ahead inflation expectations. Overall we find relatively tight distributions for our statistics of interest. Our model slightly overfits the autocorrelation of euro area inflation expectations which is 0.83. The simulation also shows that the correlation between central bank and private sector, i.e. firms' inflation forecasts, is strongly positive with a mean of 0.97 and in the lower tail of the distribution we find values around 0.9. Thus, our analysis provides support that the dispersed information model is in line with the stylized facts from Figure 1. In contrast, the full information solution of the model is inconsistent with these facts as the model implies zero disagreement about any future macroeconomic variable and complete alignment of inflation expectations between the central bank and the private sector.

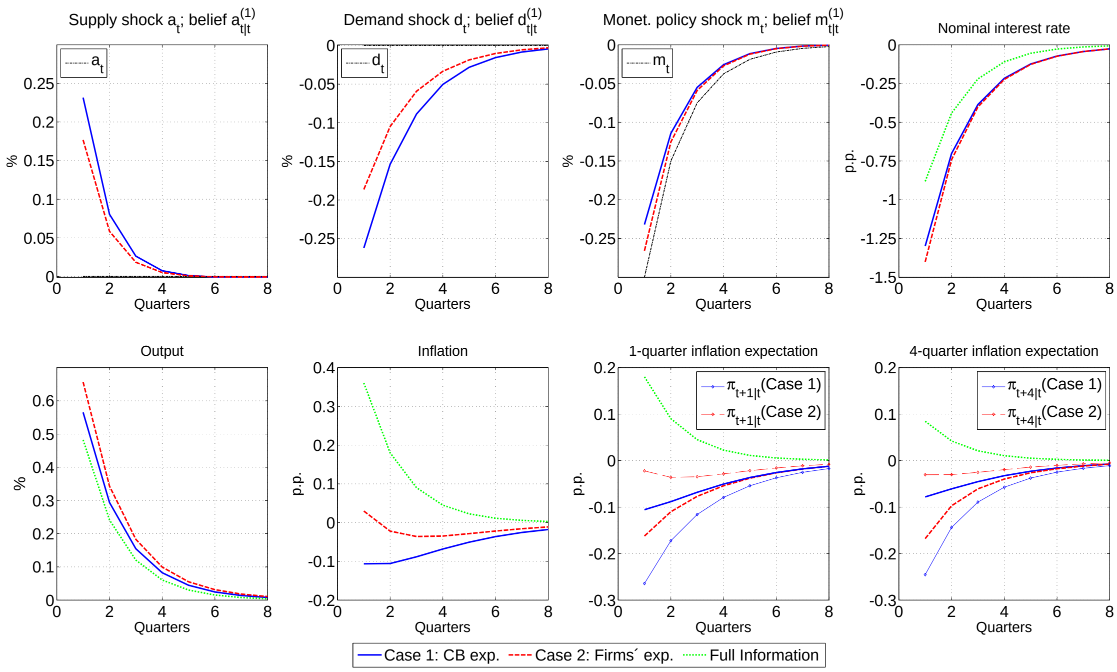
3.2 Impulse response functions

In this section we illustrate how the macroeconomic effects change in the presence of dispersed information where disagreement and misalignment of inflation expectations endogenously occur. Figure 2 shows the impulse response functions to a one standard deviation expansionary monetary policy innovation (30 bps) where the central bank responds to their own forecasts (solid line) and the case where the central bank responds to private-sector expectations (dashed line).[4] The top panel shows the evolution of the true shock and the response of first-order beliefs as well as the nominal interest rate. The first-order beliefs indicate that firms misperceive the shock for a mix of expansionary monetary policy, a positive supply shock and a contractionary demand shock.

The bottom panel shows the effects on output, inflation, and inflation expectations. While under full information (dotted line) expansionary monetary policy increases output,

[4] For completeness Appendix A shows the IRFs to a productivity and a demand shock.

Figure 2: Expansionary monetary policy shock: Case 1 vs. Case 2

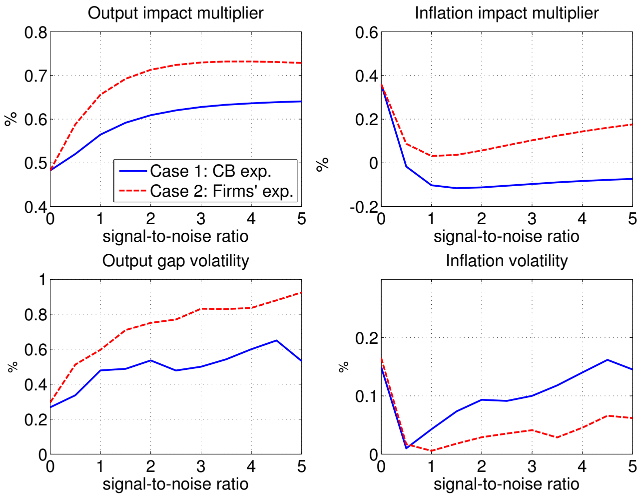


Notes: Impulse responses to a 30 basis points expansionary monetary policy shock. The diamonds denote inflation expectations from the sector (either private, i.e. firm sector or central bank) that the monetary authority did not respond to. Inflation and the nominal interest rate are annualized.

inflation as well as inflation expectations, inflation actually declines under dispersed information for both Taylor rule specifications. With private sector, i.e. firms' expectations in the Taylor rule the monetary policy shock is perceived relatively more expansionary and, thus, the effect on output is much stronger. In addition, the shock is also relatively less perceived for a contractionary demand shock and in combination with the evolution of beliefs about productivity and monetary policy, the inflation response is relatively weaker when monetary policy responds to private sector expectations.

One-quarter and one-year inflation expectations in each case are markedly different (comparison of solid lines). The diamonds indicate the evolution of expectations that the central bank did not actively respond to, but the other sector holds, e.g. the blue diamonds indicate how private sector first-order expectations evolve in the economy where the central bank responds to their own forecasts. Therefore, these two charts also indicate that if the central bank had set the nominal rate in response to the expectations of the other sector, output and inflation would have reacted differently. Hence, it is crucial for macroeconomic dynamics which expectations the central bank actually responds to and how much information the nominal rate conveys to the firms. Qualitatively, the model is consistent with the stylized fact from Figure 1 that inflation expectations from both sectors differ, but co-move strongly.

Figure 3: Sensitivity to signal-to-noise ratios



Notes: Impact multiplier on output and inflation after a 100 bps monetary policy shock with signal-to-noise ratios for TFP varied between 0 and 5. We keep the S-N ratio for demand twice the value of the TFP S-N ratio as in our baseline calibration (see Table 1). Output and inflation volatility are conditional on monetary policy shocks and computed based on 1000 simulations with 150 periods.

3.3 Second moments and sensitivity to signal-to-noise ratios

In our baseline calibration we find that output gap is more volatile if the central bank responds to firms' expectations (0.60 vs. 0.48), while inflation is less volatile when the central bank responds to their expectations (0.04 vs. 0.01). The impact multipliers on output and inflation in Figure 2 also illustrate this finding. To investigate the sensitivity of our results further we consider whether changing the signal-to-noise ratios affects our results.

In Figure 3 we show the impact multipliers on output and inflation (top panel) and the conditional volatilities of output gap and inflation (bottom panel) for signal-to-noise ratios between 0 (i.e. full information) and 5. A signal-to-noise ratio of one serves as a reference point as it exactly matches the calibration in Section 3.2. For any signal-to-noise ratios on our grid the impact multiplier on output is higher when the central bank responds to firm's expectations (Case 2). The impact multiplier on inflation in Case 2 is always relatively closer to full information (i.e. a S-N ratio of zero), but less pronounced as in Case 1. Therefore, the results illustrated by Figure 3 confirm our findings from the previous section.

In addition, the figure illustrates that the price puzzle is not an inherent feature of our model, but that under dispersed information the inflation response is ambiguous. Hence, dispersed information could be one explanation why the empirical literature sometimes finds a price puzzle according to which contractionary monetary policy leads to an increase in prices and inflation.

Our results in Section 3.2 are also robust regarding the output gap and inflation volatility for alternative signal-to-noise ratios. The bottom panel shows that the lowest output gap variance conditional on monetary policy shocks occurs when the central bank responds to their own expectations (Case 1) for all signal-to-noise ratios. In contrast, for almost all signal-to-noise ratios inflation volatility is lower when the central bank responds to firms' expectations (Case 2). Therefore, in future research, we plan to conduct a welfare analysis to examine the output gap-inflation trade-off more closely.

4 Conclusion

This paper presents a dispersed information model that matches two stylized facts from survey data: (i) an average disagreement about inflation expectations among professional forecasters in the euro area of 0.2 percentage points and (ii) official central bank forecasts and private sector forecasts about inflation co-move, but are not fully aligned. In matching these two stylized facts with a dispersed information model, expansionary monetary policy leads to a stronger decline in inflation and inflation expectations when the central bank responds to their own forecasts rather than those from the private sector.

In future research we plan to conduct a welfare analysis to determine whether the central bank should respond to their own forecasts to communicate the future path of the macroeconomy instead of responding to private sector expectations. Revealing more information through the nominal interest rate can have pervasive effects such that private sectors' misperception about the public signal might lead to welfare losses. This class of dispersed information models can also be applied to match the empirical fact that disagreement about country-specific inflation expectations is heterogeneous in a monetary union (e.g. EMU). Our results suggest that in a monetary union framework macroeconomic dynamics are also sensitive to which expectations the central bank responds to.

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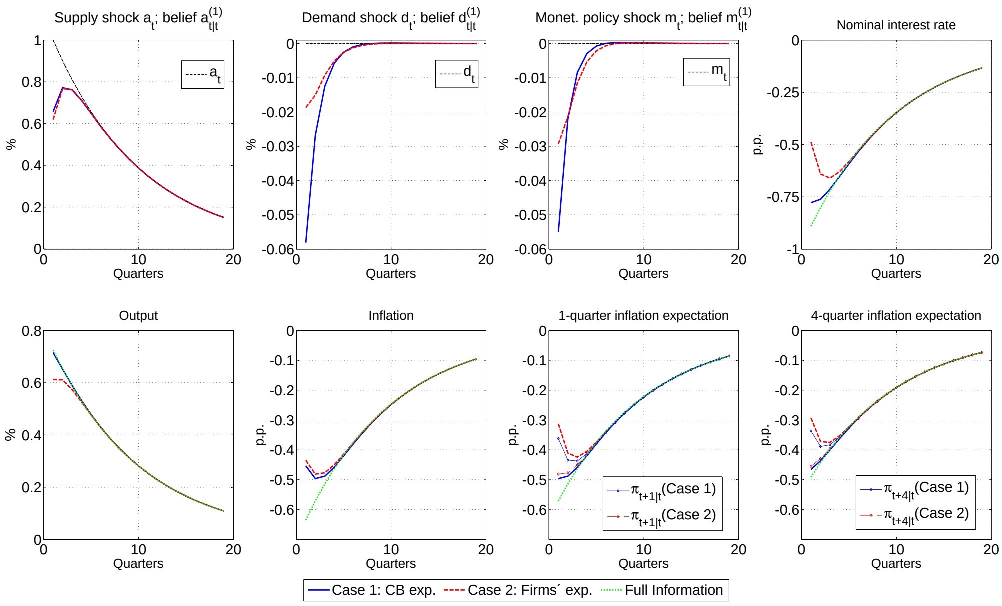
Appendix

A Further results

A.1 Impulse response functions: supply shock

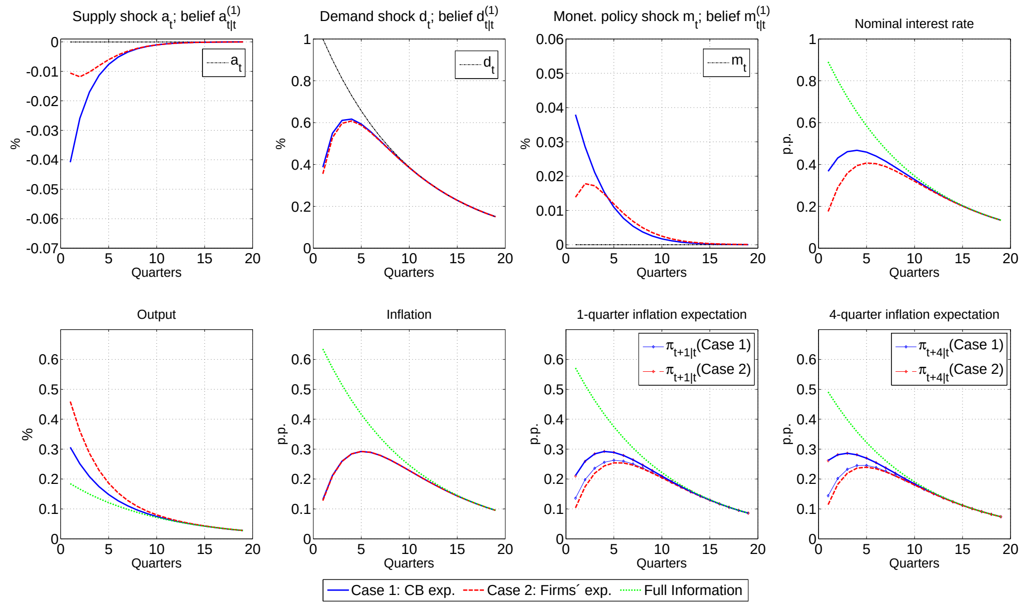
Figure 4: Expansionary supply shock: Case 1 vs. Case 2



Notes: Impulse responses to a one percent expansionary supply shock. Case 1: Taylor rule which responds to central bank expectations. Case 2: Taylor rule which responds to private sector expectations. The triangles represent inflation expectations from the sector (either private sector or central bank) that the monetary authority did not respond to. Inflation and the nominal interest rate are annualized.

A.2 Impulse response functions: demand shock

Figure 5: Expansionary demand shock: Case 1 vs. Case 2



Notes: Impulse responses to a one percent expansionary demand shock. Case 1: Taylor rule which responds to central bank expectations. Case 2: Taylor rule which responds to private sector expectations. The triangles represent inflation expectations from the sector (either private sector or central bank) that the monetary authority did not respond to. Inflation and the nominal interest rate are annualized.

B Model

B.1 Timing of events

At stage 1 shocks are realized and the central bank sets the interest rate for the current period. At stage 2 firms update their information set by observing (i) idiosyncratic technology, (ii) idiosyncratic demand and (iii) the interest rate set by the central bank. Firms then set their prices based on their information set. At stage 3 households become perfectly informed about the realization of shocks and decide about consumption, demand for assets and labour supply. At this stage firms hire domestic labour to produce the goods demanded by households, given the price they have set at stage 2. At stage 3 the fiscal authority collects either lump-sum taxes or pays transfers to households and goods, labour and financial markets clear.

B.2 Derivation of dispersed information New Keynesian Phillips curve

Firm j which re-optimizes prices solves the problem as stated by equation (8) in the main text:

$$\max_{\widetilde{P}_t} E_t(j) \left[\sum_{s=0}^{\infty} (\theta\beta)^s \lambda_{t,t+s} \left(\overline{\pi}\widetilde{P}_t(j) - MC^n(j)_{t+s} \right) Y_{t,t+s}(j) \right],$$

subject to the firm's resource constraint, where $\lambda_{t,t+s} = \beta \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} \frac{D_{t+s}}{D_t} \frac{P_t}{P_{t+s}}$ denotes the stochastic discount factor and MC^n the nominal marginal costs. $E_t(j)$ is the expectation operator conditional on firm j's information set $\mathbb{I}_{j,t} = \{\log A_\tau(j), \log D_\tau(j), R_\tau, P_\tau(j) : \tau \leq t\}$. In the main text we set the gross steady state inflation rate $\overline{\pi} = 1$. We substitute the following equations:

$$\begin{aligned} Y_t(j) &= C_t(j), \\ Y_t(j) &= \left(\frac{P_t(j)}{P_t}\right)^{-\nu} C_t, \end{aligned}$$

so that

$$\max_{\widetilde{P}_t} E_t(j) \left[\sum_{s=0}^{\infty} (\theta\beta)^s \lambda_{t,t+s} \left(\overline{\pi}\widetilde{P}_t(j) - MC^n(j)_{t+s} \right) \left(\frac{\widetilde{P}_t(j)}{P_{t+s}}\right)^{-\nu} C_{t+s} \right]. \tag{12}$$

From the Calvo (1983) price setting we have that

$$p_t = \theta\left(p_{t-1} + \log\overline{\pi}\right) + (1-\theta)\int_0^1 \widetilde{p}_t\,(j)\,dj. \tag{13}$$

Then using the following definitions

$$\begin{aligned}\widehat{\widetilde{p}}_t\,(j) &= \widetilde{p}_t\,(j) - p_t \\ \widehat{\pi}_t &= p_t - p_{t-1} - \log\overline{\pi}\ ,\end{aligned}$$

the linearized price index becomes

$$\begin{aligned}\frac{\widehat{\pi}_t}{1-\theta} &= -p_{t-1} - \log\overline{\pi} + \int_0^1 \widetilde{p}_t\,(j)\,dj \\ \Leftrightarrow \frac{\widehat{\pi}_t}{1-\theta} &= p_t - p_{t-1} - \log\overline{\pi} + \int_0^1 \widehat{\widetilde{p}}_t\,(j)\,dj\end{aligned}$$

so that

$$\int_0^1 \widehat{\widetilde{p}}_t\,(j)\,dj = \frac{\theta}{1-\theta}\widehat{\pi}_t. \tag{14}$$

In addition, the linearized real marginal costs are given by

$$\widehat{mc}_t\,(j) = \widehat{w}_t - \widehat{p}_t - \widehat{a}_t\,(j)\,.$$

Using equation (5) of the main text we have

$$\widehat{mc}_t\,(j) = \widehat{w}_t - \widehat{p}_t - \left(\widehat{a}_t + \eta_t^a\,(j)\right).$$

Using the labour-leisure condition we obtain

$$\widehat{mc}_t\,(j) = \varphi\widehat{n}_t + \gamma\widehat{y}_t - \widehat{a}_t - \eta_t^a\,(j)\,.$$

Integrating across firms the average expectations of real marginal costs are yields:

$$\begin{aligned}\widehat{mc}_{t|t}^{(1)} &= \varphi\widehat{n}_{t|t}^{(1)} + \gamma\widehat{y}_{t|t}^{(1)} - \widehat{a}_t \\ \widehat{mc}_{t|t}^{(1)} &= (\varphi+\gamma)\widehat{y}_{t|t}^{(1)} + \varphi\hat{a}_t^{(1)} - \hat{a}_t\end{aligned} \tag{15}$$

Solving the price setting problem (12) leads to the following first-order condition:

$$E_t(j)\left[\sum_{s=0}^{\infty}(\theta\beta)^s\,\lambda_{t,t+s}\left((1-\nu)\,\overline{\pi} + \nu\frac{MC_{t+s}^n\,(j)}{\widetilde{P}_t\,(j)}\right)Y_{t+s}(j)\right] = 0.$$

We can rewrite this equation in the following way:

$$E_t(j)\left[\lambda_{t,t}\left((1-\nu)\,\overline{\pi}+\nu\frac{MC_t^n(j)}{\widetilde{P}_t(j)}\right)Y_t(j)+\right.$$
$$\left.\sum_{s=1}^{\infty}(\theta\beta)^s\,\lambda_{t,t+s}\left((1-\nu)\,\overline{\pi}+\nu\frac{MC_{t+s}^n(j)}{\widetilde{P}_t(j)}\{\textstyle\prod_{\tau=1}^{s}\pi_{t+\tau}\}\right)Y_{t+s}(j)\right]=0.$$

In steady state the terms in round brackets are zero. Consequently, the terms outside the round brackets are not relevant for the following derivation:

$$E_t(j)\left[\left((1-\nu)\,\overline{\pi}+\nu\overline{MC}\,(j)\exp\left(\widehat{mc}_t\,(j)-\widehat{\widetilde{p}}_t\,(j)\right)\right)+\right.$$
$$\left.\sum_{s=1}^{\infty}(\theta\beta)^s\left((1-\nu)\,\overline{\pi}+\nu\overline{MC}\,(j)\exp\left(\widehat{mc}_{t+s}\,(j)-\widehat{\widetilde{p}}_t\,(j)+\textstyle\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}\right)\right)\right]=0\ ,$$

for $\widehat{\widetilde{p}}_t(j)=\widetilde{p}_t(j)-p_t$. Differentiating this expression gives:

$$E_t(j)\left(\widehat{mc}_t\,(j)-\widehat{\widetilde{p}}_t\,(j)+\sum_{s=1}^{\infty}(\theta\beta)^s\left(\widehat{mc}_{t+s}\,(j)-\widehat{\widetilde{p}}_t\,(j)+\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}\right)\right)=0\ ,$$

which we can rewrite as

$$\widetilde{p}_t\,(j)=(1-\theta\beta)\,E_t(j)\left(\widehat{mc}_t\,(j)+\frac{1}{1-\theta\beta}p_t+\sum_{s=1}^{\infty}(\theta\beta)^s\left(\widehat{mc}_{t+s}\,(j)+\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}\right)\right). \quad (16)$$

Forwarding the equation by one period gives:

$$E_t(j)\left(\widetilde{p}_{t+1}\,(j)\right)=(1-\theta\beta)\,E_t(j)\left(\frac{1}{1-\theta\beta}p_{t+1}+\frac{1}{\theta\beta}\sum_{s=1}^{\infty}(\theta\beta)^s\,\widehat{mc}_{t+s}\,(j)+\sum_{s=1}^{\infty}(\theta\beta)^s\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau+1}\right).$$

The equation can be written as

$$\sum_{s=1}^{\infty}(\theta\beta)^s\,E_t(j)\left(\widehat{mc}_{t+s}\,(j)\right)=\frac{\theta\beta}{(1-\theta\beta)}\left[E_t(j)\left(\widetilde{p}_{t+1}\,(j)\right)-E_t(j)\left(p_{t+1}\right)\right]-\theta\beta\sum_{s=1}^{\infty}(\theta\beta)^s\sum_{\tau=1}^{s}E_t(j)\left(\widehat{\pi}_{t+\tau+1}\right). \quad (17)$$

Rewriting equation (16) yields

$$\begin{aligned}\widetilde{p}_t\,(j)\ &=\ (1-\theta\beta)\left(E_t(j)\left(\widehat{mc}_t\,(j)\right)+\frac{1}{1-\theta\beta}E_t(j)\left(p_t\right)+\sum_{s=1}^{\infty}(\theta\beta)^s\,E_t(j)(\widehat{mc}_{t+s}\,(j))\right)\\ &\quad+(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s\,E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}.\end{aligned}$$

Substituting in (17), yields

$$\begin{aligned}\widetilde{p}_t(j) &= (1-\theta\beta)\left(E_t(j)\left(\widehat{mc}_t(j)\right)+\frac{1}{1-\theta\beta}E_t(j)(p_t)\right) \\ &\quad +\theta\beta\left[E_t(j)\left(\widetilde{p}_{t+1}(j)\right)-E_t(j)(p_{t+1})\right]-\theta\beta(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s\sum_{\tau=1}^{s}E_t(j)\left(\widehat{\pi}_{t+\tau+1}\right) \\ &\quad +(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}.\end{aligned}$$

This can be written as

$$\begin{aligned}\widetilde{p}_t(j) &= (1-\theta\beta)\left(E_t(j)\left(\widehat{mc}_t(j)\right)+\frac{1}{1-\theta\beta}E_t(j)(p_t)\right) \\ &\quad +\theta\beta\left[E_t(j)\left(\widetilde{p}_{t+1}(j)\right)-E_t(j)(p_{t+1})\right]-(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^{s+1}\sum_{\tau=1}^{s}E_t(j)\left(\widehat{\pi}_{t+\tau+1}\right) \\ &\quad +(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}.\end{aligned} \tag{18}$$

Rewrite the last term:

$$\begin{aligned}(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau} &= (1-\theta\beta)\left((\theta\beta)E_t(j)\widehat{\pi}_{t+1}+\sum_{s=2}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau}\right) \\ (1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau} &= (1-\theta\beta)\left((\theta\beta)E_t(j)\widehat{\pi}_{t+1}+\sum_{s=1}^{\infty}(\theta\beta)^{s+1}E_t(j)\widehat{\pi}_{t+1}\right. \\ &\quad \left.+\sum_{s=1}^{\infty}(\theta\beta)^{s+1}E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau+1}\right),\end{aligned}$$

and

$$\begin{aligned}(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau} &= (1-\theta\beta)(\theta\beta)E_t(j)\widehat{\pi}_{t+1}+(\theta\beta)^2E_t(j)\widehat{\pi}_{t+1} \\ &\quad +(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^{s+1}E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau+1} \\ (1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^s E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau} &= (\theta\beta)E_t(j)\widehat{\pi}_{t+1}+(1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^{s+1}E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau+1}.\end{aligned}$$

Plug this into (18), we have

$$\begin{aligned}\widetilde{p}_t(j) &= (1-\theta\beta)\left(E_t(j)(\widehat{mc}_t(j)) + \frac{1}{1-\theta\beta}E_t(j)(p_t)\right) \\ &\quad +\theta\beta\left[E_t(j)(\widetilde{p}_{t+1}(j)) - E_t(j)(p_{t+1})\right] - (1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^{s+1}\sum_{\tau=1}^{s}E_t(j)(\widehat{\pi}_{t+\tau+1}) \\ &\quad +(\theta\beta)E_t(j)\widehat{\pi}_{t+1} + (1-\theta\beta)\sum_{s=1}^{\infty}(\theta\beta)^{s+1}E_t(j)\sum_{\tau=1}^{s}\widehat{\pi}_{t+\tau+1}.\end{aligned}$$

which becomes

$$\widetilde{p}_t(j) = (1-\theta\beta)E_t(j)(\widehat{mc}_t(j)) + E_t(j)(p_t) + \theta\beta\left[E_t(j)(\widetilde{p}_{t+1}(j)) - E_t(j)(p_{t+1})\right] + (\theta\beta)E_t(j)\widehat{\pi}_{t+1}.$$

Given our definition of inflation as $\widehat{\pi}_t = p_t - p_{t-1} - \log\overline{\pi}$, we have that

$$\begin{aligned}\widetilde{p}_t(j) &= (1-\theta\beta)E_t(j)(\widehat{mc}_t(j)) + E_t(j)(p_t) + \theta\beta\left[E_t(j)(\widetilde{p}_{t+1}(j)) - E_t(j)(p_{t+1})\right] \\ &\quad +(\theta\beta)E_t(j)(p_{t+1} - p_t - \log\overline{\pi}),\end{aligned}$$

$$\Leftrightarrow \widetilde{p}_t(j) = (1-\theta\beta)E_t(j)(\widehat{mc}_t(j)) + (1-\theta\beta)E_t(j)(p_t) + \theta\beta E_t^j(\widetilde{p}_{t+1}(j)) - (\theta\beta)\log\overline{\pi}. \tag{19}$$

Then integrating (19) across firms, we obtain the average reset price:

$$\widetilde{p}_t = (1-\theta\beta)\widehat{mc}_{t|t}^{(1)} + (1-\theta\beta)p_{t|t}^{(1)} + \theta\beta\widetilde{p}_{t+1|t}^{(1)} - (\theta\beta)\log\overline{\pi}. \tag{20}$$

Combine equation (13) with the following relationship:

$$\widetilde{p}_t = \int_0^1 \widetilde{p}_t(j)\,dj, \tag{21}$$

yields:

$$p_t = \theta(p_{t-1} + \log\overline{\pi}) + (1-\theta)\widetilde{p}_t. \tag{22}$$

Next, we substitute the following equation into equation (22)

$$p_t = \widehat{\pi}_t + p_{t-1} + \log\overline{\pi}, \tag{23}$$

and we obtain

$$\widehat{\pi}_t + p_{t-1} + \log\overline{\pi} = \theta(p_{t-1} + \log\overline{\pi}) + (1-\theta)\widetilde{p}_t \text{ and, forwarding by one period:}$$

$$\widetilde{p}_{t+1} = \frac{\widehat{\pi}_{t+1}}{(1-\theta)} + p_t + \log\overline{\pi}. \tag{24}$$

Plug (20) into equation (22) gives

$$p_t = \theta\left(p_{t-1} + \log\overline{\pi}\right) + (1-\theta)\,\widetilde{p}_t \tag{25}$$

$$p_t = \theta\left(p_{t-1}\right) + \left(\theta - (1-\theta)\left(\theta\beta\right)\right)\log\overline{\pi} + (1-\theta)\left((1-\theta\beta)\,\widehat{mc}^{(1)}_{t|t} + (1-\theta\beta)\,p^{(1)}_{t|t} + \theta\beta\widetilde{p}^{(1)}_{t+1|t}\right) \tag{26}$$

Next, we substitute (23) and (24) into equation (26)

$$\widehat{\pi}_t = -(1-\theta)\left(p_{t-1} + \log\overline{\pi}\right) + (1-\theta)\left(1-\theta\beta\right)\widehat{mc}^{(1)}_{t|t} + (1-\theta)\,p^{(1)}_{t|t} + \theta\beta\widehat{\pi}^{(1)}_{t+1|t},$$

and for $p_t = \widehat{\pi}_t + p_{t-1} + \log\overline{\pi}$, we have

$$\widehat{\pi}_t = (1-\theta)\left(1-\theta\beta\right)\widehat{mc}^{(1)}_{t|t} + (1-\theta)\,\widehat{\pi}^{(1)}_{t|t} + \theta\beta\widehat{\pi}^{(1)}_{t+1|t}. \tag{27}$$

Under full information we obtain $\widehat{\pi}_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}\widehat{mc}_t + \beta\widehat{\pi}_{t+1|t}$, the well known Phillips curve. However, under imperfect information we have to take expectations of (27) and averaging across firms to get

$$\widehat{\pi}^{(k)}_{t|t} = (1-\theta)\left(1-\theta\beta\right)\widehat{mc}^{(k+1)}_{t|t} + (1-\theta)\,\widehat{\pi}^{(k+1)}_{t|t} + \theta\beta\widehat{\pi}^{(k+1)}_{t+1|t}.$$

Repeatedly substituting equation (27) for $k \geq 1$ yields the dispersed information Phillips curve, which is equation (9) in the main text.

B.3 The equilibrium system

A general representaion of the dispersed information model is given by:

$$\begin{aligned}
\Gamma_0 s_t &= \Gamma_1 E_t s_{t+1} + \Gamma_2 X_{t|t}^{(0:k)} \\
X_{t|t}^{(0:k)} &= M X_{t-1|t-1}^{(0:k)} + N\varepsilon_t \\
s_t &= [\widehat{y}_t, \widehat{\pi}_t, \widehat{r}_t, \widehat{\pi}_{t+1|t}, \widehat{\pi}_{t+2|t+1}, \widehat{\pi}_{t+3|t+2}, \widehat{\pi}_{t+4|t+3}, \widehat{\pi}_{t+4|t}]' \\
X_{t|t}^{(0:k)} &= \left[\widehat{a_{t|t}^{(s)}}, \widehat{m_{t|t}^{(s)}}, \widehat{d_{t|t}^{(s)}} : 0 \leq s \leq k\right]' ,
\end{aligned}$$

where $\widehat{\pi}_{t+ik|t+ij} = \frac{P_{t+ik}}{P_{t+ij}}$. The core equilibrium system is comprised by three linearized equations: the standard consumption Euler equation, the dispersed information Phillips curve, and the interest rate rule:

$$\begin{aligned}
\gamma\widehat{y}_t &= \hat{d}_t - E_t\hat{d}_{t+1} + E_t\gamma\widehat{y}_{t+1} +_t \widehat{\pi}_{t+1} - \hat{r}_t \\
\widehat{\pi}_t &= (1-\theta)(1-\beta\theta)\sum_{k=1}^{\infty}(1-\theta)^{k-1}\left((\gamma+\varphi)\hat{y}_{t|t}^{(k)} - (1+\varphi)\hat{a}_{t|t}^{(k-1)}\right) + \beta\theta\sum_{k=1}^{\infty}(1-\theta)^{k-1}\widehat{\pi}_{t+1|t}^{(k)} \\
\hat{r}_t &= \phi_\pi E_t\widehat{\pi}_{t+1} + \phi_y E_t\left(\hat{y}_{t+1} - \frac{1+\varphi}{\gamma+\varphi}\hat{a}_{t+1}\right) + \widehat{m}_t .
\end{aligned}$$

The remaining five equations are definitions for inflation expectations at various horizons.

Following Melosi (2014), we rewrite the Phillips curve as a function of exogenous state variables $X_{t|t}^{(0:k)}$:

$$\begin{aligned}
&\hat{\pi}_t = \mathbf{a_0} X_{t|t}^{(0:k)} \\
&\Leftrightarrow \hat{\pi}_t = (1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s 1_1^T(\gamma+\varphi)\left(v_0 T^{(s+1)} X_{t|t}^{(0:k)}\right) \\
&-(1-\theta)(1-\beta\theta)\sum_{s=0}^{k-1}(1-\theta)^s(1+\varphi)\left(\gamma_a^{(s)\prime} X_{t|t}^{(0:k)}\right) \\
&+\beta\theta\sum_{s=0}^{k-1}(1-\theta)^s 1_2^T\left(v_0 M T^{(s+1)} X_{t|t}^{(0:k)}\right) \\
&\Leftrightarrow \hat{\pi}_t = \left[(1-\theta)(1-\beta\theta)\left((\gamma+\varphi)\varpi m_1 - (1+\varphi)\left(\sum_{s=0}^{k-1}(1-\theta)^s\gamma_a^{(s)\prime}\right)\right) + \beta\theta\varpi m_2\right] X_{t|t}^{(0:k)} ,
\end{aligned}$$

where we use the following definitions:

$$1_1^T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$1_2^T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\varpi = 1_{1xk}$$
$$m_1 = \begin{bmatrix} 1_1^T v_0 T^{(1)} \\ (1-\theta)\, 1_1^T v_0 T^{(2)} \\ \vdots \\ (1-\theta)^{k-1} 1_1^T v_0 T^{(k)} \end{bmatrix}, m_2 = \begin{bmatrix} 1_2^T v_0 M T^{(1)} \\ (1-\theta)\, 1_2^T v_0 M T^{(2)} \\ \vdots \\ (1-\theta)^{k-1} 1_2^T v_0 M T^{(k)} \end{bmatrix}.$$

Furthermore, we use $\gamma_a^{(s)} = \begin{bmatrix} 0_{1x3s} & (1,0,0) & 0_{1x3(k-s)} \end{bmatrix}'$ and $T^{(s)}$, which is an operator that truncates the order of beliefs such that $s_{t|t}^{(s)} = v_0 T^{(s)} X_{t|t}^{(0:k)}$ and is defined as follows:

$$T^{(s)} = \begin{bmatrix} 0_{3(k-s+1)x3s} & I_{3(k-s+1)} \\ 0_{3sx3s} & 0_{3sx3(k-s+1)} \end{bmatrix}.$$

Using $\hat{\pi}_t = \mathbf{a_0} X_{t|t}^{(0:k)}$ allows us to cast the equilibrium system into a set of first-order difference equations in the following form:

$$\Gamma_0 s_t = \Gamma_1 E_t s_{t+1} + \Gamma_2 X_{t|t}^{(0:k)} \tag{28}$$

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & \gamma^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & -1 \end{bmatrix}, \Gamma_1 = \begin{bmatrix} 1 & \gamma^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_y & \phi_\pi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_2 = \begin{bmatrix} 0_{1x2} & \frac{1-\rho_d}{\gamma} & 0_{1x3k} \\ a_{0,11} & a_{0,12} & a_{0,13} \\ -\phi_y \rho_a \frac{1+\varphi}{\gamma+\varphi} & 1 & 0_{1x3k+1} \\ 0_{5x1} & 0_{5x1} & 0_{5x3k+1} \end{bmatrix}_{8x(3k+3)}$$

The above equilibrium system summarizes our Case 1 Taylor rule specification where the nominal rate responds to central bank expectations. To solve for Case 2 where the Taylor rule responds to the first-order expectations of the private sector we replace the interest rate rule in the third row of equation (28) by:

$$\begin{aligned} \hat{r}_t &= \phi_y y_{t+1|t}^{gap,(1)} + \phi_\pi \pi_{t+1|t}^{(1)} + m_t \\ \hat{r}_t &= \mathbf{b}_0 X_{t|t}^{(0:k)} \end{aligned} \tag{29}$$

$$\Leftrightarrow \hat{r}_t = \phi_\pi \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} v_0 M T^{(1)} X_{t|t}^{(0:k)}$$
$$+\phi_y \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} v_0 M T^{(1)} X_{t|t}^{(0:k)} - \phi_y \rho_a \frac{1+\varphi}{\gamma+\varphi} \gamma_a^{(1)\prime} X_{t|t}^{(0:k)} + \begin{bmatrix} 0 & 1 & 0_{1x(3k+1)} \end{bmatrix} X_{t|t}^{(0:k)}.$$

B.4 Solution algorithm

We solve the dispersed information model following Nimark (2011) and Melosi (2014). Note that alternative methods to solve dispersed information models have been proposed by Mackowiak and Wiederholt (2009) and Rondina and Walker (2014). As shown in Appendix B.3 we cast the structural model into the form:

$$\Gamma_0 s_t = \Gamma_1 E_t s_{t+1} + \Gamma_2 X_{t|t}^{(0:k)} \tag{30}$$
$$X_{t|t}^{(0:k)} = M X_{t-1|t-1}^{(0:k)} + N\epsilon_t \tag{31}$$

The solution algorithm is based on four steps:

1. Set $i = 1$ and guess the matrices $M^{(i)}, N^{(i)}$, and $v_0^{(i)}$.

2. Use a rational expectations solver on equation (30) and (31) to solve for the policy function matrix $v_0^{(i+1)}$ where $s_t = v_0^{(i+1)} X_{t|t}^{(0:k)}$. We truncate the order of the average expectation at $k = 10$.

3. Update the endogenous policy signal

$$\hat{r}_t = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} v_0^{(i+1)} X_{t|t}^{(0:k)}$$

   and solve for the firm's signal extraction problem using the Kalman filter to obtain the matrices $M^{(i+1)}$ and $N^{(i+1)}$ as specified in Appendix B.5.

4. We iterate on steps $2-4$ until convergence:

$$\left\| M^{(i)} - M^{(i+1)} \right\| < \varepsilon, \left\| N^{(i)} - N^{(i+1)} \right\| < \varepsilon, \left\| v_0^{(i)} - v_0^{(i+1)} \right\| < \varepsilon$$

   for $\varepsilon < 1e-6$.

B.5 Evolution of higher-order expectations

Following Melosi (2014), this section shows how to obtain the evolution of the hierarchy of expectations described by

$$X_{t|t}^{(0:k)} = MX_{t-1|t-1}^{(0:k)} + N\epsilon_t \ , \tag{32}$$

where $\begin{bmatrix} \epsilon_t^a & \epsilon_t^m & \epsilon_t^d \end{bmatrix}'$. For brevity we define $X_t = X_{t|t}^{(0:k)}$. The general form of the firms' state space model with the state and measurement equation, respectively, is given by:

$$X_t = MX_{t-1} + N\epsilon_t \tag{33}$$

$$Z_t(j) = DX_t + Q\eta_{j,t} \ , \tag{34}$$

where $D = \begin{bmatrix} d_1 & d_2 & (1_3^T v_0)' \end{bmatrix}'$, with

$d_1{}' = \begin{bmatrix} 1 & 0_{1x3(k+1)-1} \end{bmatrix}, d_2{}' = \begin{bmatrix} 0 & = & 1 & 0_{1x3(k)} \end{bmatrix}, 1_3^T = \begin{bmatrix} 0 & 0 & 1 & 0_{1x5} \end{bmatrix}, \eta_{j,t} = \begin{bmatrix} \eta_{j,t}^a & \eta_{j,t}^d \end{bmatrix}'$

and

$$Q = \begin{bmatrix} \tilde{\sigma}_a & 0 \\ 0 & \tilde{\sigma}_d \\ 0 & 0 \end{bmatrix}.$$

We solve the firms' filtering problem by applying the Kalman filter. Firm j's first-order expectation about the state vector is denoted $X_{t|t}(j)$ and the conditional covariance matrix is $P_{t|t}$:

$$X_{t|t}(j) = X_{t|t-1}(j) + P_{t|t-1}D'F_{t|t-1}^{-1}\left(Z_t(j) - Z_{t|t-1}(j)\right) \tag{35}$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}D'F_{t|t-1}^{-1}DP'_{t|t-1} \ , \tag{36}$$

where

$$P_{t|t-1} = MP_{t-1|t-1}M' + NN' \ , \tag{37}$$

and the matrix $F_{t|t-1} = E(Z_t Z_t' | Z^{t-1})$ is obtained from:

$$F_{t|t-1} = DP_{t|t-1}D' + QQ' \ . \tag{38}$$

Thus, from combining these equations we obtain:

$$P_{t+1|t} = M\left[P_{t|t-1} - P_{t|t-1}D'F_{t|t-1}^{-1}DP'_{t|t-1}\right]M' + NN' \ . \tag{39}$$

Therefore, the evolution of higher-order expectations of firm j about the unobserved state vector $X_{t|t}(j)$ is:

$$\begin{aligned} X_{t|t}(j) &= X_{t|t-1}(j) + K_t\left[DX_t + Q\eta_{j,t} - DX_{t|t-1}(j)\right] && (40)\\ K_t &= P_{t|t-1}D'F_{t|t-1}^{-1} \ , && (41) \end{aligned}$$

where K_t denotes the Kalman-gain matrix. Using that $X_{t|t-1}(j) = MX_{t-1|t-1}(j)$ we can rewrite the hierarchy of higher-order expectations:

$$X_{t|t}(j) = (M - KDM)X_{t-1|t-1}(j) + K_t\left[DMX_{t-1} + DN\epsilon_t + Q\eta_{j,t}\right] \ . \tag{42}$$

Integrating over all firms we obtain the law of motion of the average expectation about $X_{t|t}^{(1)}$:

$$X_{t|t}^{(1)} = (M - KDM)X_{t-1|t-1}^{(1)} + K_t\left[DMX_{t-1} + DN\epsilon_t\right] \ . \tag{43}$$

Note, that $X_t = X_{t|t}^{(0:k)} = \left[X_t^{(0)}, X_{t|t}^{(1:k)}\right]'$ and, therefore, the evolution of the true states is given by:

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & 0 \\ 0 & \rho_m & 0 & 0 \\ 0 & 0 & \rho_d & 0 \end{bmatrix}}_{R_1} X_{t-1|t-1}^{(0:k)} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_d \end{bmatrix}}_{R_2} \varepsilon_t \ .$$

Assuming common knowledge in rationality, i.e. agents form model consistent rational expectations (see Nimark (2008)), we construct matrices M and N:

$$\begin{aligned} M &= \begin{bmatrix} R_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0_{3x3} & 0_{3x3k} \\ 0_{3kx3} & (M - KDM)_{(1:3k,1:3k)} \end{bmatrix} + \begin{bmatrix} 0 \\ KDM_{(1:3k,3(k+1))} \end{bmatrix}, \\ N &= \begin{bmatrix} R_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ KDN_{(1:3k,1:3)} \end{bmatrix} \end{aligned}$$

Following Nimark (2011), the last row and/or column of the matrices have been cropped to make the matrices conformable (i.e. implementing the approximation that expectations of order $k > \bar{k}$ are redundant). The steady-state Kalman gain matrix is denoted K.