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## M-PRESS-CreditRisk: A holistic micro- and macroprudential approach to capital requirements


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# Non-technical summary


### Research Question

In the aftermath of the global financial crisis banking supervisors developed enhanced approaches to taming the risks of banking system failure. New tools were introduced to address systemic risk, especially macroprudential capital buffers. Stress testing, too, gained momentum, helping supervisors to gauge the potential impact of adverse developments. This paper suggests a model – M-PRESS-CreditRisk – that aligns these two aspects and presents a framework for setting capital requirements that takes systemic credit risk into accounts.

### Contribution

M-PRESS-CreditRisk has two main advantages: First, it provides an advanced portfolio model for credit risk which captures extreme losses in the banking system that may materialize during crisis-like, systemic events (tail risk). Second, our framework brings together for the first time different prudential instruments that address credit risk either at the bank level or system level, suggesting a coherent approach to their calibration. For this purpose, the paper combines the portfolio model with a macroeconomic model which generates multi-risk-factor, multi-country stress scenarios.

### Results and Policy Application

M-PRESS-CreditRisk provides estimates for the banks' own funds needed to support credit risk in the banking sector, including forward-looking loan loss provisioning, minimum capital requirements and capital buffers that address systemic risk. In particular, it delivers a measure of systemic credit risk and banks' individual contributions to it. We illustrate how the framework works using a sample of 12 systemically important German banks. The results show that the actual capital levels and the capital needs calculated with regard to systemic credit risk may differ considerably.

# Nichttechnische Zusammenfassung

## Fragestellung

Als Lehre aus der globalen Finanzkrise entwickelten die Bankenaufseher neue Methoden zur Eindämmung der Risiken aus einem Versagen des Bankensystems. So wurden neue, auf das systemische Risiko abzielende Instrumente – insbesondere makroprudenzielle Kapitalpuffer – eingeführt. Außerdem haben Stresstests erheblich an Bedeutung gewonnen, weil sie helfen, den Einfluss negativer Entwicklungen abzuschätzen. Das hier vorgestellte Modell M-PRESS-CreditRisk bringt diese beiden Aspekte zusammen und präsentiert einen Rahmen zur Bestimmung der Kapitalanforderungen unter Berücksichtigung des systemischen Kreditrisikos.

## Beitrag

M-PRESS-CreditRisk hat zwei wesentliche Vorzüge. Erstens bietet es ein fortgeschrittenes Portfoliomodell für Kreditrisken, das extreme Verluste, die sich während systemischer Krisen materialisieren können, erfasst. Zweitens führt es in einem in sich schlüssigen Modellierungsrahmen erstmals Instrumente zusammen, die das Kreditrisiko auf Banken- und auf Systemebene adressieren, und zeigt einen einheitlichen Ansatz zu ihrer Kalibrierung auf. Dazu wird das Portfoliomodell mit einem makroökonomischen Modell verknüpft, das Stressszenarien für mehrere Risikofaktoren und Länder generiert.

## Ergebnisse und Anwendungsbezüge

M-PRESS-CreditRisk erlaubt eine Abschätzung erforderlicher Eigenmittel zur Abfederung des Kreditrisikos im Bankensystem, einschließlich der zukunftsgerichteten Risikovorsorge, der Mindestkapitalanforderung und der Kapitalpuffer für systemische Risiken. Insbesondere liefert das Modell ein Maß für das systemische Kreditrisiko und für Beiträge einzelner Banken hierzu. Die Anwendung wird am Beispiel von zwölf deutschen systemrelevanten Banken veranschaulicht. Die Ergebnisse zeigen, dass die tatsächliche Kapitalausstattung und der errechnete Kapitalbedarf unter Berücksichtigung des systemischen Kreditrisikos auseinanderfallen können.

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**Abstract**

M-PRESS-CreditRisk is a new top-down macro stress testing framework that can help supervisors gauge banks' capital adequacy related to credit risk. For the first time, it combines calibration of microprudential capital requirements and macroprudential buffers in a unified, coherent framework. Its core element is an advanced credit portfolio model – SystemicCreditRisk – built upon a rich, non-linear dependence structure for interconnected bank portfolios. Incorporating numerous sector/country-specific systematic factors, the model focuses on credit default concentration risk as a major source of large losses that may have systemic impact. A test run using a sample of 12 systemically important German banks provides measures for systemic credit risk and the banks' contributions to it in both baseline and stress scenarios. Capital requirements calibrated to the results combine elements of Pillar 1 and Pillar 2, whereas macroprudential buffers can internalize the system's tail risk. The maximum model-based combined requirements range between 6.3% and 27.2% of credit RWA depending on the bank. A comparison with the reported capital figures suggests that there appears to be enough capital in the banking system, but its distribution might be suboptimal from a systemic point of view as the capital level of a number of banks might need improvement.

**Keywords:** Systemic Credit Risk, Tail Risk, Stress Testing, Microprudential Capital Requirements, Systemic Risk Buffer, O-SII Buffer, Hierarchical Archimedean Copula.

**JEL classification:** C15, C23, C63, G21, G28.

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# 1 Introduction

In the aftermath of the global financial crisis which started in 2007-08 banking supervisors developed enhanced approaches to taming the risks of banking system failure and introduced with Basel III a macroprudential overlay to bank regulation. The aim of the traditional microprudential regulation is making sure that individual banks[1] can support their standalone risk; this is achieved with minimum capital requirements. Complementary to that, the macroprudential overlay targets resilience of the banking sector as a whole by internalizing the risk of system-wide distress, using a toolkit of capital buffers that address systemic risk.[2]

Systemic risk is generally defined as the potential loss of economic value or confidence in a substantial portion of the financial system, triggered by a low-probability event which might have significant adverse effects on the real economy (Smaga, 2014; Deutsche Bundesbank, 2013b; Gerlach, 2009). It can be described along two dimensions: time and cross section (Deutsche Bundesbank, 2011; Caruana, 2010). The time dimension of systemic risk is reflected, for instance, in financial market actors' pro-cyclical behavior that may exaggerate economic fluctuations. The counter-cyclical capital buffer is the macroprudential tool designed to address these risks (Detken, Weeken, Alessi, Bonfim, Boucinha, Castro, Frontczak, Giordana, Giese, Jahn, Kakes, Klaus, Lang, Puzanova, and Welz, 2014). In the cross section, on the other hand, shocks may become systemic via many channels of risk propagation such as correlated exposures, balance sheet contagion through asset devaluation and feedback effects of asset fire sales, direct contagion in an interbank-network and funding risk due to runs by depositors and short-term wholesale lenders (Cont and Schaanning, 2016; Clerc, Giovannini, Langfield, Peltonen, Portes, and Scheicher, 2016; Gorton and Metrik, 2012). Macroprudential tools available in the EU that address structural, non-cyclical systemic risks include the systemic risk buffer and the buffer for other systemically important institutions (O-SII).[3]

Despite considerable headway already made, the question about a specific level of capital that banks should have to sufficiently cover systemic risk remains a subject of scientific debate.[4] Thus, not least for pragmatic reasons, supervisory macro stress testing became widely used as a tool for capital needs assessment under adverse macroeconomic scenarios. The supervisory stress tests can generally be conducted for micro- and macroprudential purposes. For microprudential purposes, a set of scenarios is given to banks. The banks use these scenarios for their institute-specific calculations accomplished along strict methodological guidance and provide the supervisor with the results. In the EU,

[1] We use the word "bank" as a generic term for both individual banks and banking groups.

[2] BCBS (2011); Capital Requirements Directive IV (CRD IV).

[3] Articles 133 and 131(5) of the CRD IV, respectively. The latter represents the Union's implementation of the Basel framework for dealing with domestic systemically important banks (BCBS, 2012). SIIs are large and complex organizations whose failure might create major disruptions in the functioning of the financial system (see also Freixas and Rochet, 2013).

[4] A number of academics voice doubts about whether the Basel III reforms go far enough. Hellwig (2013) and Admati and Hellwig (2013) recommend that banks have equity in the range of 20% to 30% of their total assets. Furthermore, Admati, DeMarzo, Hellwig, and Pfleiderer (2013) argue that setting capital requirements substantially above the currently existing levels bears more social benefits than social costs. Dagher, Dell'Ariccia, Laeven, Ratnovski, and Tong (2016) estimate capital rates sufficient to have withstood past banking crisis in a range between 15% and 23% of risk-weighted assets (RWA), depending on the assumptions.

the European Banking Authority (EBA) has carried out these so called bottom-up stress tests since 2011 (EBA, 2016). The goal of such exercises is to primarily estimate capital needs for single institutions.

For macroprudential purposes supervisors often develop their own models in order to run so called top-down stress tests in a unified manner, i.e. independently of any possible divergence in the banks' internal models. For instance, the ECB, BoE, OeNB, and the Board of Governors of the Federal Reserve System (FRB) have developed comprehensive stress testing frameworks.[5] Common to these frameworks is initially that they incorporate first-round effects of macroeconomic shocks on the capital adequacy of individual banks. Additionally, they incorporate second-round effects such as contagion in the interbank network and asset fire sales, feedback effects to the real economy, liquidity or funding risk, which is important from the macroprudential perspective. Therefore, macroprudential stress tests go beyond capital adequacy assessment from the point of view of an individual institution; their primary goal and the value added is to take into account the externalities of systemic risk to be able to judge about capital adequacy for the whole banking system.

The latter is also the angle of the growing strand of literature on top-down supervisory macroprudential stress testing where our paper steps in. We argue, that large credit losses represent a major risk for a banking system. Hence, modeling portfolio credit risk should be a key component of every stress testing framework. However, for reasons of simplicity, some approaches do not capture the non-linear character of portfolio credit risk in a proper way nor do they incorporate an externality important from the macroprudential point of view, namely the risk related to rare, extreme credit losses in the banking system, usually referred to as tail risk. For instance, both the BoE's and the ECB's stress testing frameworks as well as stress tests regularly conducted by the FRB focus on expected losses under the macroeconomic stress scenarios but do not account for unexpected losses which are the main source of risk (Dees, Henry, and Martin, 2017; FRB, 2016; Hirtle and Lehnert, 2014; Henry and Kok, 2013; Burrows, Learmonth, and McKeown, 2012; Alessandri, Gai, Kapadia, Mora, and Puhr, 2009). Other authors – Busch, Koziol, and Mitrovic (2015) and Düllmann and Kick (2014) – go further and adopt a Merton-type portfolio model for solvency stress testing. Yet, such models only account for linear correlation between the systematic risk factors and are not designed to properly address the tail risk externality.

The closest to our model is Systemic Risk Monitor (SRM) of the OeNB (Boss, Krenn, Puhr, and Summer, 2006; Elsinger, Lehar, and Summer, 2006a). Pointing to the minor importance of the linearized losses for assessing credit risk, SRM instead applies the CreditRisk+ model[6] in order to obtain entire loan loss probability distributions. Using a latent common economic shock, SRM introduces correlation between the banks' portfolios, which provides a clue as to how a macroprudential approach to credit risk assessment in a banking system may proceed.

At this juncture, our paper aims at improving the macroprudential supervisory stress testing toolkit for modeling the first-round effects due to the credit risk. We concen-

[5] Discussing the whole range of the existing approaches is beyond the scope of this paper. An overview of the current macro stress testing methodologies, practices and principles as well as related challenges may be found in Dent, Westwood, and Segoviano (2016); Kapinos, Mitnik, and Martin (2017); Jobst, Ong, and Schmieder (2013); IMF (2012); Foglia (2009); Sorge (2004).

[6] CreditRisk+ is a reduced-form model for assessing portfolio credit risk thats introduced by Credit Swiss First Boston in 1997.

trate on the channel of correlated credit exposures and develop a new, advanced portfolio model: SystemicCreditRisk. To make it operational for supervisory needs, we embed the portfolio model into a top-down macro stress testing framework abbreviated as M-PRESS-CreditRisk, which refers to micro- and macroprudential requirements under systemic stress with focus on credit risk.

We consider SystemicCreditRisk to be the major contribution of our paper. We tailor this stochastic portfolio model to the supervisory needs of assessing the risk of potentially extreme system-wide credit losses (tail risk). Originating from correlated credit exposures, tail risk may materialize in an adverse, crisis-like macroeconomic scenario. This is captured by modeling systematic risk that simultaneously affects borrower probabilities of default (PDs) in a given industry sector and/or country. Additionally, borrowers from different sectors and countries are interconnected using a rich and heterogeneous dependence structure for the systematic part of credit risk that allows for non-linear dependence. Importantly, this structure links the banks' portfolios together and, in effect, accounts for default concentration risk – a major source of potentially large losses in a banking system.

This allows us to go beyond a microprudential analysis for individual banks, also assessing risks from the systemic perspective. Thus, our second contribution consists in developing a holistic micro- and macroprudential approach to capital adequacy assessment. To the best of our knowledge, this is the first paper that brings together both aspects of prudential capital regulation: making sure that individual banks can support their standalone risk and making the banking sector as a whole more resilient by internalizing the risk of system-wide distress. To accomplish this task, we suggest a coherent approach to inform the calibration of the respective capital instruments, including the systemic risk buffer and the O-SII buffer.

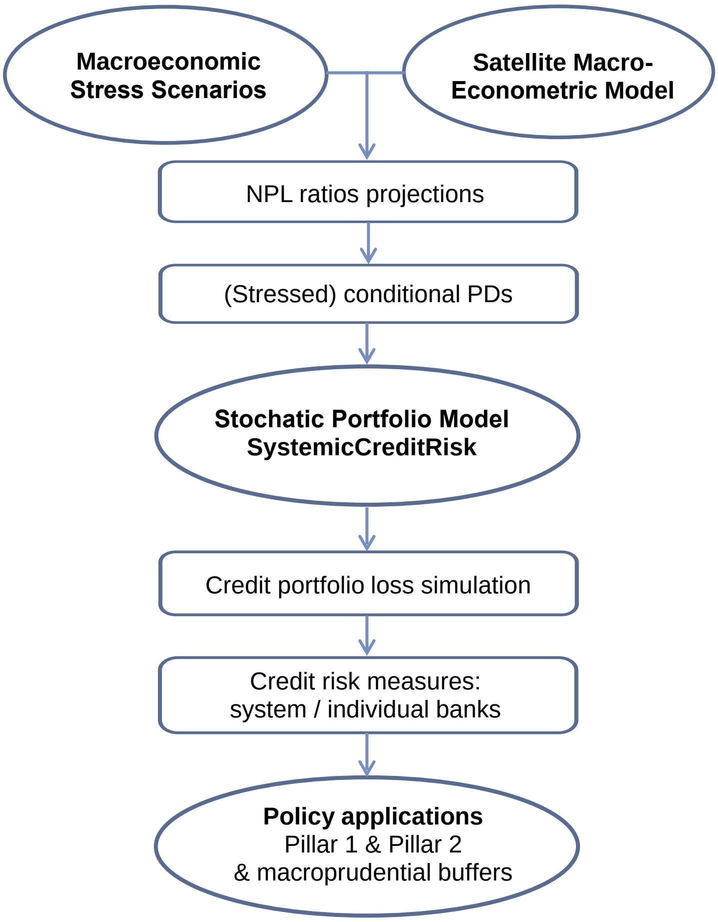
Our stress testing framework M-PRESS-CreditRisk combines an advanced portfolio model with other modules that generate and translate macroeconomic scenarios into borrower PDs and provide output that may serve policy purposes, as shown in Figure 1.1.

The first building block generates relevant international **macroeconomic stress scenarios** that could affect banks' multinational credit exposure. For the purpose of this paper we apply NiGEM as a scenario-generating engine.[7] The second building block represents a **satellite macro-econometric model** that links these scenarios with a measure of the credit quality of bank portfolios represented by the ratio of non-performing loans (NPLs). In a first step, we estimate the country-level relationship between macroeconomic variables also obtained from NiGEM and the NPL rates obtained mostly from the International Monetary Fund (IMF) database. In a second step, we apply this relationship in order to translate the macroeconomic scenarios into the future NPL ratio projections.

The **stochastic portfolio model SystemicCreditRisk** is the third and central building block of our framework. It develops and estimates the joint probability distribution (based on a "heavy-tailed" copula) for the systematic part of credit risk common to different sectors and/or countries. This distribution is constrained to a "stressed" region due to a link to the NPL ratio projections under the stress scenarios, which results in stressed borrower PDs. Using granular information about the banks' credit exposures, this block simulates probability distributions of portfolio losses for each individual bank

[7] NiGEM is frequently used in central banks, but any other model that can deliver international macroeconomic stress scenarios can be utilized here.

Figure 1.1: M-PRESS-CreditRisk: Modeling framework




and the entire system. The resulting portfolio loss distributions deliver all relevant credit risk metrics, including measures of systemic credit risk and the banks contributions to it. The fourth and last building block – **policy applications** – develops a novel coherent approach to capital adequacy assessment. In more detail, it shows how both microprudential capital requirements of Pillar 1 and 2 and macroprudential buffers can be calibrated to the respective risk metrics derived from the simulated portfolio loss distributions.

To test the new framework, we carry out an analysis for 12 systemically important German banks, using unique supervisory data of the German credit register as of 2013Q4. In this illustrative study we concentrate on macroeconomic shocks to Greece, Italy, Portugal and Spain (GIPS) and, additionally, include a scenario that generates macroeconomic stress at the global level. We observe heterogenous results: The maximum model-based combined capital requirements range between 6.3% and 27.2% of credit RWA depending on the bank. On aggregate, it appears to be enough capital in the system to support the system's tail risk, and a number of individual banks seem to be very well capitalized in accordance with the model. However, the other banks' capital level seems to need improvement. These findings suggest that the distribution of capital in the system might be suboptimal from the macroprudential point of view (i.e. when the banks' contributions to the system-wide credit risk are taken into account). Moreover, our results and conclusions based on them are robust to econometric approaches and estimation techniques (satellite model) and the choice of the time period (portfolio model).

Needless to say, our numerical results should be treated cautiously. First, while the results of each model are subject to uncertainty, the linkage of the models may elevate the overall degree of uncertainty (see Danielsson, James, Valenzuela, and Zer, 2016). Second, our present assessment concentrates on the credit risk that arises from correlated exposures, not yet taking into account the systemic risk propagation via direct contagion and other channels mentioned above. Third, the microprudential capital requirements obtained in our framework and the regulatory requirements based on the banks' internal models are not directly comparable because of many differences in the underlying assumptions.[8]

This paper is organized as follows. Section 2 introduces our novel portfolio model. Section 3 describes the data sets used. Section 4 presents our stress test scenarios. Section 5 designs the satellite macro-econometric model and discusses estimation results. Section 6 focuses on the calibration of SystemicCreditRisk, the sampling procedure, and the relevant risk metrics. Section 7 makes suggestions for calibration of capital tools. Section 8 concludes with a summary.

# 2 SystemicCreditRisk

## 2.1 Basic set-up of the portfolio model

SystemicCreditRisk joins the ranks of default-only type of credit portfolio models, i.e. it only considers a borrower's default as a credit event.[9] The target variable is the credit portfolio loss (PL). This is defined as the sum of losses on exposures to $N$ individual borrowers that comprise the portfolio and is calculated as

$$PL = \sum\nolimits_{i=1}^{N} EAD_i \cdot LGD_i \cdot D_i. \tag{2.1}$$

EAD represents the credit exposure at the moment of a borrower $i$'s default (i.e. exposure at default). LGD or loss given default is the percentage of the EAD that will be potentially lost in the event of the borrower's default. D denotes the default indicator that equals one if the borrower defaults and zero otherwise. D is a random, Bernoulli-distributed variable. Its distribution is governed by one parameter set to the borrower's probability

[8]The Basel II formula for minimum capital requirements under the IRB approach is based on the assumption of a perfectly diversified credit portfolio and, thus, neglects credit concentration risk. This approach only accounts for one source of systematic risk in the portfolio and does not allow default correlations to be stronger in the downswing (tail dependence). Our model relaxes those assumptions and, therefore, leads to a more conservative estimation of portfolio credit risk (see Puzanova, 2011). Additionally to the microprudential layer of requirements, our model accounts for correlation between bank portfolios and for the banks' impact on the system-wide risk, which is not part of the stand-alone assessment of the individual banks' risk under Basel II. We also assess credit risk in adverse macroeconomic scenarios and, based on that, calibrate macroprudential capital buffers, which were not part of the official capital requirements at the time of consideration.

[9]Related benchmark models in the financial industry include the Merton-type structural model of Portfolio Manager™ launched by KMV in 1993 and the reduced-from model CreditRisk+™ introduced by Credit Swiss First Boston in 1997. Among the "mark-to-market" models that additionally take into account the borrowers' credit rating migration, the well-known examples are CreditMetrics™, published by J. P. Morgan in 1997, and CreditPortfolioView™ by McKinsey and Company, which followed in 1998.

of default (PD). All variables mentioned refer, as is usual in this type of model, to a risk horizon of one year into the future.

When we consider the credit portfolio of an individual bank, the sum in Equation (2.1) runs over all borrowers of this bank. For the system's portfolio, on the other hand, the sum is taken over the borrowers of all banks contained in the sample. If a firm $i$ borrows from different banks, $D_i$ is the same for all these banks, but $EAD_i$ and $LGD_i$ represent bank-specific figures.

D is the only random variable in Equation (2.1); EADs, LGDs and PDs are treated as fixed values (calibrated in Section 6.1). Since $D_i$ are stochastic, so is PL. Thus, PL has a probability distribution. This is determined by the joint probability distribution of $\{D_i\}_{i=1,\dots,N}$. Therefore, in order to approximate the probability distribution of PL using Monte Carlo sampling, we need to make assumptions about the stochastic structure that governs mutual dependencies among the default indicators. In this respect, there are essentially two approaches. One method ties default indicators to the underlying economic factors that may lead to borrowers default, such as the obligors' asset value (structural models). The other method specifies the dependence structure of default indicators directly, not going into details on the economic reasons behind a default, such as negative equity when the asset value drops below the value of the borrower's debt (reduced-form models). In this paper, we exploit the second method. For more information on the two approaches, we recommend Duffie and Singleton (2003) and Gordy (2000).

## 2.2 Hierarchical dependence structure

Building SystemicCreditRisk we adopt the central assumption of CreditRisk+ (CSFB, 1997) which is that PDs are realizations of random variables dependent on systematic background factors.[10] Following this general idea, we assume that the variability of PDs over time relates to a number of stochastic systematic factors that reflect business cycle effects. "Bad" realizations of the systematic factors increase default rates in the economy and, therefore, cause larger losses for banks (recessions). "Good" realizations decrease default rates (upswings). This is consistent with the hypothesis that joint default risk varies with economic conditions, as investigated empirically by Das, Freed, Geng, and Kapadia (2006) among others.

We introduce one systematic factor for each group of borrowers, as described below. This factor applies multiplicatively to the borrowers' PDs and scales them up and down. More formally, the stochastic probability of default of a borrower $n$ in group $m$ is a function of the fixed PD as estimated by the lender and the group-specific stochastic systematic factor $S_m$.[11] This is given as

$$PD_{mi}(S_m) = PD_{mi} \cdot S_m, \quad \text{with} \quad E[S_m] = 1, \quad m = 1, \dots, M. \tag{2.2}$$

[10] This assumption arises from the observed variability of actual default rate statistics over time. Consider, for instance, insolvency statistics by sector published by the German Federal Statistical Office (a proxy for the rate of borrowers' defaults): Whereas mean historical insolvency rates over the period 1995–2012 ranged in a sectoral cross section from 0.2 % for electricity, gas and water supply companies to 2.5 % for financial intermediaries and insurance companies, their standard deviation over this time period ranged between 0.1 % and 1.2 %, respectively.

[11] The implicit assumption here is that the lender estimates PDs in a through-the-cycle manner (as opposed to a point-in-time estimate which incorporates information on the current state of the economy).

A common systematic factor $S_m$ refers to systematic risk that affects all borrowers in group $m$. This is normalized to have unity expectation so that $E[PD_{mi}(S_m)] = PD_{mi}$ holds. That is, on average, the random PDs equal their estimates reported by lenders. In terms of default correlation, Equation (2.2) generally implies

$$\operatorname{corr}\big(D_{mi}, D_{nj}\big) = \frac{PD_{mi}PD_{nj}}{\sqrt{PD_{mi}(1-PD_{mi})}\sqrt{PD_{nj}(1-PD_{nj})}} \cdot \operatorname{cov}(S_m, S_n)$$

for two borrowers in different groups $m$ and $n$. For two borrowers in the same group $m$, $\operatorname{var}(S_m)$ replaces $\operatorname{cov}(S_m, S_n)$. That is, the (co-)variation of systematic factors governs linear dependence among the borrowers' defaults.

Introducing systematic factors has an important consequence from a systemic perspective. It makes the default indicators of the obligors of different banks dependent on common factors and, thus, renders the banks' portfolios interdependent. Therefore, the model captures defaults concentration risk as a major source of potentially large losses in the banking sector.[12]

As in the CreditRisk+ model, random variables $S_m$ follow a gamma distribution with expectation one and variance given by the scale parameter $\alpha_m$, as recapitulated in Appendix A.1. In the standard CreditRisk+ setting, though, the group-specific factors $S_m$ are assumed to be mutually independent, which implies that default events of the borrowers who belong to different sectors/countries are independent of each other. Such a simplifying assumption might lead to a significant underestimation of losses given that in Germany, for instance, the correlation between insolvency rates in different industries often reaches 60% to 90%.[13] A more realistic model would allow for inter-group dependence. Giese (2004) introduces two such models – hidden gamma and compound gamma[14] – which generate a uniform inter-group correlation and a uniform inter-group covariance, respectively. In both models the maximum possible correlation depends on either largest or smallest sector variance. Parameters involved can, in principle, be fitted to an externally given covariance matrix of risk factors. However, a good fit is rarely possible and the fitting procedure is far from being straightforward, since $M + 1$ parameters of the systematic factors have to be fitted to an empirical covariance matrix with $M(M + 1)/2$ degrees of freedom (see also Glogova and Warnung, 2006, pp. 80-81).

For SystemicCreditRisk we design a novel, conceptually appealing model for the inter-group dependence. Its parameters can be estimated without constraints using conventional statistical techniques and historical data from the German credit register. We

[12] As the global financial crisis demonstrated, contagion between different parts of the market can be another source of systemic risk, especially due to loss of confidence (see Allen and Carletti, 2013, p. 125). However, at this stage of the model development we do not consider contagious effects via direct linkages between the banks. We instead give priority to an advanced portfolio model. The impact of systematic factors on common exposure of banks does indeed have far more severe consequences for the probability of simulations bank defaults than direct contagion through the interbank market, as pointed out in Elsinger et al. (2006a, p. 1311) and Elsinger, Lehar, and Summer (2006b, p. 154) and references therein.

[13] Correlation was estimated for the insolvency statistics published by the German Federal Statistical Office (see Footnote 10) in a simplified manner without adjusting for the autocorrelation. Similar findings for Germany for the period 1962–1993 are reported by Lesko, Schlottmann, and Vorgrimler (2004).

[14] Puzanova (2013) suggests another applications of compound-gamma distributed systematic factors as an extension to a KMV-type credit portfolio model.

suggest joining the group-specific factors by a copula function.[15] Using copulas has the advantage that we can control dependence characteristics by choosing a copula with appropriate properties such as

1. A heterogeneous dependence structure.
2. Upper tail dependence.
3. Feasible parameter estimation and sampling in high-dimensional problems.

The first property allows differentiating between groups that are closely related to each other and groups that are only weakly related to each other. The second characteristic means that large, i.e. "bad", realizations of different systematic factors are likely to occur simultaneously. Consequently, default probabilities of the borrowers tend to increase simultaneously, mimicking clustering default events during a recession, when credit concentration risk materializes in large losses. The third, pragmatic criterion implies that even for a large number of systematic factors, estimation and sampling from the copula have to remain feasible.

Archimedean copulas with their simple, closed-form analytical representation are typical candidates for modeling high-dimensional joint distributions (our third criterion). Whereas these copulas themselves are not suitable for modeling heterogeneous dependence structures (our first criterion), the nested or hierarchical Archimedean copulas (HACs) fulfill this requirement.[16] It is possible to construct a HAC consistent with the second criterium, using a Gumbel generating function.

A Gumbel copula has one parameter $\theta \in [1, \infty)$ that controls the strength of dependence between random variables, including the asymptotical upper tail dependence (see definition A.2). This parameter is closely related to Kendall's rank correlation $\tau$ given in definition A.1.[17] For a bivariate Gumbel copula (a copula that joints two random variables), $\tau$ has a simple analytical expression in terms of the dependence parameter: $\tau = (\theta - 1)/\theta$. This shows that higher values of $\theta$ lead to stronger dependence of the marginal distributions. $\theta = 1$ corresponds to the independence copula.

The dependence model that we suggest has a hierarchical structure. Its first hierarchy level comprises single sectors, countries and regions, i.e. one-dimensional marginal distributions of the systematic factors. At the next level, some of the sectors, countries and regions are coupled together, for instance, two or three at a time. They a combined by low-dimensional marginal Gumbel copulas with large dependence parameters. Some of the sectors, countries and regions remain loose at this level. They may join in at the next, higher level of hierarchy, where marginal copulas with smaller dependence parameters may combine them either with each other or with the copulas of their peers that have been already coupled together at the previous level. And so on until, at the very top, the last remaining elements are joined together by a copula with the smallest dependence parameter (possibly equal to 1). The entire structure has a mathematical representation

[15] Appendix A.2 provides the basic theoretical background for copulas. For further reading, we recommend Aas (2004) and Okhrin, Okhrin, and Schmid (2013).

[16] For an application of HAC within a structural credit risk framework and for its impact on the portfolio tail risk in comparison to a Gaussian model see Puzanova (2011).

[17] Conceptually, a measure of rank correlation is more appealing than the Pearson's linear correlation in the non-Gaussian context: see Appendix A.2.

that can be easily pieced together using expressions in the form of Equation (A.7) in Appendix A.2.

The assumptions of gamma marginal distributions and a dependence function in the form of HAC with a Gumbel generator completely determine the joint distribution function of the systematic factors. Relevant parameters must be fitted to the available data, as Section 6.2 will describe.

## 2.3 Portfolio loss distribution and risk metrics

Having completed the description of our portfolio model, we now outline the general procedure for sampling portfolio loss variable which delivers PL distributions at the bank and system level:

1. Draw $1,000$ joint realizations of $M$ systematic factors using their copula.
2. Based on that, obtain conditional PDs as given in Equation (2.2).
3. For each joint realization of the systematic factors, draw $10,000$ realizations of the individual default indicators for each single borrower using the conditional PDs.
4. For each draw, calculate portfolio loss according to Equation (2.1).
5. Calculate the empirical probability distribution of PLs for each single bank and for the entire system; the system's loss equals the sum of losses of all banks in a given simulation run.

The overall number of simulation runs amounts to ten million – a solid basis for the empirical PL distributions.

To express credit risk in numbers, we can obtain the following metrics from the PL distributions of every single bank and of the entire system: expected loss, value-at-risk and expected shortfall. Furthermore, from the simulation results we can also derive each bank's contribution to the system's expected shortfall. In the following paragraph, we define these risk metrics in plain words and refer to Puzanova and Düllmann (2013, p. 1245 f.) and references therein for more technical details:

- Expected loss (EL) is defined as the expected value of PL. It is estimated as the mean of the simulated PL distribution.
- Value-at-risk (VaR) equals a predefined percentile $q$ of the PL distribution. VaR is estimated as the level of PL that was only exceeded in $(100 - q)\%$ of simulation runs.
- Expected shortfall (ES) is defined as the expected value of PL beyond the VaR. We calculate the ES as the mean over $(100 - q)\%$ of the largest realizations of PL.[18]
- A bank's individual contribution to the system's ES is estimated as the mean PL of the bank in those $(100 - q)\%$ of simulation runs, in which the system's PL exceeds the VaR-threshold.

[18] For simplicity, we do not adjust ES for the discontinuity of the discrete probability distribution of losses; see Puzanova and Düllmann (2013, p. 1245).

# 3 Data

To study the impact of macroeconomic stress scenarios on the credit portfolios of systemically important German banks, we combine four data sets: (1) macroeconomic variables, (2) NPL ratios, (3) the Bundesbank's credit register and (4) supervisory reporting data.

We generate the relevant international macroeconomic stress scenarios using NiGEM. From the NiGEM database, we additionally obtain time series of macroeconomic variables and use them for the empirical estimation of the satellite macro-econometric model. Working with the same database ensures consistency of estimation, projection and stress testing.

The country-specific NPL ratios are largely obtained from the International Monetary Fund (IMF) database on Financial Soundness Indicators (FSI). In the IMF FSI database NPLs are defined as non-performing loans to total gross loans.[19] This data is enriched with NPL ratio time series from the World Bank and, in some cases, from the National Central Banks. We use quarterly rather than annual observations, which allows for NPL ratio projections that may vary during the year. The recent crisis, in particular, demonstrated how quickly events in financial markets can be reflected in changes of NPL ratios.

The Bundesbank's credit register is our main data source for individual exposures of German banks to their borrowers.[20] Therefore, we build our portfolio model on this database. The credit register contains information on large exposures that reached or exceed €1.5 million anytime during the reporting period. We concentrate on the 12 major banks that, at the time of our analysis, the German Federal Financial Supervisory Authority (BaFin) identified (along with a few other banks) as systemically important institutions.[21] Nevertheless, no aggregation at group level takes place. We instead consider individual credit exposures of all banks belonging to a group, omitting the intra-group claims. For each borrower, the credit register contains information on the country code and industry branch. The credit register thus covers both domestic and foreign exposures of individual banks to other banks, other financial institutions, non-financial firms, house-

[19] However, there is no single definition of NPLs across countries. For a detailed discussion of the definition of NPLs according the IMF FSI database, see also Beck, Jakubik, and Piloiu (2013). The IMF summarizes the recommendations for considering loans (and other assets) as NPLs in the IFS Compilation Guide. According to the Guide, loans (and other assets) become non-performing when (1) payments of interest and/or principal are more than 90 days past due, or (2) interest payments equal to 90 days or more have been capitalized, refinanced, or delayed by agreement, or payments are less than 90 days overdue, but there are doubts that payments will be made in full (IMF, 2006). The Basel II rules and to the EBA standards introduced in 2013 use a broader definition of non-performing exposures that applies to all on-balance sheet loans and debt securities (except those held for trading) as well as to some off-balance sheet exposures. Additionally, non-performing exposures include defaulted and impaired exposures, or exposures more than 90 days past due, or, if the full repayment of principal and interest without realization of collateral is unlikely, regardless of the number of days past due (BIS, 2016; EBA, 2013).

[20] Details on the credit register can be found in Schmieder (2006), and in published work by Ongena, Tümer-Alkan, and von Westernhagen (2015, 2012); Bednarek, Dinger, and von Westernhagen (2015), for example. Additional documents on the credit register provides tab "Tasks → Banking supervision → Lending business" at `www.bundesbank.de/en/`.

[21] The list of systemically important German institutions may change every year. BaFin publishes an updated list at `https://www.bafin.de/SharedDocs/Downloads/DE/Eigenmittel_BA/dl_asri_institute_ba.pdf`. See also the list published by the EBA at `http://www.eba.europa.eu/risk-analysis-and-data/other-systemically-important-institutions-o-siis-/`.

holds and governments. We consider them all, excluding only loans to the central banks. Furthermore, we obtain information on borrowers' PDs as estimated from internal rating systems by the lenders and information about the credit risk mitigation techniques such as collateral and guarantees.

Finally, we obtain data on capital and RWA for the banks under consideration from the supervisory reporting system maintained by the Bundesbank and the BaFin for regular banking supervision. All bank-specific information is as of 2013Q4.

Below we present some descriptive statistics on large German banks and briefly elaborate on their credit exposure to the GIPS countries, on which our application example focuses. Section 6 devoted to the calibration of the portfolio model will present more details on the country- and sector-breakdown.

The 12 systemically important German banks we focus on in our analysis cover 60% from total assets of the whole banking industry as of 2013Q4; see Table A.1. Their exposure and RWA reported in the credit register total €2,309 billion and to €641 billion, respectively. Thus, the data from the credit register accounts for about 78% of the banks' total RWA for credit risk, which equals €817 billion. On average, they maintain core equity tier 1 capital (CET1) ratio of nearly 16%.

Table A.2 additionally provides aggregated figures on the banks' portfolio structure with respect to the GIPS countries. Whereas the overall foreign exposure accounts for 55% (€1,272 billion) of the total exposure, exposure to the borrowers located in the GIPS countries makes up 12% (€147 billion). In this context, Italy (€76 billion) and Spain (€58 billion) account for a bulk of the GIPS exposure. Exposure to Portugal and Greece is far less significant (€10 billion and €2 billion, respectively).

# 4 Scenario Design

## 4.1 Modeling with NiGEM

The present paper designs relevant international macroeconomic stress scenarios using NiGEM. NiGEM is a global structural macroeconomic model developed and maintained by the National Institute of Economic and Social Research (NIESR). It is often used for policy analysis and stress testing purposes by various institutions, such as international organizations and central banks (including the ECB; Dees et al., 2017). It combines structural models for a large number of individual economies by linking them through trade, prices and financial markets. Thus, the model captures important mechanisms for the transmission of macroeconomic shocks across countries.[22] Additionally, we use the baseline scenario designed by the NIESR that supplies projections as to how macroeconomic variables could develop in the future in the absence of any additional shocks.

[22]In contrast to purely empirical models (e.g. vector autoregressive or VAR models) NiGEM imposes an explicit structure on economies and exhibits "New Keynesian" characteristics by combining nominal rigidities and forward-looking elements. On the other hand, it does not model the microeconomic optimization decisions which are the foundation of dynamic stochastic general equilibrium (DSGE) models. For an overview of NiGEM, see, in particular, Barrell, Becker, Byrne, Gottschalk, Hurst, and van Welsum (2004). A brief model description and a policy application are also provided by Catte, Cova, Pagano, and Visco (2011), for example. For more recent information on the model structure, see `http://nimodel.niesr.ac.uk/`.

We define various stress scenarios using shocks to certain variables that induce endogenous reactions of real GDP, unemployment as well as short and long-term interest rates. One key advantage of this model-based approach is that the derived responses of the variables which are relevant to bank stress are model-consistent and not determined exogenously. The short-term (three-month) interest rates, in particular, develop in accordance with NiGEM's default monetary policy rules, the key objective of which is to stabilize inflation. The long-term interest rates are, in turn, derived from the expected short-term interest rates over the next ten years. The deviations of the relevant variables from the baseline (as a percentage or percentage points) show the extent of macroeconomic stress generated. The levels for important macroeconomic variables represent the sum of the baseline levels and the deviations in a stress scenario simulation. Thus, they also depend on the underlying NIESR projections. In macroeconomic stress tests, sufficiently strong stress should be generated within a plausible macroeconomic setting. The scenarios presented in this paper which focus on specific, intuitive macroeconomic stories have been designed in order to illustrate how our model works. The consideration of even larger individual shocks might have compromised the plausibility of the scenarios. However, it would be straightforward to combine the individual scenarios in order to generate even greater macroeconomic stress (at the expense of an intuitive storytelling).

In all stress scenarios, GDP declines and the unemployment rate increases in the GIPS countries compared to the baseline. However, the implications for the interest rate spread, i.e. the premium on long over short-term interest rates, differ considerably depending on the specific stress scenario applied. As a rule, the shocks for the GIPS countries are calibrated to the same relative magnitude (e.g. as a percentage of GDP). Furthermore, the shocks always hit these countries simultaneously, which means that direct effects and spill-over are included in the simulations. Originating in a small number of countries, the shocks are transmitted to macroeconomic variables in other countries in the NiGEM world. In all cases, the simulations of stress scenarios start in 2015Q1 and allow us to make a projection over a four-year time horizon until 2018Q4, based on the baseline scenario assumed in NiGEM.[23] The baseline starts as early as 2014Q1. In order to generate sufficiently strong stress, each scenario represents a combination of several macroeconomic shocks. Subsequent shocks are implemented by stacked simulations so that new shocks are not anticipated by economic agents.

## 4.2 Macro stress scenarios for the GIPS countries

We concentrate on macroeconomic shocks to the GIPS countries. During the sovereign debt crisis in the euro area the GIPS countries were subject to severe macroeconomic stress, which also affected other economies adversely. While we acknowledge that the past may not be a good guide to the future, we have selected those countries in order to illustrate how stress propagated through our model might potentially affect the German financial sector. Additionally, we include a scenario that generates macroeconomic stress at the global level. Overall, we simulate three stress scenarios for the GIPS countries in NiGEM: an oil crisis, a financial crisis and a scenario of fiscal contraction. They are mutually complementary insofar as they differ particularly in terms of the development of the interest rate spread. Moreover, the scenarios also differ in terms of the persistence of

[23]NiGEM version v2.14-b and the corresponding baseline APR14-B were used for simulation purposes.

the assumed shocks and the degree to which other economies are affected by the assumed shocks. Figure A.1 presents the resulting deviations of the macroeconomic variables of interest from their baseline path for the GIPS countries.

**Oil crisis scenario**. In the oil crisis scenario, an exogenous surge in the nominal oil price is assumed, which might reflect an escalation of geopolitical tensions, for example.[24] Eventually, the oil price peaks at US$120 above the baseline, before returning to the baseline. This oil price shock raises inflation sharply at the global level, triggering a vigorous tightening in monetary policies so that short-term interest rates rise steeply relative to long-term rates. Real global output falls below the baseline due to the immediate weakness in aggregate demand and the deterioration in supply-side fundamentals. As the oil price surge ultimately proves to be transitory, however, the variables return to their original path in the long term.

**Financial crisis scenario**. In this scenario, a crisis in the financial markets of the GIPS countries is generated by a combination of several shocks which, eventually turn out to be temporary. First, a term premium is used to raise the long-term interest rates relative to short-term interest rates. This corresponds *ceteris paribus* to a decline in the prices of the affected government bonds. As the yields on government bonds form the basis of enterprises' financing costs, this dampens private investment. Additionally, a premium is also introduced directly into enterprises' user costs of capital with the aim of capturing a specific deterioration in their financing conditions. The overall rise in user costs of capital in the GIPS countries resembles the crisis in 2008-09. At the same time, a slump in stock prices is simulated via the risk premium contained in stock prices, while house prices are lowered exogenously. Overall, a pronounced weakness in aggregate demand (especially in private investment) is generated in the GIPS countries which dampens consumer prices there over the longer term. However, the financial shocks entail a depreciation of the euro, the short-term inflationary impact of which affects the entire euro area and needs to be dampened from the perspective of monetary policy. As a result, the euro area-wide short-term interest rate rises slightly above the baseline initially before it falls below it. A decline in real (long-term) interest rates stimulates investment in other euro-area countries counteracting a shortfall in demand due to reduced exports to GIPS countries. Lower euro-area demand and the depreciation of the euro also transmit the impact of the shocks to economies outside the euro area.

**Fiscal contraction scenario**. The scenario of fiscal contraction simulates ambitious efforts to improve public finances in the GIPS countries combining cuts to (real) government consumption and hikes in the income tax rate. To capture mixed economic developments in the euro area, nominal hourly wages in Germany are raised exogenously above the baseline. At the euro-area aggregate level, this basic inflationary shock counteracts the disinflationary impact of fiscal consolidation in the GIPS countries. As a result, there is virtually no change in short-term and long-term interest rates in the euro area. Nevertheless, real aggregate output in the GIPS countries is severely dampened in this setting. In Germany, real GDP is somewhat lower compared to the baseline, as the exogenous rise in wages constitutes a negative supply-side shock.[25]

[24] The plunge in the oil price since mid-2014 demonstrates that the oil market has been prone to severe price shifts. Moreover, US recessions have been regularly preceded by strong increases in the oil price; (see Hamilton, 2011).

[25] See also Deutsche Bundesbank (2013a).

# 5 Multi-Country Macro-Econometric Model

## 5.1 Dynamic country panel model

Designing our satellite macro-econometric model, we follow the current literature on the determinants of NPLs (Beck et al., 2013; Louzis, Vouldis, and Metaxas, 2012) and use the dynamic panel approach in order to estimate the impact of macroeconomic variables on the NPL ratios. The estimation procedure examines changes in the NPL ratios rather than NPL ratio levels.[26]

The regression explains changes in the NPL ratios at country level in terms of changes in the most important macroeconomic variables that may theoretically exert an influence on the NPLs. The explanatory variables taken into consideration relate, first, to the real economy (the growth rate of real GDP and the change in the unemployment rate)[27] and, second, to the financial markets (the change in the short-term and long-term interest rates), with the exact definitions of the variables to be found in Table A.3[28]. Our sample consists of 24 EU countries available in NiGEM plus the four large economies of Australia, Canada, Japan and the United States (see also the upper panel of Table A.7; we exclude Croatia, Cyprus, Luxembourg, Malta, and Norway because of lack of data in NiGEM). This is a balanced panel data set with a total of 1,008 country-quarter observations for the period from 2005Q1 to 2013Q4.[29] Table A.4 provides descriptive statistics for the variables entering the regression.

We use the two-step system generalized method of moments (GMM) approach proposed by Blundell and Bond (1998) to estimate the following linear dynamic model:

$$\Delta \mathrm{NPL}_{it} = \sum_{k=1}^{p} \alpha_k \Delta \mathrm{NPL}_{i(t-k)} + \beta' (L) \, \Delta \mathrm{Macro}_{it} + \eta_i + \upsilon_{it}, \tag{5.1}$$
$$t = q+1, \cdots, T_i, \quad i = 1, \cdots, N,$$

where the variable $\Delta\mathrm{NPL}_{it}$ is the change in the NPL ratios at time $t$ compared to the prior period and $\Delta\mathrm{NPL}_{i(t-k)}$ is the lagged change in the NPL ratios, $\Delta\mathrm{Macro}_{it}$ is a vector of explanatory macroeconomic variables, $\beta'(L)$ is a vector of associated polynomials in the lag operator and $q$ is the maximum lag length in the model. $\Delta\mathrm{Macro}_{it}$ consists of the

[26]First, the levels are very persistent. Second, we are predominantly interested in the influence of macroeconomic factors on changes in NPL ratios. Last but not least, considering changes rather than levels, we can better deal with differences in the definition of the NPLs in individual countries.

[27]The NPL ratios used in our model refer to total gross loans for a two-fold reason. First, a recent Bundesbank study for Germany, France, Italy and Spain shows on the one hand that there is a pronounced cyclical relationship between the annual growth rates of real loans to non-financial corporations and the corresponding growth rates of real GDP Deutsche Bundesbank (2015). Second, Minh, Dinh, Mullineaus, and Muriu (2012) show, using the UK data, that unemployment appears to be the major macroeconomic factor influencing loan losses for mortgage loans.

[28]We do not include variables that describe the credit cycle in our regression since they are not available in NiGEM. However, there is evidence that loans to non-financial corporations follow the business cycle with a lagging pattern. Moreover, on the demand side, large firms may prefer financing through corporate bonds rather than through bank borrowing during periods when capital market conditions are favorable (ECB, 2013).

[29]NPL ratios enter the regression starting in 2005Q1, whereas macroeconomic variables are used from 2004Q2 onwards to account for the maximum lag length of three quarters.

growth rate of real GDP ($\Delta\mathrm{GDP}_{it}$), the change in the unemployment rate ($\Delta\mathrm{UN}_{it}$), the change in the short-term ($\Delta\mathrm{SR}_{it}$) and long-term ($\Delta\mathrm{LR}_{it}$) interest rates. $T_i$ is the number of time periods for which macroeconomic variables are available for the $i$th country and $N$ is the number of countries. In our case we have a balanced panel and $T_i$ equals 39 quarters for every $i$ and $N$ equals 28 countries. $\eta_i$ denotes the time-invariant unobservable effects – the country-specific effects – and $\upsilon_{it}$ denotes the error term.

A number of studies (Nkusu, 2011; OeNB, 2002; Demirgüç-Kunt and Detragiache, 1998) describe the relationship between the macroeconomic variables and NPL ratios as well as the expected signs of the corresponding regression coefficients as follows:

- In an economic upturn, there are few corporate insolvencies and credit defaults. As a result, growth in real GDP will be negatively correlated to changes in NPL ratios. By contrast, a higher unemployment rate is likely to be associated with an increase in NPL ratios.

- A rise in the long-term interest rate implies higher funding costs for enterprises and households and therefore impairs their ability to repay loans. A large increase in the short-term interest rate can be passed on to borrowers as well, especially due to rising funding costs of banks. Additionally, a very large increase in the short-term interest rate may lead to an inverted yield curve, which indicates the periods of economic recession. Accordingly, an increase in both short-term and long-term interest rates should be positively correlated with the changes in NPL ratios.

To account for the NPL persistence, we include two lags of the changes in NPL ratios, expecting positive coefficients. We consider the macroeconomic variables to be endogenous to the model.[30] The two-step Blundell-Bond estimation for small samples requires a Windmeijer correction of the standard errors (Windmeijer, 2005). We apply a two-step robust estimation using Stata.

## 5.2 Results of dynamic country panel analysis

Table A.5 shows the results of the two-step Blundell-Bond estimation, including two tests for autocorrelation as well as a test for overidentifying restrictions.[31] Moreover, in order to assess the cumulative impact of each explanatory variable on the change in the NPL ratios, the table additionally reports the long-run coefficients defined as $\beta^{\mathrm{LR}} = \sum_{k=1}^{3} \beta_k / (1 - \sum_{k=1}^{2} \alpha_k)$. The coefficients for the lagged NPL ratio are positive and statistically significant. This means that an increase in the change of the NPL ratios in the last two quarters is likely to be followed by an increase in the current quarter.

[30] However, the number of groups in the cross-section imposes certain restrictions on the number of instruments used for the endogenous variables. Too many instruments could lead to implausible results in the specification tests conducted by the Blundell-Bond estimator (particularly in the Hansen test); see Roodman (2009b). Therefore, in our estimation procedure, we limit the number of instruments according to the rule that the overall number of instruments for the endogenous variables should be as close as possible to the number of groups in cross-section.

[31] Although, normally, in the dynamic panel regression, the R-sq is not reported we can use the R-sq within from the corresponding fixed-effect estimator which is equal to 0.45 as a guide to the overall goodness-of-fit for our model. We also run the two-step Arellano-Bond estimation as a robustness test. However, the results remain mostly the same.

The persistence of the NPL ratio growth is also confirmed by Beck et al. (2013). Overall, the macroeconomic variables in our model are fairly good at explaining the change in the NPL ratios; they show the expected signs and a significant impact.[32] Our results are generally in line with the findings of other recent studies (Beck et al., 2013; Louzis et al., 2012; Nkusu, 2011).

The impact of the macroeconomic variables also appears to be significant in economic terms.[33] Overall, for both the entire country sample and the GIPS countries, the economic effects from our empirical model show that if all macroeconomic factors are "stressed" simultaneously and their values increase by two standard deviations, the change in NPL ratios also increases by more than two standard deviations. In terms of economic significance, the declining real GDP growth rate is the main driving force behind the deterioration in credit quality for the entire country sample. It is worth noting that the change in the unemployment rate exerted a clear, economically significant influence on the change in NPL ratios. For the GIPS countries, the economic effect from the increase in the unemployment rate slightly outweighs the economic effect from the decline in real GDP. For the GIPS countries, the economic effects from the change in the long-term interest rates seem to play an important role as well.

Two diagnostic autocorrelation tests unambiguously attest the consistency of the dynamic Blundell-Bond estimator (see also Roodman, 2009a). As expected, the AR(1) test rejects the null hypothesis that there is no first-order autocorrelation in the differentiated error terms. The more important AR(2) test cannot reject the null hypothesis that there is no second-order autocorrelation in the differentiated error terms.

The third diagnostic Hansen test for overidentifying restrictions generally shows that the instruments used are valid. It does not reject the null hypothesis which assumes that the instruments do not correlate with a certain set of error terms.

Since we aim at using our model to project the NPL ratios as part of stress testing, Figure A.2 demonstrates, using the GIPS countries as an example, the projections of the NPL ratios for the period from 2015Q1 until 2018Q4 for the baseline and for all three stress scenarios. With the exception of Portugal, the oil crisis scenario has the greatest impact on the NPL ratios in the GIPS countries. In contrast to the financial crisis and the fiscal contraction scenarios, the oil crisis scenario comprises a global shock affecting all economies (albeit to varying degrees), creating a slump in aggregate demand at the global level. Moreover, the inflationary impact of higher oil prices causes central banks to raise short-term interest rates vigorously (in line with pre-set monetary policy rules). This response in interest rates poses additional stress on the financial system beyond the

[32] It is evident from the correlation matrix (not reported here) that there is a problem of multicollinearity between the lagged change in the unemployment rate and growth in real GDP lagged by one or two quarters. Despite this problem, the lagged change in the unemployment rate stays significant in the model. A robustness check using Arellano-Bond, fixed-effects and OLS estimations has shown that the lagged change in the unemployment rate proves consistently significant. We therefore decide to retain the lagged change in the unemployment rate in the model, as the removal of an important explanatory variable may result in model misspecification and a biasing of the coefficients for the variables remaining in the model (Wooldridge, 2000). Additionally, while an adverse shock to aggregate demand lowers real GDP and raises the unemployment rate, the degree of persistence of these responses differs in NiGEM, as the wage mechanism facilitates a return of the labor market to equilibrium.

[33] The economic effect of a macroeconomic variable is calculated as two times its standard deviation multiplied by its estimated long-run coefficient. It is a usual practice in different stress-testing models to take two standard deviations in order to evaluate the stress levels.

impact of lower macroeconomic activity.

In the case of the oil price shock, the deviation of the NPL ratios from the baseline in the period from mid-2016 to mid-2017 for Greece distinctly exceeds the 3pps mark with the maximum value reaching $3\frac{2}{3}$pps, whereas it reaches nearly 3pps for Spain. The reaction of Italy and Portugal is less strong with the largest deviation from the baseline amounting to around $1\frac{1}{2}$pps for Italy and to roughly 1pp for Portugal. The maximum absolute values of the NPL ratios during the same time span are the highest for Greece at slightly above 36%, followed by Italy, Spain and Portugal with values at around 20%, 18% and 12%, respectively.

The second strongest reaction of the GIPS countries is observed in the financial crisis scenario. During the period from end-2016 to end-2017, the deviation of the NPL ratios from the baseline reaches the highest value of over 3pps for Greece and of around $2\frac{2}{3}$pps for Spain. Here too, Italy and Portugal show a less strong reaction with the highest values at around $1\frac{1}{2}$pps. The maximum absolute values for the NPL ratios for Greece are slightly above 35%, followed by Italy, Spain and Portugal with values, again, at about 20%, 18% and 13%, respectively.

In the scenario of fiscal contraction we see a rather moderate reaction for the GIPS countries. During the period from end-2016 to end-2017, the largest deviation of the NPL ratios from the baseline is observed for Greece at slightly below 2pps, followed by Spain at just over 1pp. The deviation of the NPL ratios from the baseline for Italy and Portugal remains at the level of around 1pp for both countries. The maximum absolute values for the NPL ratios are just over 34% for Greece, at around 19.5% for Italy, just over 16% for Spain and at around 12.5% for Portugal.

# 6 Calibration of the Portfolio Model

## 6.1 Risk parameters

In order to calibrate the SystemicCreditRisk model described in Section 2, we first define risk parameters EADs, LGDs and PDs using the data from the German credit register. The EAD is approximated by the total exposure of a bank to a borrower. Bank exposure to borrowers is defined fairly broadly. It includes not only loans but also bonds and other securities in the banking book as well as derivatives and other off-balance sheet items.[34] Taking into account collateral posted, the LGD is fixed to 35% for the secured part of the exposure and to 45% for the unsecured part. Additionally, the LGD is adjusted for the effect of guarantees provided by other banks. The PDs originate from internal rating systems of the lenders and undergo a few corrections. Appendix A.3 provides more technical details on EADs, LGDs and PDs.

[34]The following items are not reported in the credit register: exposures to German central and local governments and communities (supposed to have zero PD), securities in the trading book, undrawn loan commitments, shares in other enterprises. For a detailed definition of credit exposure and exceptions, see Sections 19 and 20 of the German Banking Act, respectively (the version applied before 01.01.2014).

## 6.2 Copula of the systematic factors

The next step in the calibration of the portfolio model consists in the estimation of the joint probability distribution of systematic factors for German sectors, foreign countries, and remaining geographic regions.

The credit register assigns each corporate borrower to a country and an industry sector according to the Nomenclature statistique des activités économiques dans la Communauté européenne (NACE) classification and separates remaining retail borrowers into two groups: households and non-profit organizations. Based on that, we introduce $M = 69$ group-specific systematic factors. This includes, on the one hand, 28 sectors for German borrowers. Foreign borrowers, on the other hand, are only grouped either by country or by region of their residence (since there are not always enough borrowers per group to allow a finer breakdown). Thus, in addition to German sectors, we consider 32 single foreign countries (Table A.7) and nine remaining geographic regions. The regions are defined in accordance with the definition of country groups used in the balance of payments statistics.

Tables A.6 and A.7 provide sectoral, country and regional breakdown of exposure of the 12 banks as well as the historic average share of loans in each of these groups. This overview shows that approximately half of large loans by major German banks have been extended to German borrowers and that the domestic exposure is concentrated in the financial sector (on average, exposure to banks and other financial intermediaries has made up around 75% of the reported large domestic loans).[35]

We calibrate the joint distribution of systematic factors in two steps. First, we estimate parameters of the marginal gamma distributions of the group-specific systematic factors. Then, we fit their copula to the data. For this purpose, we need historical data that reflects (joint) variability of defaults per sector, country and region. In the CreditRisk+ context, related literature suggests using historical default rates; see, for example, Boegelein, Hamerle, Knapp, and Rösch (2004). Such data is very scarce, though.[36] Fortunately, since 2008, the credit register has contained, on borrower level, data on specific provisions for impairment losses. Using them, we can approximate annualized default

[35]Around 60% of the domestic exposure are concentrated in the banking sector. Therefore, we argue, it is essential to consider this exposure. Other studies with a similar top-down stress-testing set-up also choose portfolio structure which includes exposure to financial institutions (Castre, Fitzpatrick, and Sydow, 2009; Riksbank, 2006). Some authors (Düllmann and Kick, 2014, who focus on export-oriented sectors) argue that exposure to banks is different in terms of the amount secured by collateral and in terms of maturity structure; thus, they exclude it from their analysis. However, in our study, we take into account the amount of collateral posted. In this respect we do not differentiate between exposure to banks and other exposures. Additionally, there is also empirical evidence that the average length of contracts in the interbank market is well above one year (Bluhm, George, and Krahnen, 2016). In the central bank models such as the BoE's RAMSI, the ECB's stress-testing approach and the OeNB's SRM, the interbank market appears to be an integral part of the top-down stress-testing framework as well. However, these models focus mere on network effects and contagion in the interbank market. Admittedly, considering credit exposure to banking sector within the scope of our portfolio model is a first step towards a more-sophisticated stress-testing framework.

[36]Since the actual default rate history accumulated by banks is not available to researchers, published statistics on insolvency rates are often used instead. However, data restrictions do not allow us to pursue this estimation strategy. Although the German Federal Statistical Office publishes annual data on insolvency rates, the sector breakdown differs from the NACE classification used in the credit register. Moreover, it is difficult to obtain insolvency statistics for all countries and regions included in the model.

rates per sector/country/region for each quarter. To accomplish this, we take four consecutive quarters at a time and calculate the share of borrowers with specific provisions in the respective group. By doing so, we take account of lending from all German banks, as reported in the credit register.[37]

The default statistics serve as a data basis for parameter calibration. First, we use them for setting the parameters of the marginal gamma distributions of the group-specific systematic factors. For each factor, we apply Maximum Likelihood Estimation (MLE) for truncated samples to the historical group-specific default rates. We assume that samples are right truncated with a known point of truncation that equals the maximum observed value of default statistics for this group. The estimation procedure boils down to MLE that does not simply use a gamma probability distribution function $f_g(x;\alpha)$ to define its objective function, but also takes into account the cumulative probability of observing the data up to the point of truncation $f_g(x;\alpha)/F_g(x_{tr};\alpha)$; see Hughes (1962, p. 21 f.). In some rare cases, the MLE may fail. Here, we suggest using sample moments for pinning down distribution parameters. The estimated gamma distributions are then normalized to have the mean of one. Tables A.6 and A.7 provide a schematic overview of the resulting gamma distributions.

We then use the default statistics to calibrate the structure and dependence parameters of the copula that joins all systematic factors. To estimate parameters $\theta$ at different levels of the hierarchy of the Gumbel HAC copula, we utilize routines from the "HAC package" in R programming environment (R Core Team, 2014). The HAC package documentation by Okhrin and Ristig (2012, p. 4 f.) describes the multi-stage MLE procedure implemented, and Appendix A.2 briefly summarizes the main steps of the estimation algorithm. MLE is performed such as to neglect small differences in the strength of dependence between different systematic factors and, on the other hand, with the goal of preserving a reasonably granular copula structure.[38]

Figures A.3 to A.5 present the estimated HAC in all details and show its 44 dependence parameters. For reasons of clear presentation, we divide the HAC into many parts: sub-copulas C[1] to C[5] and a "top" copula. Here, we briefly describe the estimated dependence structure. Sub-copulas C[1] and C[4] mostly join German sectors. The estimated parameters suggest that, historically, default statistics approximated based on the data for specific provisioning by German banks were closely associated for the following

[37]Similar approaches have been pursued in the related literature before; see, for instance, Memmel, Gündüz, and Raupach (2012) for German borrowers. The authors approximate historical default statistics by the rates of write-downs obtained from the German borrowers statistics. The quarterly borrowers statistics collected by the Bundesbank contain information on loans to domestic borrowers as well as write-downs (write-ups) aggregated at the level of different industries and groups of retail borrowers. Information on foreign borrowers is not collected, however, which precludes consideration of the borrowers statistics in our study.

[38]In order to check the robustness of estimation results for the joint distribution function of the systematic factors (their marginal distributions and the copula), we have additionally performed estimation that includes the most recent data up to 2015Q4. The results (not reported here) show that there are some changes in dependence parameters and in the composition of groups (sub-copulas) of German sectors and/or foreign countries. These changes in the overall dependence structure, however, do not affect the results for banks' capital requirements considerably.

sectors in Germany:

> ((((construction (S19) and housing companies (S20)) and
> (households (S26) and food service (S22))) and other real estate (S21)) and
> other services (S25),

as shown in C[1], and, as shown in C[4], for:

> (((metals and metal products (S6) and machinery (S7)) and
> ((wood, furniture, paper, printing (S9) and textiles (S10)) and
> (renting and leasing (S14) and equity investment (S18)) and
> computers, electronic (S8))) and a number of others).

On the other hand, the arguments of sub-copulas C[3] and C[5] are mostly countries and regions. C[3] includes Sweden, Norway and other countries in Europe (1031) as well as Spain and remaining countries in the Americas (1036). C[5] joins Slovenia and Poland, Croatia and remaining countries in Europe (1032), Malta and Cyprus, Denmark, Netherlands, Finland, Italy, Portugal and Greece as well as Luxembourg and Austria. The sub-copula C[2] has mixed arguments, including most of the remaining sectors in Germany and Canada and the USA and other main countries in the Americas (1035), as well as remaining countries in Asia (1038) and Africa (1034). At the top level, we see that in the past the default statistics for Bulgaria evolved virtually independently of the rest of the portfolio and even more so in the case of Japan, Latvia and Hungary.

Additionally, figures presented in Appendix A.4 show as an example small segments of the copula that represent pairwise realizations for a number of selected variables. Figure A.6 presents 500 pairwise realizations for seven selected industry sectors in Germany. It demonstrates for the financial sector that the copula produces the strongest link between equity investment firms and other non-bank financial intermediators (including money market fonds). The link to the rates of bank defaults is substantially weaker. The interdependence between systematic factors for the sector "insurance companies" and for all three other groups of financial intermediaries is of a medium-scale order of magnitude. Interestingly, the positive association between construction, housing companies, and other real estate activities is next to perfect. Next, Figure A.7 shows a number of selected countries. It illustrates a strong default correlation for the USA and Canada and a relatively close association with defaults on the Australian exposure. As for the group of countries on which we focus our scenario analysis, the HAC generates a tight dependence between systematic factors for Greek, Italian and Portuguese credit exposures. Interestingly, credit losses in Spain are more closely related to the group of countries we described first. Between both groups, no considerable dependence can be detected.

The combination of the univariate gamma marginal distributions and the hierarchical Gumbel-copula makes up the joint $M$-dimensional distribution function of the systematic factors. This completes our definition of the dependence structure in the proposed portfolio model. What remains to be described is the link between the portfolio model and the macroeconomic scenarios.

## 6.3 The link between NPL ratio projections and the portfolio model

To embed the portfolio model SystemicCreditRisk into the stress testing framework, we perform a percentile mapping between the NPL ratios and the systematic factors that will conduct the effect of macroeconomic projections to the borrower PDs.[39] For this purpose, we first fit gamma distributions to the country-specific observations of the NPL ratios (similarly to the procedure applied for the systematic factors). Then, given a realization of the NPL ratio for a country in a macro scenario, we calculate to which percentile of the fitted gamma distribution this realization corresponds. This percentile serves in the following simulation as the lower threshold for the systematic factor(s) for this country. Conditioning on the percentiles means that acceptable realizations of the systematic factors can only lie above the specified thresholds. Thus, sampled systematic factors cannot be "better" than suggested by the scenario, but they can be "worse". The percentile mapping is done for those 28 countries for which NPL ratio projections are available.[40] Even if no percentile mapping is possible for the remaining five countries and geographic regions, the corresponding sampled systematic factors will depend on stress scenarios, too. The reason for this is that realizations of all systematic factors have to exceed their respective thresholds simultaneously. This constrains the area of the HAC from which sampling is performed. That is, all systematic factors are sampled from a small, "stressed" part of the copula.

Figure A.8 illustrates the percentiles of the estimated gamma distributions that correspond to NPL ratio projections per country in each scenario. For countries such as France, Netherlands or Germany, where historical NPL ratios over the observation period were not very high, the stressed, large values of NPL ratios lie in the far right tail of the distribution. That is, they represent rare events for these countries. By contrast, for GIPS countries with a historic record of high NPL ratios, realizations from stress scenarios seem to be more commonplace, so the NPL ratio percentiles are lower.

## 6.4 Simulation of portfolio losses

The NPL ratio projections determine from which part of the estimated copula systematic factors can be drawn. The sampled simulated factors will affect borrower PDs and, ultimately, the distribution of credit portfolio losses in accordance with the simulation procedure described in Section 2.3.

For our simulation study, we use the data from the credit register as of the end of 2013 and assume a static balance sheet. Thus, we sample portfolio losses (PL) quarter-by-quarter, scenario after scenario based on the same portfolio composition. The time span of simulation covers 20 quarters from 2014Q1 until 2018Q4.[41]

[39]A direct link between the macro model and the portfolio model is problematic. On the one hand, NPL ratios are not reported in the credit register. On the other hand, there is no breakdown of NPL ratios by sector in Germany. Similarly, the data on NPL ratios covers only a limited sub-sample of countries reported in the credit register.

[40]In Germany, one country-specific percentile threshold applies to all domestic sectors.

[41]For the year 2014 only the baseline scenario is used. Stress scenarios start in 2015Q1. The first realization of all stress scenarios coincides with the baseline, however. Therefore, for the stress scenarios, we only perform sampling starting with 2015Q2.

First, we draw joint realizations of the 69 systematic factors using their constrained copula. The resulting sample only contains realizations that lie above the "stressed" NPL ratio thresholds. Figure A.9 demonstrates the magnitude of the systematic factors in the oil crisis scenario for a number of selected sectors and countries. The systematic factors scale up PDs for the borrowers in Italy, Greece and Portugal as high as by a factor of approximately 3 to 4. For Spain and, for instance, the United Kingdom, the scaling factors of around 2 apply. The same is true of the German real estate sector (excluding housing companies) and the banking sector. For other domestic financial intermediaries, the oil crisis scenario affects only a moderate increase in PDs of some 50%. Similar scaling factors apply to French borrowers' PDs.

In the next simulation step, we draw default indicators for all borrowers using PDs conditional on a realization of systematic factors and calculate the banks' portfolio losses. Figure A.10 shows a few examples of PL distributions after performing ten million simulation runs. Comparing PL distribution under the baseline and the oil crisis scenario, we see that not only the magnitude of simulated losses but also sometimes the shape of distribution change considerably under the stress scenario, shifting both bank portfolios and the system's portfolios into a more dangerous area of high credit risk.

Based on the sampled distribution, we calculate relevant risk measures as described at the end of Section 2.3. For the simulations under the baseline and stress scenarios we focus on the system-wide tail risk represented by the system's ES. In order to calculate the system's ES, we fix the percentile threshold at a lower of $q = 99\%$, which accounts in a non-formal way for a limited probability of scenario occurrence.[42]

For the purpose of policy application introduced in the next section, we additionally compute the banks' individual EL and VaR based on an unconditional simulation of credit portfolio losses, i.e. without regard to any macroeconomic scenarios or NPL ratio projections. This means that the area of the copula, from which systematic factors are sampled, is not constrained by any percentile conditions; we draw realizations from the entire HAC.[43] Running the unconditional simulation for individual banks, we take the regulatory threshold for credit-VaR calculation given as $q = 99.9\%$.[44]

Both scenario-based and unconditional risk metrics described above can be utilized (alongside other supervisory information and risk assessment results) for assessing micro- and macroprudential capital needs, as the next section describes. In contrast to a mere stand-alone risk assessment for every single bank, this approach incorporates a systemic view of capital adequacy.

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[42]In this context, $q$ reflects the regulator's tolerance towards the probability of a systemic event, as pointed out by Puzanova and Düllmann (2013). In our case, the latter is given by $100 - q = 1\%$ conditional on a scenario. Why use ES and not VaR at the system's level? Consider that VaR only represents *minimum loss* in the worst $(100-q)\%$ of cases. Thus, it might be an appropriate risk measure from the bank owners' point of view because their liability given the bank's default is limited. The regulator, however, has to care about the losses that a bank failure may incur, i.e. in formal terms, the expected losses given that the losses exceed the VaR threshold. This is exactly the definition of ES that represents *expected loss* in the worst $(100 - q)\%$ of cases. Therefore, ES is a more appropriate measure of externalities than VaR.

[43]To ensure a better coverage of the entire copula (as opposed to only a small part of the copula conditional on NPL ratio scenarios), we increase the number of joint drawings of the systematic factors by factor 10 (10,000 drawings). In turn, we reduce the number of simulation runs for default indicators by the same factor (1,000 drawings) to ensure that the sampling remains computationally feasible.

[44]This means that the regulator tolerates a bank's unconditional probability of default of $100-q = 0.1\%$.

# 7 Results and Policy Applications

## 7.1 Overall assessment

This section presents an overall assessment of the capability of the system as a whole to withstand severe macroeconomic shocks. Figure A.11 summarizes the results obtained for the system's portfolio that consists of all credit exposures of the 12 banks. It presents EL and ES at the system level calculated under the unconditional and scenario-based simulations using the SystemicCreditRisk model. These risk metrics suggest that, if we only consider shocks to the banks' credit portfolios, the system's ES in the unconditional simulation amounts to no more than 13% of the aggregated credit RWA and 62% of the aggregate common equity tier 1 (CET1) of the 12 banks. In the scenario analysis, the system's ES never exceeds 18% of the reported credit RWA or 88% of the available CET1 capital. Thus, in aggregate, there is seemingly enough capital to support the system's tail credit risk.

But the heterogenous results at the bank level presented below tend to tell another story. Although a number of banks appear very well capitalized to withstand even the maximum stress imposed by the scenarios, other banks' capital level might need improvement in order to sufficiently reflect tail risk externality. This indicates that the distribution of capital in the banking system is not necessarily optimal from a systemic point of view.[45] This results remain stable also when we include more resent data from the credit register for the purpose of model calibration and portfolio loss simulations.

## 7.2 Micro- and macroprudential capital requirements

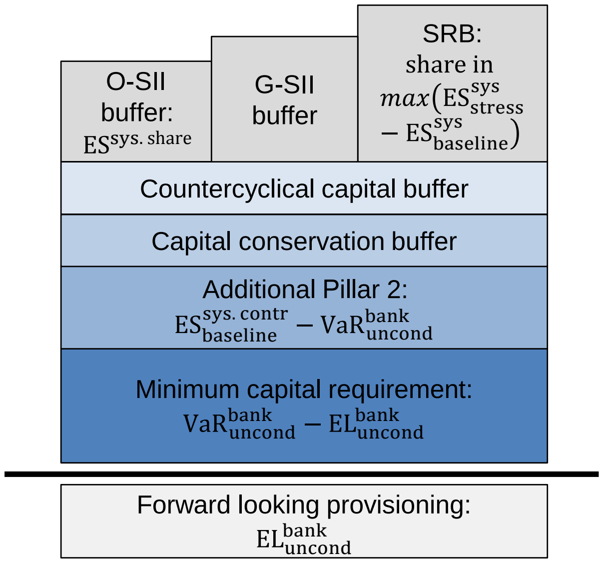
This section shows how to calibrate micro- and macroprudential capital requirements for individual banks based on the risk measures provided by the model and reports model-based requirements for the 12 banks under consideration.

The M-PRESS-CreditRisk framework features a holistic micro- and macroprudential approach to capital adequacy assessment in the banking sector. Properly adapted, it can inform decisions both on CET1 capital requirements of Pillar 1 and Pillar 2 and on macroprudential buffers such as the systemic risk buffer (SRB) and the buffer for other systemically important institutions (O-SII). Figure 7.1 outlines these model applications embedded into the overall structure of capital requirements, and the next paragraphs walk us through them. Additionally, the model provides information that can be used for building loan loss provisions in a forward-looking way.[46]

[45] A reallocation of capital according to the banks' systemic risk contributions can reduce risk in the banking system, especially by decreasing the probability of multiple defaults; see Gauthier, Lehar, and Souissi (2012).

[46] From the range of macroprudential capital tools, we omit the counter-cyclical buffer because it is tied, in the first place, to aggregate credit growth; see Detken et al. (2014). On the other hand, we do not consider the capital surcharge for global systemically important institutions. According to the Basel methodology (BCBS, 2013), setting this buffer requires detailed information on a wide range of the worlds' major banks, which is not available to us. For both these instruments, Puzanova and Düllmann (2013) suggest an alternative: a credit portfolio-based approach. Another tool not considered here is the capital conservation buffer. This is defined as a fixed percentage (2.5%) of an institution's risk-weighted assets and, therefore, does not require any modeling.

Figure 7.1: Model-based CET 1 capital requirements and loan loss provisions



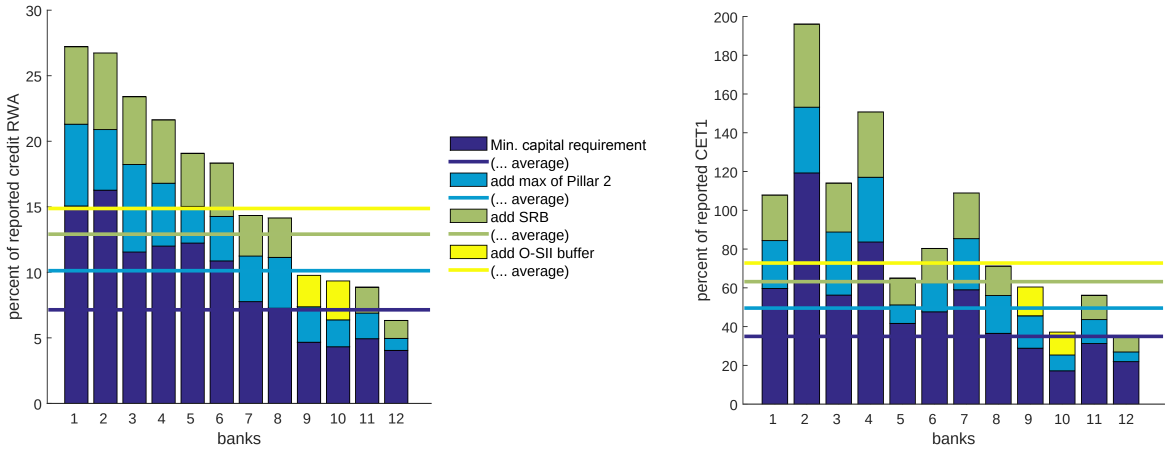

**Forward-looking provisioning** should cover expected future losses. The model developed in this paper suggests that a cushion against the losses expected one year ahead equals the unconditional expected loss of a bank, denoted in the figure as $EL^{bank}_{uncond}$. The metric is computed as the mean of a bank's credit portfolio loss distribution obtained from an unconditional simulation. In our example EL ranges from around 1% to roughly 4% of the banks' credit exposure reported in the credit register. At the aggregated level, it amounts to about 2% of the credit exposure of the system.

The aim of forward-looking provisioning is to absorb expected losses. Minimum capital requirements and capital buffers, on the other hand, are intended to protect individual banks and the system as a whole from losses that exceed the expected amount, i.e. from unexpected losses. Unexpected losses are less likely to materialize, but if they do, they can bring an undercapitalized institution – or, in an extreme event, even a large part of the banking system – to the brink of bankruptcy. Against this backdrop, the present paper focuses on addressing unexpected losses by means of both microprudential minimum capital requirements and macroprudential add-ons and buffers.

**Minimum capital requirements** under Pillar 1 represent the minimum amount of capital that a bank must have in order to cover unexpected losses at a one-year risk horizon calculated in accordance with regulatory standards. In general, the regulator defines unexpected credit loss as the difference between VaR (99.9% percentile) and expected loss computed for the credit portfolio of a bank in the unconditional simulation.

In our framework, this metric corresponds to the unexpected loss at the bank-level obtained from the unconditional simulation and denoted as $VaR^{bank}_{uncond} - EL^{bank}_{uncond}$. Thus calculated, the unexpected loss can be compared to the regulatory capital reported by a given bank. Any comparison should, however, account for differences in the underlying calculation prescriptions between the regulatory standards under Pillar 1 and our model-based figures. The former is computed using either pre-defined risk weights (standardized approach) or based on the assumptions of a perfectly diversified portfolio (internal rating-based approach), which neglects credit concentration risk. In contrast, our model-based

Figure 7.2: Model-based capital requirements per bank




(a) Relative to credit RWA

(b) Relative to CET1

*Note*: The figure presents the model-based capital requirements per bank as well as average values of the respective capital charge for the system. Only relative numbers in terms of actual credit RWA and CET1 capital reported by a bank as of 2013Q4 are shown. Since the scope of model-based requirements and actual reported figures differ substantially, they are not directly comparable and serve only illustrative purposes. For the Pillar 2 add-on only the maximum values over the simulation period per bank are shown. As to the systemic risk buffer (SRB) and the buffer for other systemically important institutions (O-SII buffer), only the maximum of both per bank is shown. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, Deutsche Bundesbank (German credit register and supervisory reporting system), the authors' own calculations and stochastic simulations.

minimum capital requirement for credit risk goes beyond the coverage of the Pillar 1 and additionally accounts for the concentration risk, arising from imperfect portfolio diversification with respect to economic sectors and geographical regions.[47]

As Figure 7.2 summarizes, the model-based microprudential capital requirement ranges from 4% to 16% in terms of the credit RWA of a bank and from 17% to 119% in terms of the CET1 capital, reported for the end of 2013. According to the model-based unconditional simulations, all but one bank have enough capital to back their credit risk. The average value amounts to 7% and 35%, respectively, where the average value is calculated as the sum over the unconditional unexpected losses of individual banks divided by the sum of credit RWA or of CET1 capital.

**Additional Pillar 2 capital requirements**, should cover volatility over the business cycle (EBA, 2014, §358 ff.), i.e. risks to which a bank may expect to become exposed

[47]Usually, concentration risk is addressed under Pillar 2, see BCBS (2006, § 772) and EBA (2014, Title 7). More generally, those risks to which an institution might be exposed beyond the coverage of Pillar 1 requirements should be taken into account under Pillar 2. If the risks not accounted for under Pillar 1 are addressed by any existing capital buffer requirements, including macroprudential buffers, Pillar 2 requirements should be offset against those buffers if they address the same risk (no double charge); see EBA (2014, § 346). Note, however, that different jurisdictions apply different approaches to exercising the supervisory powers under Pillar 2; compare, for instance, EBA (2014) with PRA (2015).

over a forward-looking planning horizon. We define the ES of the banking system as the relevant tail risk metric. It shows how much loss the banking system's aggregate credit portfolio is expected to incur in the event that the loss exceeds the system's VaR. The supervisor should impose an additional capital charge to back this tail risk.[48] For this purpose, the Pillar 2 capital add-on can be calculated as the banks' contribution to the system's ES in the baseline scenario minus the banks' VaR from unconditional simulation that was already addressed under Pillar 1: $(ES^{sys.contr}_{baseline} - VaR^{bank}_{uncond})^{+}$.

Depending on the macroeconomic projections, the banks' contributions to the system's ES expressed in euro vary over time. This gives rise to a time-varying Pillar 2 capital add-on. Its maximum value per bank observed during the 20 quarters for which we generate NPL ratio projections, is shown in Figure 7.2. The add-on ranges between around 1% to 6.5% of credit RWA or 5% to 34% of CET1 capital. The average figures come to 3% and 15%, respectively.

Although not a macroprudential instrument per se, the Pillar 2 add-on suggested in this paper takes a system-wide view on capital adequacy. This perspective is central to the application of macroprudential instruments, two of which we consider in the following paragraphs: the SRB and the O-SII buffer.[49]

**The systemic risk buffer** is a macroprudential capital tool available to banking supervisors in the EU.[50] It aims at mitigating long-term non-cyclical systemic risk not covered by other instruments. We argue that the SRB can be applied to ensure that even in a severe stress scenario the system as a whole can cover its tail risk. To guide decisions on the possible size and timing of the buffer, we suggest using the model-based estimates of the system's ES under stress. In particular, for each of the relevant supervisory scenarios, the euro amount of the system's ES can be calculated quarter by quarter and compared to the system's ES in the baseline scenario. The maximum positive difference to the baseline indicates the maximum excess systemic credit risk that should be covered by the aggregate SRB requirement: $\text{SRB}^{sys} = \max(ES^{sys}_{stress} - ES^{sys}_{baseline})^{+}$. The capital charge for each single bank equals to the amount of additional capital needed in the entire system multiplied by the bank's average percentage share in the system's ES: $\text{SRB}^{bank} = \text{SRB}^{sys} \cdot ES^{sys.share}$.[51] Since the ES shares remain remarkably stable, we simply take the average shares over all considered scenarios. The quarter in which the maximum difference is observed indicates the time period up to which the banks should have built up the buffer.

In our particular example, the oil crises scenario generates the greatest adverse impact on the NPL ratios and portfolio loss. Therefore, the SRB is based on this scenario. This difference between the system's ES in oil crisis and baseline scenario reaches its maximum

[48] The intention is that the owners – rather than the deposit insurance schemes or the public – bear the risk of losses.

[49] The capital regulation does not provide a definite clear cut between macro- and macroprudential instruments. From the economic perspective, however, we can argue that capital charges based on a standalone assessment of risk of individual banks serve microprudential purposes. On the other hand, instruments that address risks gauged from the systemic perspective, taking externalities into account (see Section 1), pursue macroprudential objectives.

[50] Article 133 of Directive 2013/36/EU (Capital Requirements Directive IV or CRD IV).

[51] Note that article 133(9) of the CRD IV allows the application of different SRB requirements for different subsets of institutions. If the macroprudential authority nevertheless decides to apply a level SRB charge for all relevant institutions, any positive difference between (i) $\text{SRB}^{sys}$ defined above for an individual bank and (ii) the level charge imposed can be accounted for under Pillar 2.

in the 15th simulation quarter or 2017Q3; see Figure A.12. Until this time, the banks should have (gradually) built up the SRB. The buffer ranges for individual banks from about 1.4% to 5.9% of credit RWA or from around 7% to 43% of the CET1 capital reported at the end of 2013. The average figures amount to 2.8% and 13.6%, respectively.

**The O-SII buffer** is a European supplement to the capital buffer designed in the Basel III framework for the global systemically important banks (G-SIB). It has a more local focus, and should internalize the risk which the failure of a large and interconnected institution located in the EU poses for the banking system of the EU as a whole or its member states.[52] According to article 131(5) of the CRD IV, O-SIIs may be required to maintain an O-SII buffer of up to 2% of the RWA. In Germany, for instance, the identified O-SIIs are assigned to one of four groups with gradually decreasing buffers of 2%, 1.5%, 1% and 0.5% of their total RWA.[53]

Following the currently applied methodology, we suggest an approach to how to divide the banks into the pre-defined four buckets according to their systemic importance, taking the level of the respective capital buffers as given. We gauge the banks' systemic importance based to their mean relative share in the system's ES, i.e. $ES^{sys.share}$, and apply the k-means clustering algorithm. This algorithm assigns banks with the largest $ES^{sys.share}$ into Group 1. The other groups follow as shown in Figure 7.3.[54]

**The sum of all capital requirements** suggested by our model at hand can be seen in Figure 7.2. Here, only the maximum of SRB and O-SII buffer is taken into account; not both requirements.[55] The smallest capital charge totals 6.3% of credit RWA. The corresponding bank reports nearly three times as much CET1 as required by the model.[56] The highest capital charge in terms of credit RWA totals 27.2%; the corresponding bank reported nearly 8pps less CET1 capital than the suggested combined model-based capital charge for credit risk. Four other banks, labeled bank 2, 3, 4 and 7 in the figure, also reported less capital than our maximum model-based charge, the difference being 96, 14, 50 and 9pps, respectively. Therefore, our results emphasize that the distribution of capital among the banks might not be consistent with a macroprudential risk assessment.

**Further research**. In this paper we have shown how a novel approach to model systemic credit risk can be embedded in a general macroprudential stress testing framework. Because of its flexibility, this framework can accommodate further important features left

[52]See BCBS (2013) for the G-SIB framework and article 131(5) of the CRD IV for the O-SII buffers. The latter represent the Union's implementation of the Basel framework for dealing with domestic systemically important banks (BCBS, 2012).

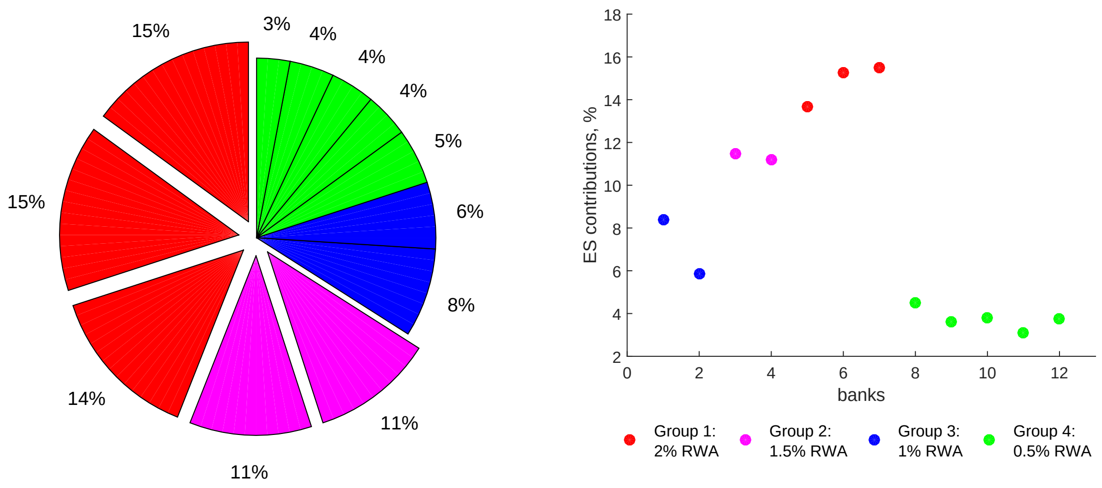
[53]`https://www.bafin.de/SharedDocs/Downloads/DE/Eigenmittel_BA/dl_methode_asri_ba.pdf`.

[54]The k-means clustering divides entities into a predefined number of groups by maximizing the distance between the clusters' means. This method was applied by Puzanova and Düllmann (2013) to the G-SIBs' contributions to the system's ES.

[55]Since both the SRB and the O-SII buffer target the same cross-sectional dimension of systemic risk (although possible from different angles), in most common cases, only the maximum of SRB and O-SII would be applicable. More precise, the maximum of the SRB, O-SII, and the G-SIB buffer would apply; but specific calculations may occur; see articles 131(14-15) and 133(4-5) of the CRD IV.

[56]The results should be interpreted with caution, bearing in mind that the model only considers a part of credit risk attributed to the exposures reported in the credit register. The calculated maximum requirement does not, however, include capital charges for the remaining credit risk, market risk, operational risk or credit valuation adjustment. We also do not consider the following capital instruments: the capital conservation buffer, the counter-cyclical capital buffer and the buffer for global systemically important institutions.

Figure 7.3: Banks grouped with respect to their expected shortfall contributions




(a) Banks' average share in the system's ES

(b) Four groups of respective O-SII buffers

*Note*: Four groups of banks are shown, clustered according to their mean ES contributions (i.e. percentage shares in the expected shortfall of the system) using the k-means algorithms. For the group with the largest ES contributions we assign the highest buffer for other systemically important institutions of 2% of the total RWA. For the following three groups we assign buffers of 1.5%, 1%, and 0.5% of the total RWA, respectively. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, Deutsche Bundesbank (German credit register and supervisory reporting system), the authors' own calculations and stochastic simulations.

for future research: a network extension to the credit risk model that will incorporate direct and indirect contagion through the interbank exposure and asset fire sales; dynamic projections of bank balance sheets and income statements; market and operational risk models. Moreover, in tandem with a capital stress test, liquidity buffers should be challenged by funding shocks in order to assess the overall resilience of the system (see Hirtle and Lehnert, 2014, p. 8). Most importantly, a comprehensive macroprudential approach to stress testing needs to include the feedback effects of a possible credit contraction and asset reallocation on the real economy. Obviously, the output risk metrics of an extended modeling framework will be different in numbers from those presented in the paper. This will not, however, affect the logic of the coherent approach to capital adequacy assessment from the micro- and macroprudential perspective which we propose.

# 8 Conclusions

This paper develops an advanced portfolio model for credit risk assessment in a banking sector called SystemicCreditRisk and embeds it into a novel stress testing framework which can inform decisions on micro- and macroprudential capital requirements under systemic stress with focus on credit risk: M-PRESS-CreditRisk. From a technical point of view we add to the literature by developing a detailed dependence structure (copula) that can capture the externality of tail risk, i.e. risk of extreme losses that materialize only rarely during crisis-like, systemic events. This is achieved by interconnecting borrowers from different industry sectors and countries in a non-linear way. This approach accounts for many simultaneous borrower defaults across banks' portfolios and, therefore, incorporates credit default concentration risk in a banking system. Consequently, portfolio loss distributions of individual banks and the entire system show a higher likelihood of large losses. In comparison with Basel II and similar approaches based on simplifying assumptions of perfectly diversified portfolios, no concentration risk, linear dependencies, absence of clustering borrower defaults and banks that are independent from each other (no systemic risk), our model considerably reduces the scope for the possible underestimation of credit risk in the banking system.

Additionally to the SystemicCreditRisk model, M-PRESS-CreditRisk consists of the module that generates multi-risk-factor, multi-country stress scenarios designed for our application in NiGEM and the satellite macro-econometric model that helps to translate these scenarios into the credit portfolio model. As to the policy applications, we show how the risk metrics derived from the combination of the portfolio model and scenario analysis can inform supervisors' judgment about the level of capital needed in the banking system to cover both expected and unexpected losses under normal and adverse macroeconomic conditions (baseline and stress scenarios, respectively). To this end, the framework provides model-based measures of both bank-level and systemic credit risk that supervisors may use in their assessment of the whole chain of capital adequacy: starting with dynamic loan loss provisioning, proceeding with minimum capital requirements, including Pillar 2 add-ons, and finishing with macroprudential buffers that address structural systemic risk.

We also illustrate how the framework may be put into practice, using a sample of 12 systemically important German banks and utilizing a baseline and three stress scenarios (oil crisis, financial crisis, and fiscal contraction). Based on the results, we argue that the actual capital levels and the capital needs calculated with additional regard to systemic credit risk may differ considerably. Moreover, although the aggregate capital figures suggest that there is enough capital in the system as a whole to withstand severe adverse macroeconomic scenarios, the analysis on the bank level indicates that the distribution of capital in the banking system might be suboptimal from a systemic point of view. This follows from the observation that a number of banks show excess capital compared to the overall requirement calibrated to the systemic risk metrics from our model, whereas another banks' capital base needs improvement. In the worst case, five banks would have difficulties satisfying the combined model-based micro- and macroprudential capital requirements.

At the end, we must emphasize that our numerical results should be treated with caution because our model is subject to a number of constraints. First, it only focuses on credit risk, whereas regulatory requirements also address other risk sources such as market and operational risk. Second, our model concentrates on only one channel of systemic risk propagation represented by the correlated bank exposures; contagion among the banks or other second-round effects are not considered here. Finally, M-PRESS-CreditRisk links a number of approaches together which introduces an additional layer of model risk.

Bearing these caveats in mind, the risk measures generated in the coherent stress testing framework of M-PRESS-CreditRisk may help supervisors think of different micro- and macroprudential capital instruments as a logically linked chain of requirements and support the calibration of bank-specific capital charges.

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# A Appendix

## A.1 Gamma distribution

A random variable $X$ is gamma distributed if its probability distribution function reads

$$f(x) = \frac{1}{\Gamma(k)\alpha^k} x^{k-1} e^{-x/\alpha}, \quad x \in [0, \infty),\ k > 0,\ \alpha > 0. \tag{A.1}$$

Parameters $k$ and $\alpha$ control the shape and scale of the distribution, respectively. The expected value and the variance of $X$ are given by $E[X] = k\alpha$ and $\text{var}(X) = k\alpha^2$. This leads to the following method of moments parameter estimators:

$$\hat{k} = \frac{\bar{x}^2}{s^2} \quad \text{and} \quad \hat{\alpha} = \frac{s^2}{\bar{x}}$$

where $\bar{x}$ and $s^2$ are the sample mean and variance, respectively. The objective function for MLE can be obtained from Equation (A.1).

Let $X$ be a gamma random variable with parameters $k$ and $\alpha$. Then, the normalized gamma variable $Y := X/E[X]$ has unity expectation, and its distribution function can be described using only the variance: $\text{var}(Y) = \text{var}(X)/E[X] = 1/k = \alpha$.

## A.2 Basics of copulas

A comprehensive introduction to the copula theory may be found in Nelsen (1999); Joe (1997). The gist of the copula concept can be explained in a few paragraphs, though. Consider a joint cumulative distribution function (cdf) of a random vector. This comprises information about each single random variable and information about the interdependencies between the random variables. We can separate these two pieces of information in a formal way. That is, each joint cdf can be expressed in terms of its univariate marginal distributions and a unique function – called copula – that describes its dependence structure. The Sklar's theorem formalizes this idea (for proof see, for instance, Nelsen, 1999, p. 18).

**Theorem A.1 (*Sklar*)**

*Let $H$ be a continuous joint cdf of a random vector $(X_1, \ldots, X_N)$ with univariate marginal cdfs $F_{X_i}$ with $i = 1, \ldots, N$. There exists a unique copula function $C : [0,1]^N \mapsto [0,1]$ such as for all $x_i \in \mathbb{R} \cup \{-\infty, \infty\}$ with $i = 1, \ldots, N$ the following holds*

$$H(x_1, \ldots, x_N) = C\big(F_{X_1}(x_1), \ldots, F_{X_N}(x_N)\big). \tag{A.2}$$

*The converse is also true: given a copula $C : [0,1]^N \mapsto [0,1]$ and continuous cdfs $F_{X_i}$ with $i = 1, \ldots, N$ the function $H$ in Equation (A.2) is an $N$-dimensional cdf with univariate margins $F_{X_1}, \ldots, F_{X_N}$.*

A copula itself is a joint cdf on the $N$-dimensional unit hypercube. It has uniform margins that represent the cdfs of the probability-integral transforms of the original random variables given by $U := F_X(X)$:

$$C(u_1, \ldots, u_N) = H\big(F_{X_1}^{-1}(u_1), \ldots, F_{X_N}^{-1}(u_N)\big), \qquad u_1, \ldots, u_N \in [0,1].$$

A copula is invariant under strictly monotone transformations of the random variables. The same is true of copula-based dependence measures. We define two such (bivariate) measures: Kendall's rank correlation and coefficient of tail dependence.

**Definition A.1 (*Kendall's rank correlation*)**

*Let $(X_1, Y_1)$ and $(X_2, Y_2)$ be two iid random vectors. Then Kendall's rank correlation, denoted $\tau$, is given by*

$$\tau(X,Y) = Pr\{(X_1 - X_2)(Y_1 - Y_2) > 0\} - Pr\{(X_1 - X_2)(Y_1 - Y_2) < 0\}. \tag{A.3}$$

*The first term in Equation (A.3) is the probability of concordance of the two random vectors. The second term is the probability of discordance.*

The invariance of Kendall's $\tau$ to transformations of the marginals is an advantage over the Pearson's product-moment correlation. The Pearson's $\rho$ only describes the linear relationship between variables; it measures how well the random variables cluster around a linear function. Therefore, $\rho$ is a natural measure of association for multivariate normally and, more generally, elliptically distributed random variables. For other multivariate distributions, measures of concordance like $\tau$ are more appropriate because they are more general and reflect the degree to which random variables cluster around a monotone function (with linear functions being only one of the special cases).

**Definition A.2 (*Coefficients of tail dependence*)**

*Let $X_1$ and $X_2$ be random variables with cdfs $F_{X_1}$ and $F_{X_2}$. The coefficients of upper and lower tail dependence of $X_1$ and $X_2$, denoted $\lambda_U$ and $\lambda_L$, respectively, are*

$$\lambda_U = \lim_{u \nearrow 1} Pr\left\{X_2 > F_{X_2}^{-1}(u) \mid X_1 > F_{X_1}^{-1}(u)\right\} \tag{A.4}$$

$$\lambda_L = \lim_{u \searrow 0} Pr\left\{X_2 \leq F_{X_2}^{-1}(u) \mid X_1 \leq F_{X_1}^{-1}(u)\right\}, \tag{A.5}$$

*provided that limits $\lambda_U, \lambda_L \in [0,1]$ exist. If $\lambda_{(\cdot)} \geq 0$, $X_1$ and $X_2$ are said to be asymptotically dependent (in the corresponding tail). If $\lambda_{(\cdot)} = 0$, $X_1$ and $X_2$ are said to be asymptotically independent (in the corresponding tail).*

According to definition A.2, extreme realizations of two asymptotically tail-dependent random variables tend to occur simultaneously. By the same token, asymptotical independence in the tails means that, regardless of whether two random variables are linearly correlated or not, extreme realizations occur independently for each variable.

Often, using copulas to construct multivariate models with desirable dependence properties is more convenient than specifying cdfs. It is especially true if a copula, but not the corresponding cdf, can be expressed in the form of a simple formula. Copulas of one special class – Archimedean copulas – have particulary simple analytical representation in terms of what is known as a generating function, denoted $\phi$, and the general inverse of the generating function, denoted $\phi^{-1}$:

$$C(u_1, \ldots, u_N) = \phi^{-1}\big(\phi(u_1) + \cdots + \phi(u_N)\big). \tag{A.6}$$

To give rise to a proper Archimedean copula, a generating function must satisfy certain criteria, as described in Nelsen (1999, p. 90, p. 122) and Joe (1997, p. 373 f.).[57]

Archimedean copulas have a number of appealing features that make them popular in finance and risk management. Their simple analytical representation facilitates their use in high-dimensional portfolio models. These copulas are flexible in capturing various dependence structures, including tail dependence, which makes them suitable for modeling extreme events. For a number of families of Archimedean copulas, efficient sampling algorithms have been developed, too.

At the same time, the permutation symmetry, stemming from the exchangeability of the summands in Equation (A.6), constrains the practical applicability of Archimedean copulas. The permutation symmetry means that the dependence among all components is identical; all sub-copulas of the same dimension are equal. For those models, where the exchangeability of the marginals poses too strong a restriction, a more general and flexible class of *nested* Archimedean copulas should be considered.

Nested Archimedean copulas are often called *hierarchical Archimedean copulas*, or HACs. A HAC represents a hierarchical structure of sub-copulas, where different sub-copulas may be defined for separate groups of marginals. For instance, a copula with five

[57] The inverse of the generator function must be (at least) $N$-monotone on the interval $[0, \infty)$. This is why Laplace transforms of positive random variables, being completely monotone functions, are often applied to construct Archimedean copulas.

arguments and a structure of sub-copulas schematically represented as $(((1,2),3),(4,5))$ takes the following form:

$$C(u_1,\dots,u_5)=\phi_4^{-1}\Big(\phi_4\circ\phi_2^{-1}\Big(\phi_2\circ\phi_1^{-1}\big(\phi_1(u_1)+\phi_1(u_2)\big)+\phi_2(u_3)\Big)+\phi_4\circ\phi_3^{-1}\big(\phi_3(u_4)+\phi_3(u_5)\big)\Big). \tag{A.7}$$

This is a partially nested HAC with three sub-copulas: one sub-copula is a fully nested HAC that joins arguments 1, 2 and 3; other two sub-copulas are the regular two-dimensional Archimedean copulas that join 1 with 2 and 4 with 5, respectively.

A HAC can adopt arbitrarily elaborate structures. It may be either fully or partially nested. The generator functions within a HAC can come either from different generator families or from just a single generator family with different parameters for its sub-copulas. In this paper, we construct a HAC making use of the generator function of the Gumbel family of Archimedean copulas.

In what follows we consider, without loss of generality, the two-dimensional Gumbel copula. This copula exhibits asymmetric dependence structure by putting more probability mass on the upper tail. Its generating function is given by $\phi(u)=(-\ln(u))^{\theta}$ and the bivariate copula function reads

$$C_{\theta}^{Gu}(u_1,u_2)=\exp\Big\{-\big([-\ln(u_1)]^{\theta}+[-\ln(u_2)]^{\theta}\big)^{1/\theta}\Big\}. \tag{A.8}$$

Parameter $1\leq\theta<\infty$ controls the degree of dependence between two random variables. For $\theta=1$ the variables are independent; they become perfectly dependent as $\theta$ grows large. For any $\theta>1$, the random variables joined by a Gumbel copula are positively associated and asymptotically dependent in the upper tail. For a Gumbel copula, the population versions of Kendall's tau is given by $\tau^{Gu}=(\theta-1)/\theta$ and the coefficient of upper tail dependence is given by $\lambda_U^{Gu}=2-2^{1/\theta}$. The lower tail dependence coefficient is zero.

As to the copula estimation, the "HAC package" in R programming environment documented in Okhrin and Ristig (2012) provides a multi-stage MLE procedure, which simultaneously determines the parameter and the structure of a HAC. The algorithm endows the estimator with the usual asymptotic properties and is less computationally intensive than a regular one-step MLE. It can be summarized as follows: Consider $M$ random variables. For all possible pairs of these variables, parameters $\theta$ of bivariate Gumbel copulas are estimated using regular MLE. The pair with the strongest dependence (the largest $\theta$) is then combined by its estimated copula to a new pseudo variable – the first node in the copula hierarchy. The program reiterates the whole procedure until all $M-1$ parameters of bivariate copula nodes are estimated. To allow for more sophisticated structures, the variables of two successive nodes are aggregated if the difference between corresponding parameters $\theta$ is smaller than a fixed small threshold which we set to 0.5. This allows for marginal copulas with more than two arguments. The chosen value of 0.5 makes it possible to neglect small differences in the strength of dependence, preserving a reasonably granular copula structure. For further information on the methods and challenges of estimating Archimedean copulas in high dimensions, we refer the reader to Hofert, Mächler, and McNeil (2013).

## A.3 Details on approximation of EADs, LGDs and PDs

If a bank has more than one credit exposures to a particular borrower, the EAD equals the total exposure to this borrower. The on-balance sheet exposure is taken at its book value. By contrast, lenders convert their off-balance sheet exposure stemming from derivatives into credit equivalents using appropriate credit conversion factors. They report other off-balance sheet credit exposure such as "guarantees extended" at its book value.

To compute LGD, we take into account the following information about credit risk mitigation techniques: (i) collateral posted by the borrower and (ii) guarantees provided by other banks. For the secured (overcollateralized) part of the exposure we assign the LGD of 35%. For the remaining unsecured part we fix the LGD at 45% (see sections 287 and 295 in BCBS, 2006). As for the guarantees, we scale down the LGD for that part of the exposure for which a guarantee is available by the ratio of the guarantor's PD to the borrower's PD. Ideally, simulation of a double default, i.e. a possible default of the guarantor given the default of the borrower, should be performed. But this is difficult to implement because no estimates for the probability of joint default of the borrower-guarantor pairs are available. Thus, we only model the borrowers's default, but scale down the corresponding LGD to account for the risk mitigation technique (see sections 306-307 in BCBS, 2006). We assume that the expected loss from a direct credit exposure to the guarantor ($g$) should be equal to the expected loss from the secured exposure to the borrower ($b$): $LGD_g \cdot PD_g = LGD_b \cdot PD_b$. Then, we compute the adjusted LGD as $LGD_b = LGD_g \cdot PD_g/PD_b$, with $LGD_g = 45\%$ if $PD_g < PD_b$. If, on the contrary, $PD_g > PD_b$, we do not consider the guarantee at all (see section 301 in BCBS, 2006). If an exposure is backed by more than one guarantor, the median guarantors' PD is taken. For the credit exposures backed by both collateral and a guarantee, we take both into account subject to the LGD floor of 0%.

The PDs are taken as reported by the lenders. They originate from the lenders' internal rating systems. For a given rating system, the PDs of two borrowers only differ from each other if the borrowers belong to different rating categories. For all PDs we impose a floor of 0.03% in accordance with section 285 BCBS (2006) and a ceiling of 30%. A few reported PDs that exceed this ceiling are set to 100%, which means a sure default of the borrower. The same borrowers may be rated differently by different banks. Therefore, if multiple PDs are available for the same borrower, we take the maximum value. For borrowers for which no estimates of PDs are available at all, we take the median PD of all borrowers belonging to the same group (sector/country/region). As an exception, missing PDs for banks are set to the median PD for the banking sector across *all* countries.

## A.4 Tables and figures

Table A.1: Descriptive statistics for the 12 systemically important German banks

| Variable | Amount (€billion or %) |
|---|---|
| Number of banks | 12 |
| Total assets | 4,551 |
| Percentage of total assets of all German banks | 60% |
| Total credit exposure from the credit register | 2,309 |
| Total RWA | 1,053 |
| Total RWA: credit risk | 817 |
| Total RWA: credit register | 641 |
| Total capital | 200 |
| Total CET1 capital | 167 |
| Total CET1 to total RWA ratio | 15.86% |

*Note*: Data as of 2013Q4. Total assets relate to on-balance sheet items. Total credit exposure includes off-balance sheet items and is given as reported to the German credit register; intra-group exposure and exposure to central banks are not considered. Total exposure can, to some extent, be subject to double-counting due to guarantees which banks provide to each other. "Total RWA: credit register" shows the sum of RWA reported in the credit register for the credit exposure used in this analysis; intra-group exposure and exposure to central banks are not considered. CET1 capital refers to core capital for solvency purpose defined in accordance with the German Banking Act. *Source*: Deutsche Bundesbank (German credit register and supervisory reporting system), the authors' own calculations.

Table A.2: Total GIPS exposure of 12 systemically important German banks

| Countries | Total exposure (€billion) |
|---|---|
| Greece | 2.28 |
| Italy | 76.03 |
| Portugal | 10.38 |
| Spain | 57.96 |
| GIPS | 146.65 |
| All foreign countries | 1,271.52 |
| Banks' total portfolio | 2,309.22 |

*Note*: Total credit exposure to the respective country based on the data from the German credit register as of 2013Q4. Intra-group exposure and exposure to central banks are not considered. Total exposure can, to some extent, be subject to double-counting due to guarantees which banks provide to each other. The total groups' exposure in the last row includes domestic and foreign exposure but excludes exposure to international organizations. Deutsche Bundesbank (German credit register), the authors' own calculations.

Table A.3: Definition of macroeconomic variables

| Variables | Definition | Unit | Data sources |
|---|---|---|---|
| $\Delta\text{NPL}_{it}$ | The change in non-performing loan ratios | Percentage points | IMF, World Bank, National Central Banks (NCBs) |
| $\Delta\text{GDP}_{it}$ | The growth rate of real GDP, seasonally adjusted | Percentage | NiGEM |
| $\Delta\text{UN}_{it}$ | The change in the unemployment rate, seasonally adjusted | Percentage points | NiGEM |
| $\Delta\text{SR}_{it}$ | The change in short-term interest rates, average over the period | Percentage points | NiGEM |
| $\Delta\text{LR}_{it}$ | The change in long-term interest rates, average over the period | Percentage points | NiGEM |

Table A.4: Descriptive statistics for regression analysis

| | # | Mean | Median | St. dev. | 10th perc. | 90th perc. |
|---|---|---|---|---|---|---|
| Entire Sample | | | | | | |
| $\Delta\text{NPL}_{it}$ | 1008 | 0.155 | 0.017 | 0.695 | -0.225 | 0.719 |
| $\Delta\text{GDP}_{it}$ | 1008 | 0.345 | 0.462 | 1.444 | -1.078 | 1.660 |
| $\Delta\text{UN}_{it}$ | 1008 | 0.050 | 0.000 | 0.553 | -0.500 | 0.694 |
| $\Delta\text{SR}_{it}$ | 1008 | -0.080 | 0.000 | 0.678 | -0.708 | 0.372 |
| $\Delta\text{LR}_{it}$ | 1008 | -0.025 | -0.043 | 0.670 | -0.500 | 0.470 |
| GIPS Countries | | | | | | |
| $\Delta\text{NPL}_{it}$ | 144 | 0.405 | 0.303 | 0.675 | -0.203 | 1.050 |
| $\Delta\text{GDP}_{it}$ | 144 | -0.129 | -0.027 | 1.119 | -1.522 | 1.018 |
| $\Delta\text{UN}_{it}$ | 144 | 0.315 | 0.200 | 0.606 | -0.400 | 1.000 |
| $\Delta\text{SR}_{it}$ | 144 | -0.053 | 0.018 | 0.462 | -0.451 | 0.331 |
| $\Delta\text{LR}_{it}$ | 144 | 0.054 | 0.046 | 1.163 | -0.584 | 0.863 |

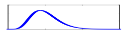
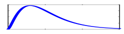
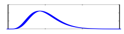
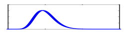
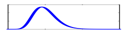
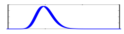
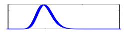
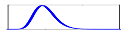
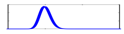
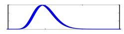
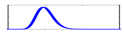
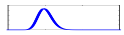
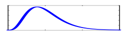
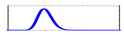
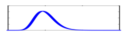
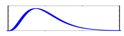
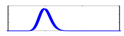
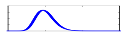
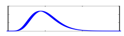
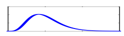
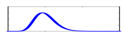
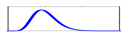
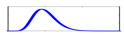
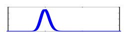
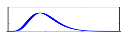
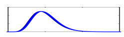
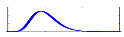
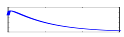
*Note*: The table provides the number of observations, mean, median, standard deviation, the 10th percentile and the 90th percentile for all variables for the entire sample and for the GIPS countries. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, the authors' own calculations.

Table A.5: Macroeconomic determinants of non-performing loans (NPLs)

| Two-step Blundell-Bond estimation: dependent variable $\Delta$NPL | | |
|---|---|---|
| | Coefficients | Standard errors |
| $\Delta \text{NPL}_{it-1}$ | 0.264*** | [0.061] |
| $\Delta \text{NPL}_{it-2}$ | 0.175** | [0.073] |
| $\Delta \text{GDP}_{it-1}$ | -0.059*** | [0.019] |
| $\Delta \text{GDP}_{it-2}$ | -0.055** | [0.023] |
| $\Delta \text{UN}_{it-1}$ | 0.252** | [0.097] |
| $\Delta \text{SR}_{it-1}$ | 0.090* | [0.049] |
| $\Delta \text{SR}_{it-2}$ | 0.035** | [0.013] |
| $\Delta \text{LR}_{it-3}$ | 0.135*** | [0.044] |
| Constant | 0.129*** | [0.032] |
| Long-run coefficients | | |
| $\Delta \text{GDP}_{it}$ | | -0.203 |
| $\Delta \text{UN}_{it}$ | | 0.449 |
| $\Delta \text{SR}_{it}$ | | 0.223 |
| $\Delta \text{LR}_{it}$ | | 0.241 |
| # of observations | | 1008 |
| # of countries | | 28 |
| AR(1), p-value | | 0.039 |
| AR(2), p-value | | 0.634 |
| Hansen, p-value | | 0.286 |

*Note*: The coefficients and standard errors in brackets are from the two-step Blundell-Bond GMM estimation with robust standard errors. ***, **, and * denote significance at the 1%, 5% and 10% levels, respectively. All variables are treated as endogenous. The number of instruments is chosen as close as possible to the number of groups. AR(1) and AR(2) are the Blundell-Bond tests for first- and second-order autocorrelation of the residuals. The Hansen test of overidentifying restrictions tests for the validity of the instruments. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, the authors' own calculations.

Table A.6: Default statistics: German sectors

| Code | Sector | Loan share | Gamma pdf |
|---|---|---|---|
| S1 | Agriculture, forestry, fishing & aquaculture | 0.18% |  |
| S2 | Electricity, gas & water supply, waste disposal, mining & quarrying | 1.54% |  |
| S3 | Manufacture of chemicals & chemical products, coke & refined petroleum products | 0.34% |  |
| S4 | Manufacture of rubber & plastic products | 0.19% |  |
| S5 | Manufacture of other non-metallic mineral products | 0.12% |  |
| S6 | Manufacture of basic metals & fabricated metal products | 0.48% |  |
| S7 | Manufacture, installation & maintenance of machinery, equipment & transport equipment | 1.26% |  |
| S8 | Manufacture & repair of computer, electronic & optical products | 0.46% |  |
| S9 | Manufacture of wood, wood products & furniture, pulp, paper & paper products, printing | 0.33% |  |
| S10 | Manufacture of textiles, wearing apparel & leather goods | 0.08% |  |
| S11 | Manufacture of food products & beverages, tobacco products | 0.29% |  |
| S12 | Wholesale & retail trade, repair of motor vehicles & motorcycles | 1.80% |  |
| S13 | Transportation & storage, post & telecommunications | 1.66% |  |
| S14 | Rental & leasing activities | 0.29% |  |
| S15 | Banks | 59.63% |  |
| S16 | Insurance companies | 0.38% |  |
| S17 | Other financial intermediation (including MMF) | 15.05% |  |
| S18 | Equity investment companies | 2.88% |  |
| S19 | Construction | 0.90% |  |
| S20 | Housing companies | 3.18% |  |
| S21 | Other real estate activities | 3.88% |  |
| S22 | Accommodation & food service activities | 0.16% |  |
| S23 | Information & communication, research & development, membership organizations, publishing activities & other | 1.36% |  |
| S24 | Health & social work activities | 0.68% |  |
| S25 | Other service activities | 0.31% |  |
| S26 | Households | 0.94% |  |
| S27 | Non-profit organizations | 0.20% |  |
| S28 | General government | 1.48% |  |

*Note*: Sectoral breakdown of lending to German borrowers; 12 systemically important German banks. Loan share describes the mean share of the respective credit exposure in the portfolio in the period from 2008Q1 to 2013Q4. The fitted normalized gamma probability distribution functions (pdf) of the corresponding systemic factors are shown on the interval $(0, 3]$. *Source*: Deutsche Bundesbank (German credit register), the authors' own calculations.

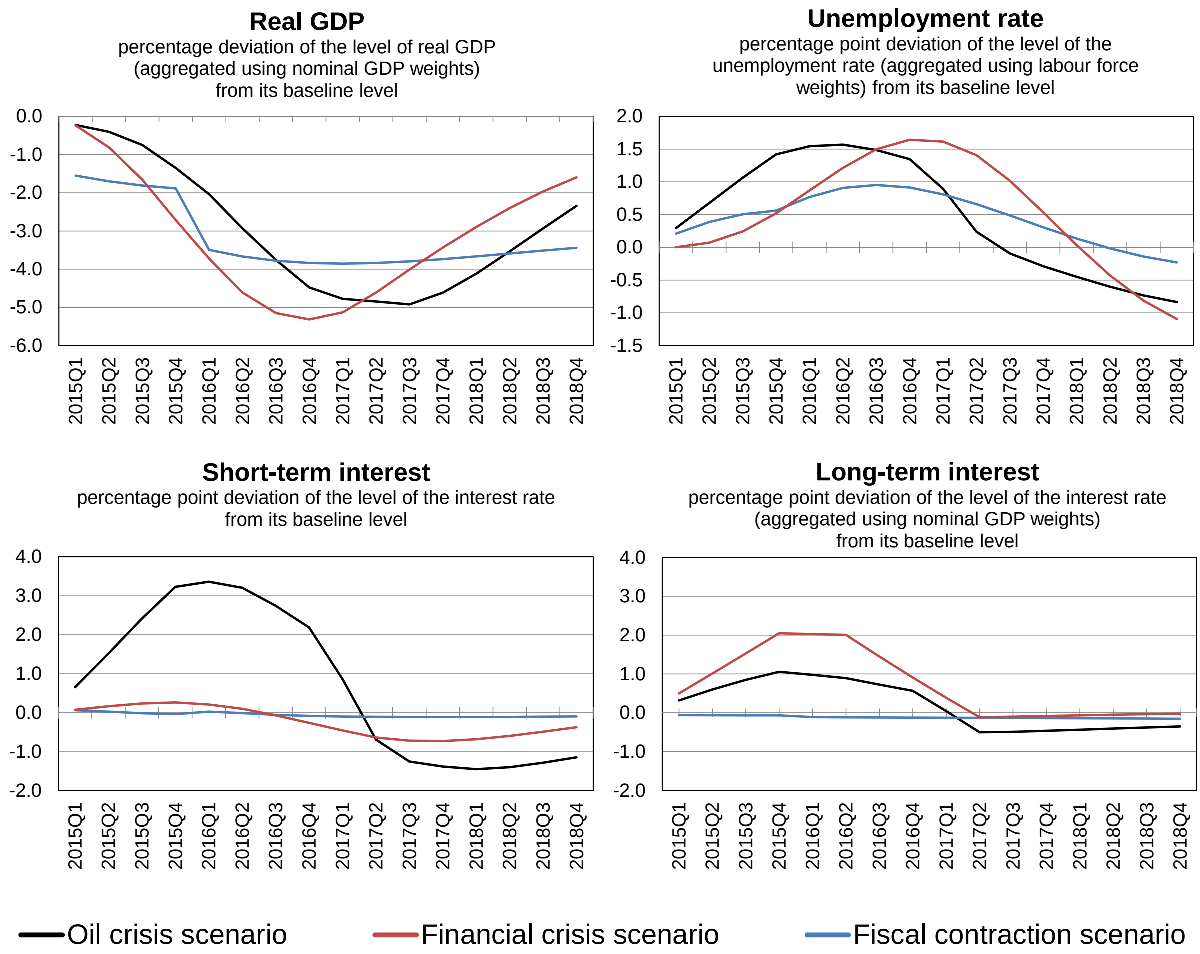
Table A.7: Default statistics: foreign countries and regions

| Code | Country | Loan share | Gamma pdf | Code | Country | Loan share | Gamma pdf |
|---|---|---|---|---|---|---|---|
| AU | Australia | 0.56% | | JP | Japan | 0.74% | |
| AT | Austria | 1.31% | | LV | Latvia | 0.03% | |
| BE | Belgium | 0.51% | | LT | Lithuania | 0.02% | |
| BG | Bulgaria | 0.03% | | LU | Luxembourg | 4.60% | |
| CA | Canada | 0.65% | | MT | Malta | 0.03% | |
| HR | Croatia | 0.70% | | NL | Netherlands | 2.49% | |
| CY | Cyprus | 0.11% | | NO | Norway | 0.32% | |
| CZ | Czechia | 0.14% | | PL | Poland | 0.60% | |
| DK | Denmark | 0.40% | | PT | Portugal | 0.44% | |
| EE | Estonia | 0.01% | | RO | Romania | 0.05% | |
| FI | Finland | 0.24% | | SK | Slovakia | 0.05% | |
| FR | France | 3.00% | | Sl | Slovenia | 0.08% | |
| GR | Greece | 0.36% | | ES | Spain | 2.22% | |
| HU | Hungary | 0.29% | | SE | Sweden | 0.51% | |
| IE | Ireland | 2.76% | | UK | UK | 5.28% | |
| IT | Italy | 2.57% | | US | USA | 10.83% | |

| Code | Region | Loan share | Gamma pdf |
|---|---|---|---|
| 1031 | Other European countries (Iceland, Liechtenstein, Switzerland, Turkey, Ukraine, Russia) | 2.57% | |
| 1032 | Remaining European countries (Andorra, Gibraltar, San Marino, Vatican, Belarussia, Moldova, etc.) | 0.08% | |
| 1033 | Main countries in Africa | 0.25% | |
| 1034 | Remaining countries in Africa | 0.06% | |
| 1035 | Main countries in the Americas | 1.83% | |
| 1036 | Remaining countries in the Americas | 0.05% | |
| 1037 | Main countries in Asia | 1.94% | |
| 1038 | Remaining countries in Asia | 0.03% | |
| 1039 | Countries in Oceania | 0.18% | |

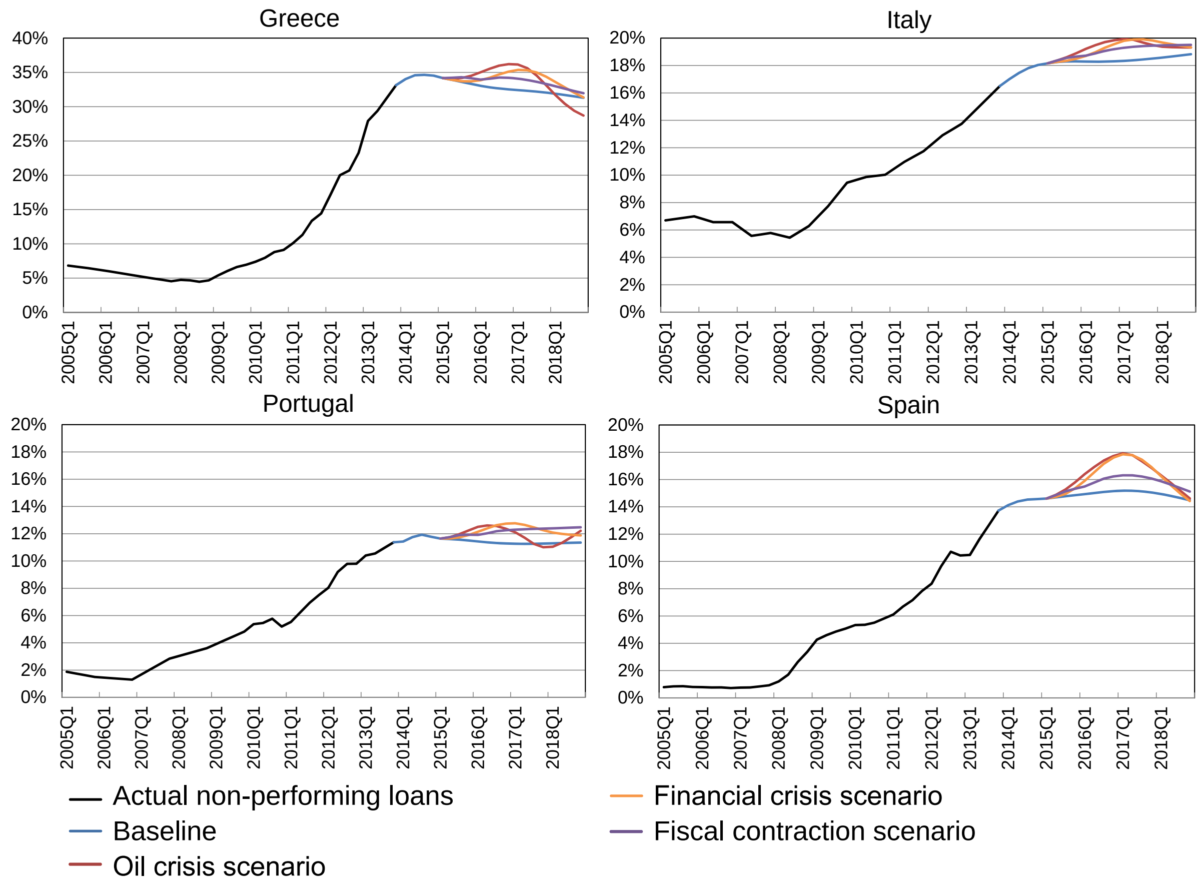
*Note*: Country breakdown of lending to non-German borrowers and reginal breakdown for loans to borrowers in the countries that are not considered separately; 12 systemically important German banks. Loan share describes the mean share of the respective credit exposure in the portfolio in the period from 2008Q1 to 2013Q4. The fitted normalized gamma probability distribution functions (pdf) of the corresponding systemic factors are shown on the interval $(0, 3]$. *Source*: Deutsche Bundesbank (German credit register), the authors' own calculations.

Figure A.1: Macroeconomic variables for the GIPS countries in stress scenarios



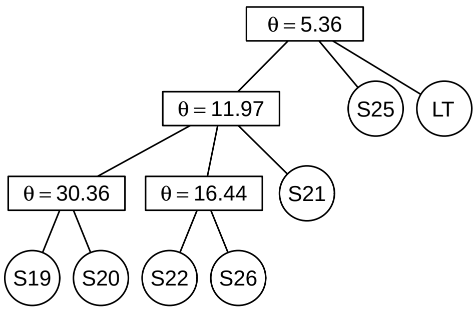

*Source*: National Institute's Global Econometric Model (NiGEM), the authors' own calculations.

Figure A.2: Projections for non-performing loans in the GIPS countries

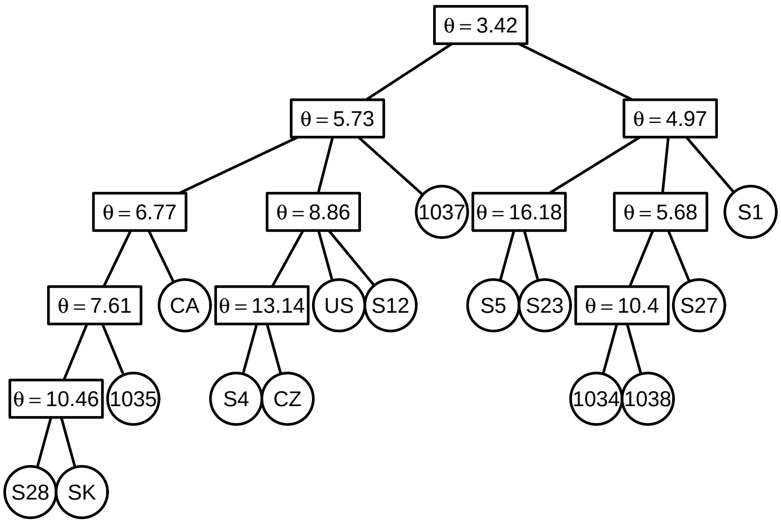



*Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, the authors' own calculations.

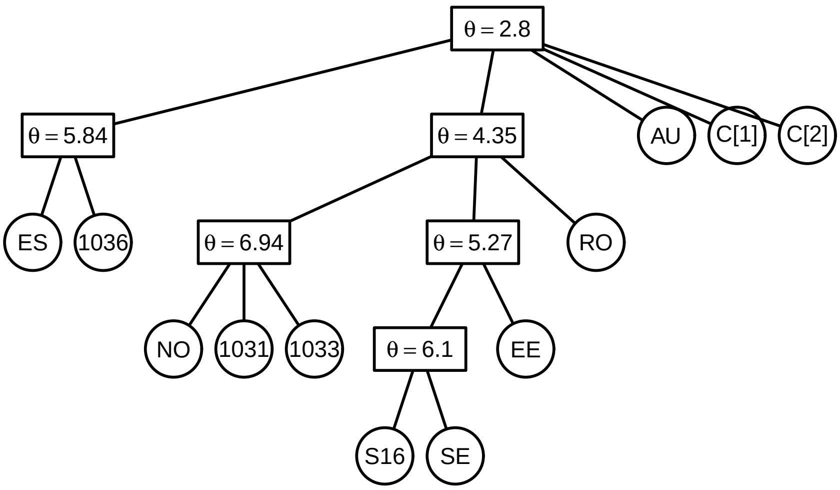
Figure A.3: Estimated HAC – lower levels, part 1




(a) C[1]



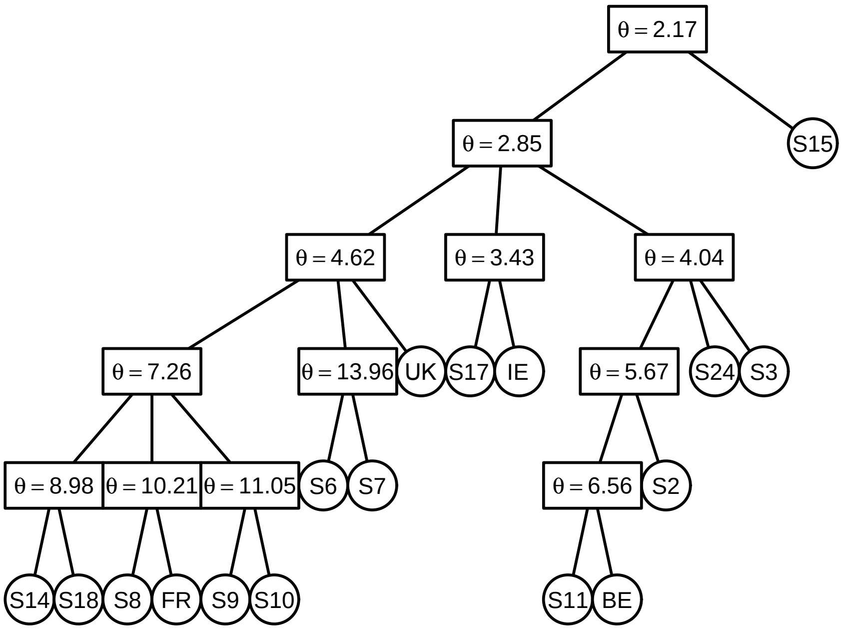

(b) C[2]



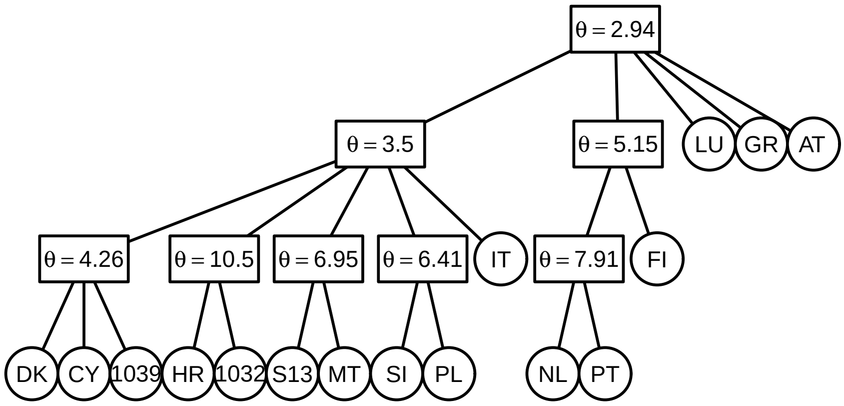

(c) C[3]

*Note*: For systematic factors associated with German sectors, other countries and remaining geographic regions, the figures show how the single arguments represented in circles are joined by sub-copulas that are denoted by rectangles which contain the relevant estimates for $\theta$. *Source*: Deutsche Bundesbank (German credit register), the authors' own calculations.

Figure A.4: Estimated HAC – lower levels, part 2



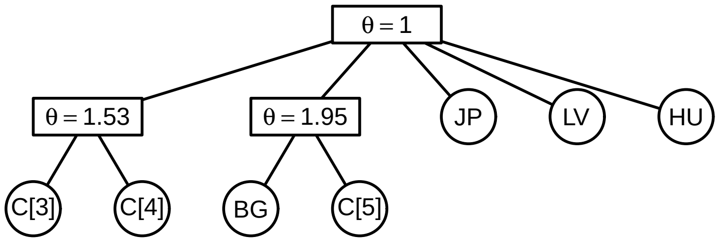

(a) C[4]




(b) C[5]

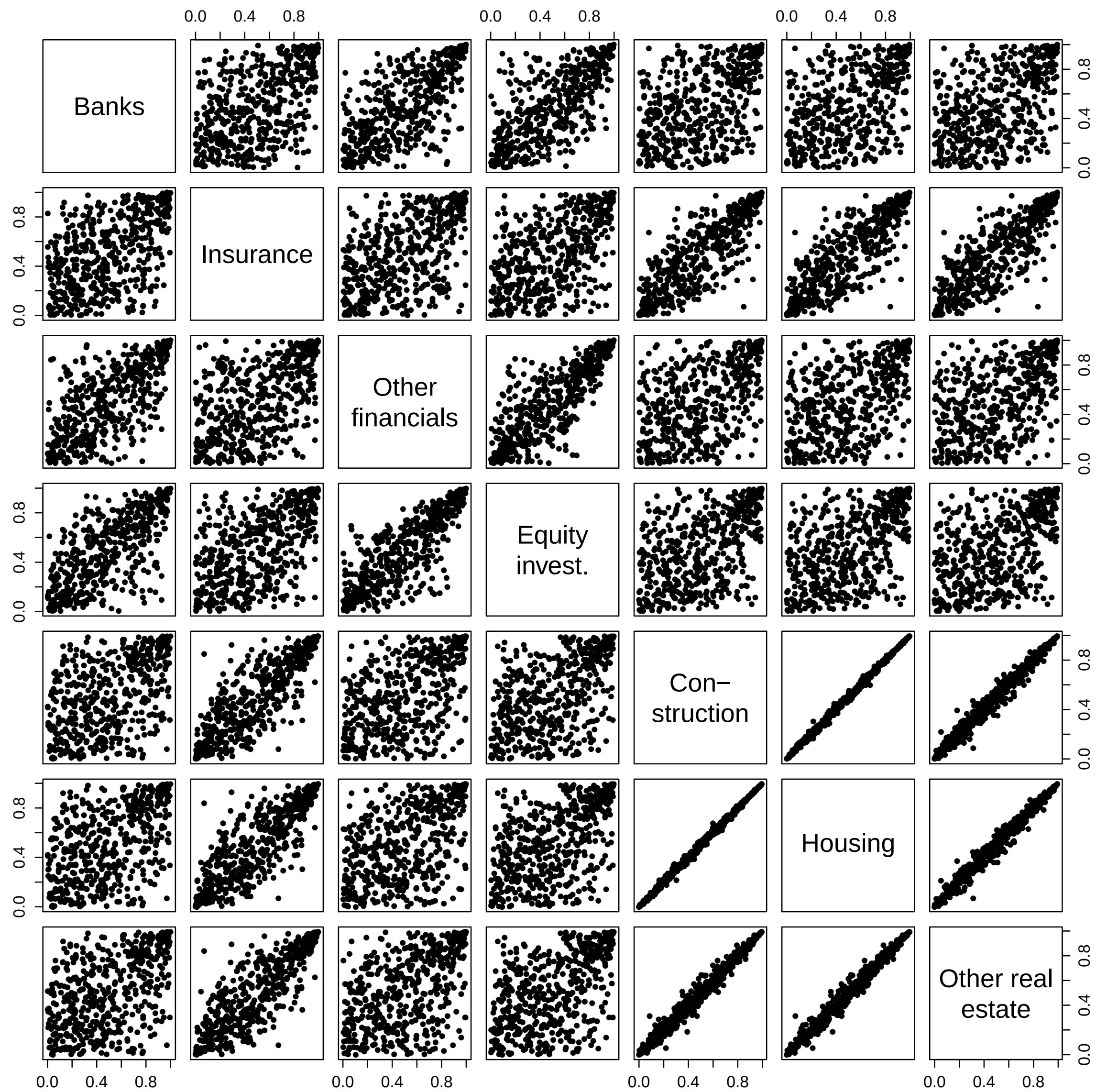
*Note*: For systematic factors associated with German sectors, other countries and remaining geographic regions, the figures show how the single arguments represented in circles are joined by sub-copulas that are denoted by rectangles which contain the relevant estimates for $\theta$. *Source*: Deutsche Bundesbank (German credit register), the authors' own calculations.

Figure A.5: Estimated HAC – top level



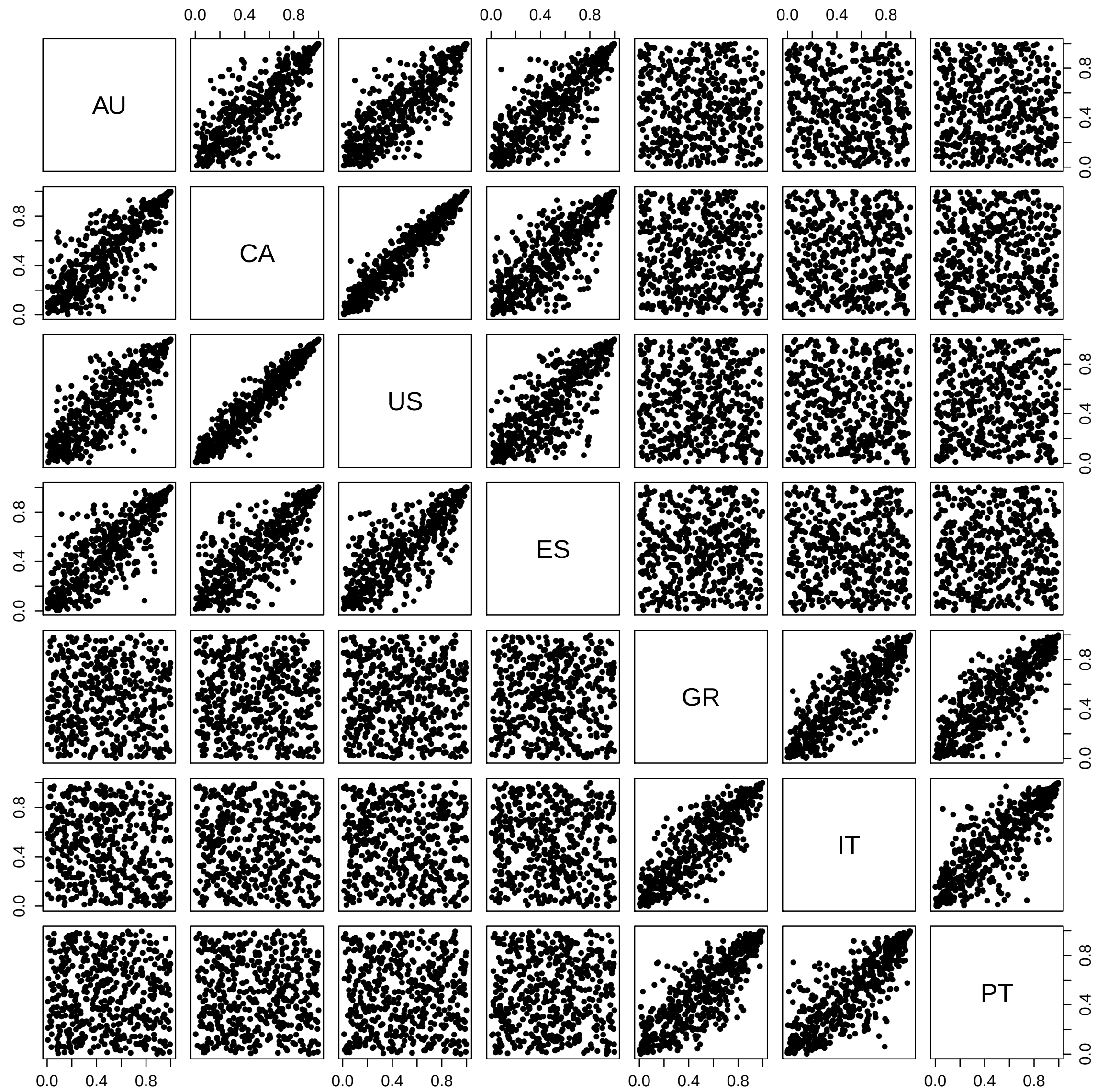

*Note*: For systematic factors associated with German sectors, other countries and remaining geographic regions, the figures show how the single arguments represented in circles are joined by sub-copulas that are denoted by rectangles which contain the relevant estimates for $\theta$. *Source*: Deutsche Bundesbank (German credit register), the authors' own calculations.

Figure A.6: Pairwise realizations from the copula of systematic factors for selected sectors in Germany



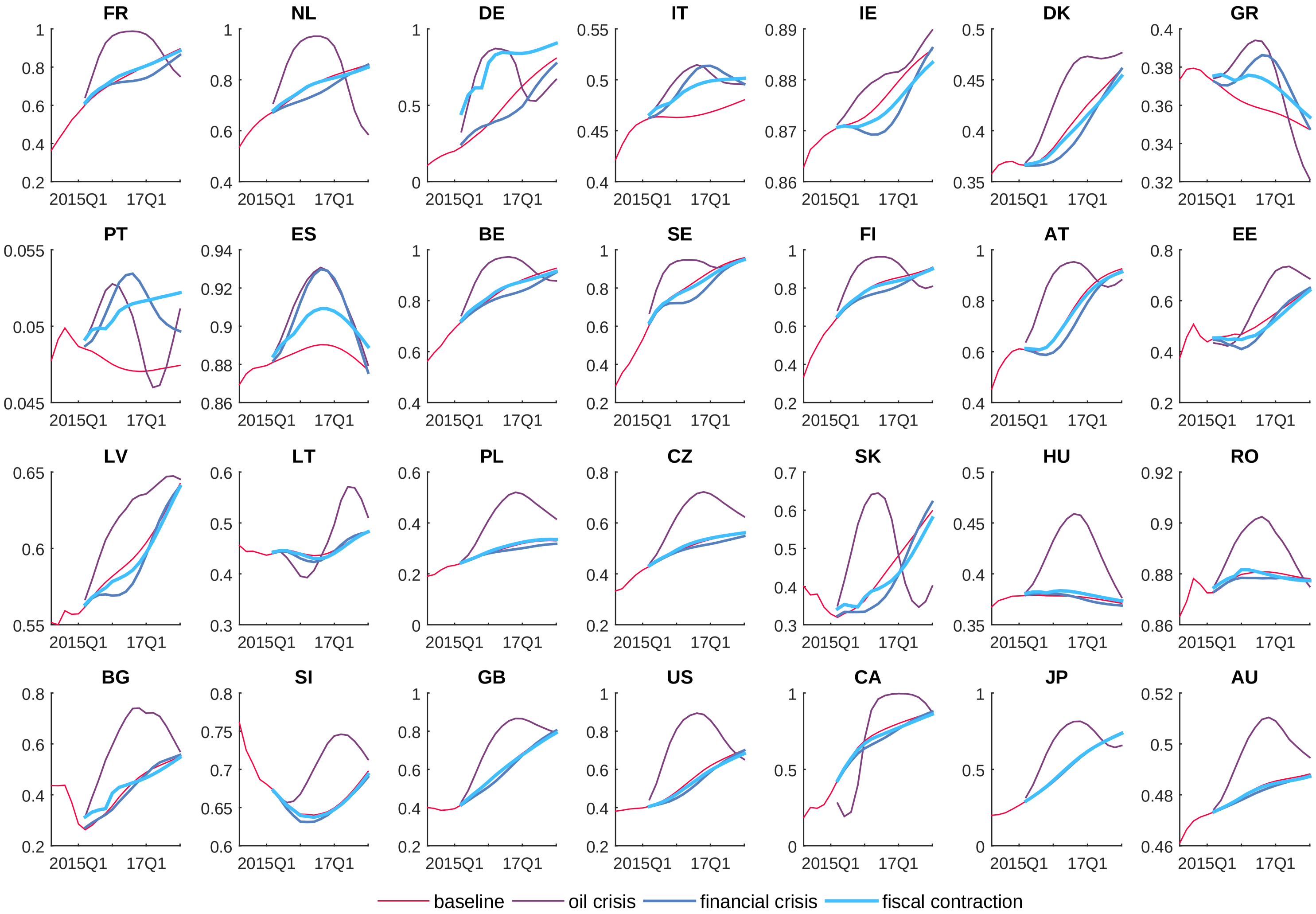

*Source*: Deutsche Bundesbank (German credit register), the authors' own calculations and stochastic simulations.

Figure A.7: Pairwise realizations from the copula of systematic factors for selected countries



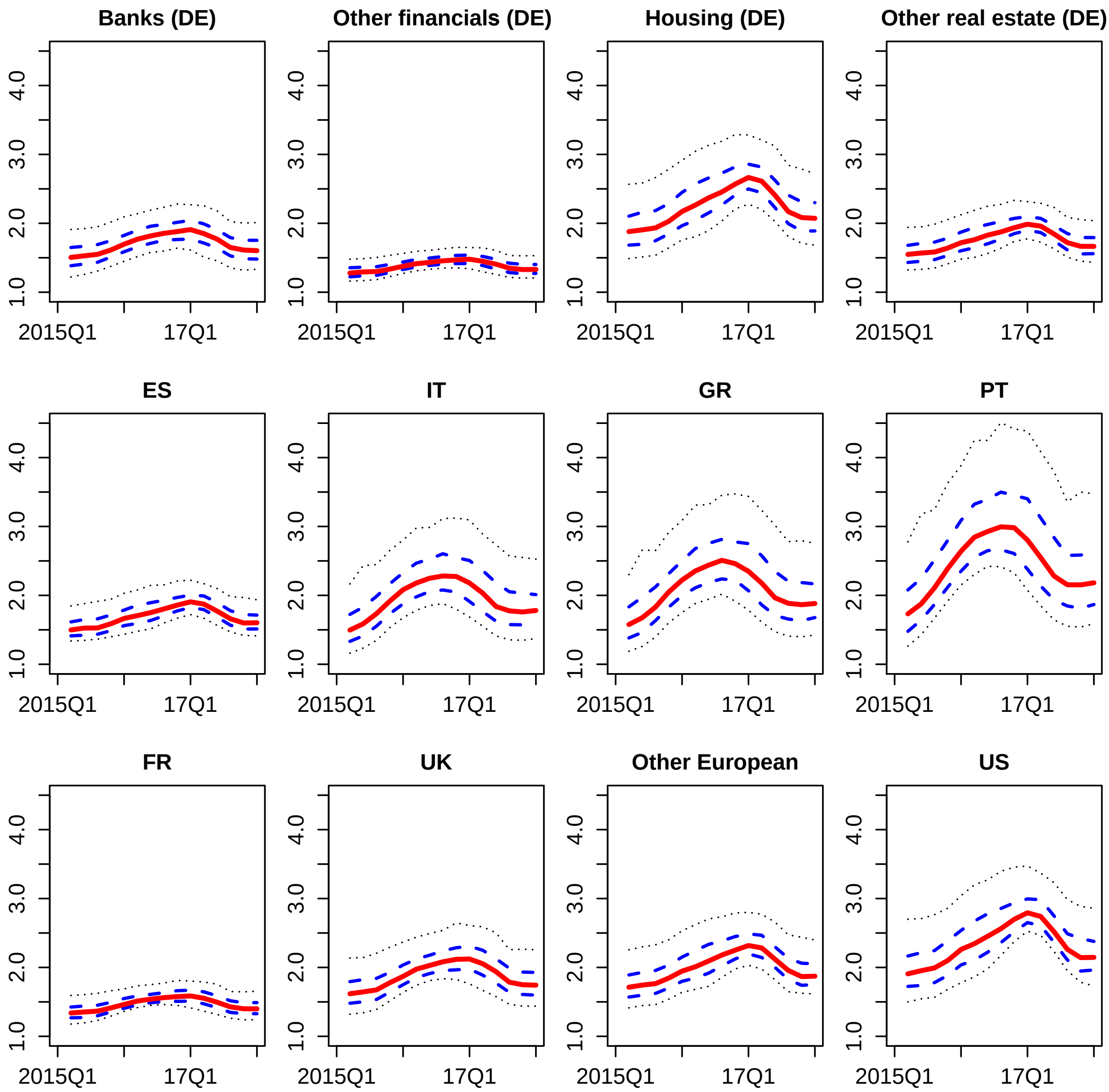

*Source*: Deutsche Bundesbank (German credit register), the authors' own calculations and stochastic simulations.

Figure A.8: NPL ratio percentiles based on fitted gamma distributions: realizations in different scenarios



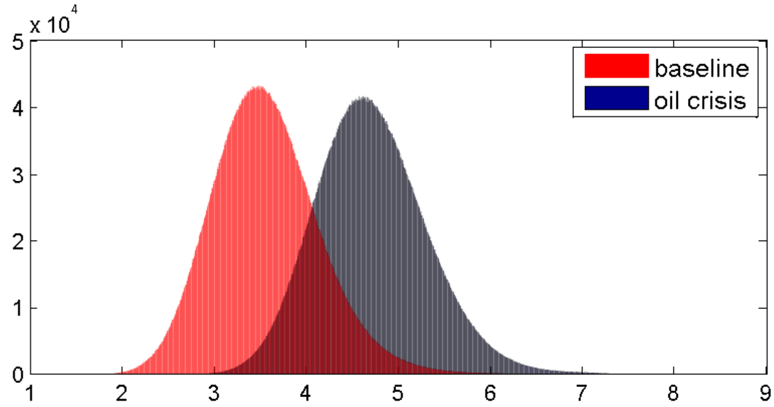

*Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, the authors' own calculations.

Figure A.9: Magnitude of simulated systematic factors: selected examples under the oil crisis scenario

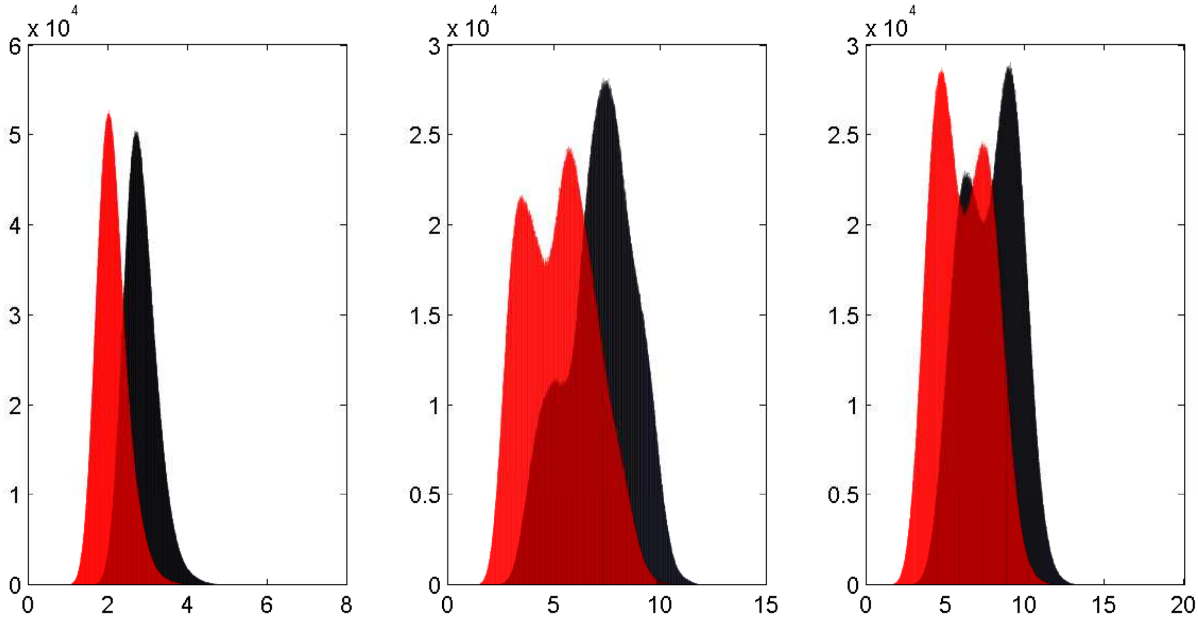



*Note*: Solid red lines show median realizations for each quarter; dashed blue lines show first and third quartiles; black dotted lines show 5% and 95% percentiles. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, the authors' own calculations.

Figure A.10: Frequency distributions of simulated portfolio loss



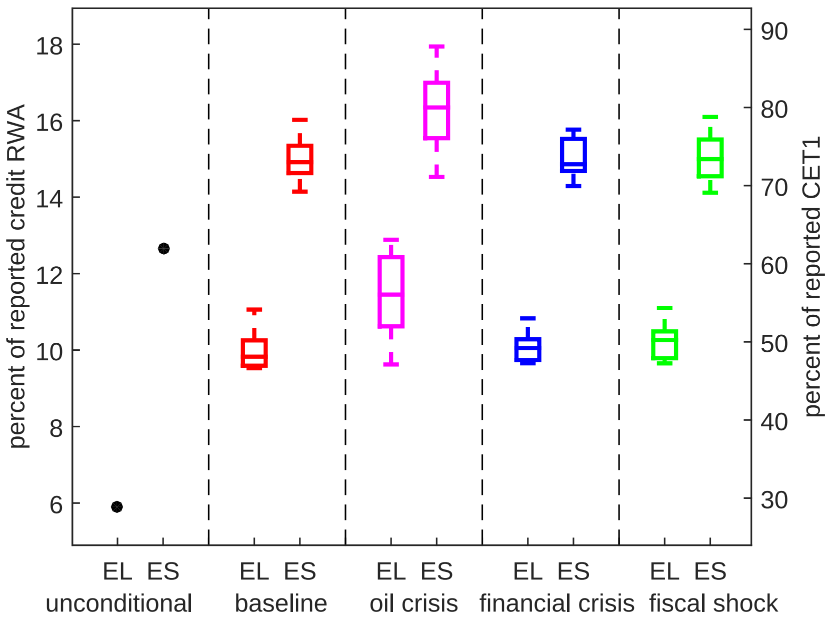

(a) System




(b) Three selected banks

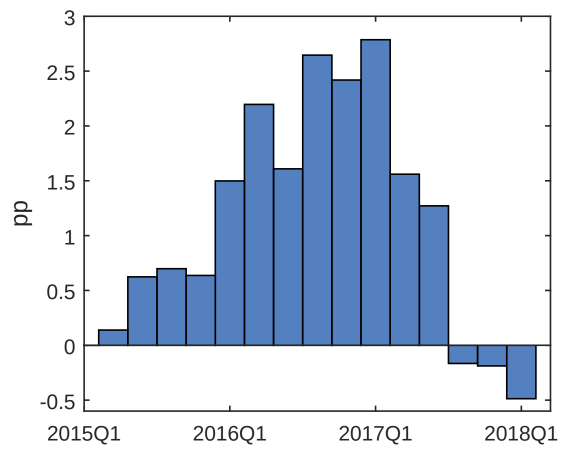
*Note*: On the x-axis: simulated portfolio loss as a percentage of portfolio exposure. The baseline example is given for 2015Q1. The example under the oil crisis scenario is given for 2017Q1, when the systems' loss reaches its maximum for this stress scenario. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, Deutsche Bundesbank (German credit register), the authors' own calculations and stochastic simulations.

Figure A.11: Expected loss and expected shortfall at the system's level




*Note*: In the unconditional simulation, expected loss (EL) and expected shortfall (ES) for the system's credit portfolio (the aggregated portfolio of the 12 banks under consideration) are represented by black dots. For the simulations based on NPL ratio scenarios, the box plots show the distribution of the portfolio EL and ES over the time period for which the scenarios are available. *Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, Deutsche Bundesbank (German credit register and supervisory reporting system), the authors' own calculations and stochastic simulations.

Figure A.12: Difference between the system' expected shortfall (as a percentage of total credit RWA) under the oil crisis and baseline scenarios




*Source*: National Institute's Global Econometric Model (NiGEM), International Monetary Fund, World Bank, National Central Banks, Deutsche Bundesbank (German credit register and supervisory reporting system), the authors' own calculations and stochastic simulations.